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Mathematical Critical Thinking: Analysis of Middle School Students' Thinking Processes in Solving Trigonometry Problems

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Abstrak

Kemampuan berpikir kritis siswa dalam matematika saat ini masih lemah. Penelitian ini bertujuan untuk mengidentifikasi karakteristik kemampuan berpikir kritis matematis siswa dalam menyelesaikan masalah trigonometri. Metode penelitian yang digunakan adalah studi kualitatif dengan desain fenomenologi, melibatkan peneliti sebagai instrumen utama. Penelitian ini menggunakan tes (masalah trigonometri), pedoman wawancara, dan observasi sebagai instrumen non-tes. Subjek penelitian terdiri dari tiga siswa kelas X yang dipilih dengan teknik purposive sampling. Uji keabsahan dilakukan melalui triangulasi metode dan teori, serta analisis data menggunakan teknik induktif. Hasil penelitian menunjukkan bahwa siswa dengan kemampuan berpikir kritis cenderung lebih efektif dan cepat dalam menyelesaikan masalah trigonometri. Mereka memiliki pendekatan unik dalam memecahkan masalah dan fokus pada pencarian jawaban. Pemikiran kritis matematis juga mempengaruhi kemampuan siswa dalam memahami dan menghubungkan informasi yang ada dalam soal. Selain itu, pemikiran kritis matematis berdampak pada pemikiran logis dan daya ingat yang baik, sehingga siswa lebih mudah dalam menentukan formula pemecahan masalah.
Kata Kunci: berpikir kritis matematis; pendidikan matematika; masalah trigonometri.

Abstract

Students' critical thinking skills in mathematics are currently still weak. This study aims to identify the characteristics of students' mathematical critical thinking abilities in solving trigonometry problems. The research method used is a qualitative study with a phenomenological design involving researchers as the main instrument. This study uses tests (trigonometry problems), interview guidelines, and observation as non-test instruments. The research subjects consisted of three class X students selected by purposive sampling technique. The validity test was carried out through method and theory triangulation and data analysis using inductive techniques. The results showed that students with critical thinking skills tended to be more effective and faster in solving trigonometry problems. They have a unique approach to solving problems and focus on finding answers. Mathematical critical thinking also affects students' ability to understand and relate the information contained in the problem. In addition, mathematical critical thinking impacts logical thinking and good memory, so students find it easier to determine problem-solving formulas.
Keywords: mathematical critical thinking; mathematics education; trigonometry problems.

I. INTRODUCTION

Mathematics plays a huge part in our everyday life. One of them is to think logically and systematically (Yayuk et al., 2020). In the educational system, mathematics is one of the key aspects of the educational curriculum in Indonesia (Efendi & Hsi, 2020). Because of that. Sole reason mathematical education is important in helping students understand and master concepts, skills, and mathematical techniques needed to solve everyday problems (Muhaimin et al., 2023). One of the strategies to actualize that part is to train systematic mathematical problems in the process of learning.

In mathematical problem solving, it is important to explore many methods and strategies that can be applied to solve problems. It can help solve problems and solidify the understanding of mathematical concepts regarding that problem (Schoenfeld, 2016). Another note to take, it is also important to utilize a wide range of problem-solving questions to train this skill (Schoenfeld, 2014). By doing this, students can build up the mathematical skills needed in everyday life and even the professional field.

The importance of mathematical problem-solving has made a trend in research. Much research highlighted and explained the process of students' problem-solving skills, and the result shows that their mathematics problem-solving skill is still low (Hasibuan et al., 2019; Muhaimin et al., 2023). Students assume that the struggle to comprehend mathematical concepts is the biggest factor in this problem (Simamora et al., 2017). The complexity of this mathematical material is none other than how abstract and hard the material itself is. One of the materials which is quite abstract in mathematics is trigonometry.

Studi Kusuma et al. (2019) discovered that most students had difficulty comprehending the concept and solving mathematical problems, especially in trigonometry. Putri et al. (2020) also stated that trigonometry is a complex mathematical content and needs logical and critical thinking skills. Aminudin et al. (2019) explained that one of the reasons behind the students' problem-solving trigonometry is their lack of critical mathematical thinking. Therefore, mathematical problems in trigonometry can be applied to enhance their critical thinking skill. Improving critical mathematical thinking is key to helping students resolve problems and understand mathematical concepts. Sachdeva & Eggen (2021) stated that critical mathematical thinking skills are the ability to comprehend, analyze, and evaluate mathematical situations logically and rationally. It involves the ability to identify and evaluate proof that has been given. So, the indicator of critical mathematical thinking skills are identifying, associating, analyzing, and solving mathematical problems (Palinussa, 2013)

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More attention has been given to developing critical mathematical thinking skills in mathematics education in the last few years. The reason for this is the students' poor critical mathematical thinking skills. Sari & Caswita (2020) research claimed that middle school students' critical mathematical thinking skills are still weak because they are unfamiliar with solving geometrical problems. Salido & Dasari (2019) also stated that the weak critical mathematical thinking skill is the lack of intensity in HOTS (High Order Thinking Skills) questions.

Researchers applied geometry problems and HOTS to do the research that had been done above, which was to discover students' critical mathematical thinking. To widen the field of research, researchers used trigonometry problems to analyze students' critical mathematical thinking. The purpose of this research is to identify the students' thinking process and understand the characteristics of critical mathematical thinking skills in completing trigonometry problems. This research is important because it gives literature contributions regarding critical mathematical thinking by identifying a specific critical thinking process in completing trigonometry problems at the educational level of middle school. Since this literature may give ideas and strategies at educational levels to improve critical mathematical thinking skills, it may also be used as a foundation to develop more effective teaching materials and teaching strategies for enhancing middle school students' critical mathematical thinking skills.

II. METHOD

This is qualitative research with a phenomenology approach. According to Creswell (2015), Phenomenology aims to discover the phenomenon or what is happening inside an individual by their experiences. This condition discovers students' thinking processes to identify critical mathematical thinking skills' characteristics to solve trigonometry problems. The research subjects used class X students from a high school in Surakarta. The subject selection used a purposive sampling technique, and three students were selected. The researcher is the key instrument of this research, alongside other instruments such as trigonometry, observation sheets, and interview guidelines. Trigonometry problems are used to find students' critical thinking skills by how they solve the problems, then the observation sheets are used to observe their movement and gestures when solving the problems, and interview guidelines are used to show and confirm students' critical mathematical thinking skills by their problem-solving. Material experts and linguists have validated the instruments of this research. Data collection in this research covered tests, interviews, and observations.

The data validity test in this research uses the triangulation method and theory. Creswell (2015) stated that the triangulation method is applied by comparing data from the same source with different techniques in this research. Some of them are tests, interviews, and observations. Subsequently, triangulation theory is applied by comparing the findings obtained with existing theories (Creswell, 2015). The inductive technique is used to analyze the data in this research, beginning with data collection from various techniques, sorted in the data reduction afterward. After the reduction, the data is provided with descriptions, and eventually, it concludes.

III. RESULT AND DISCUSSION

The researcher used identifying indicators (comprehending problems known and questioned skill), correlating (understanding the relationships among the obtained data), and solving mathematical problems (computing, formulating the right answer, and drawing conclusions) (Palinussa, 2013). Applying trigonometry problems which consist of 3 questions to recognize students' critical mathematical thinking (Figure 1).

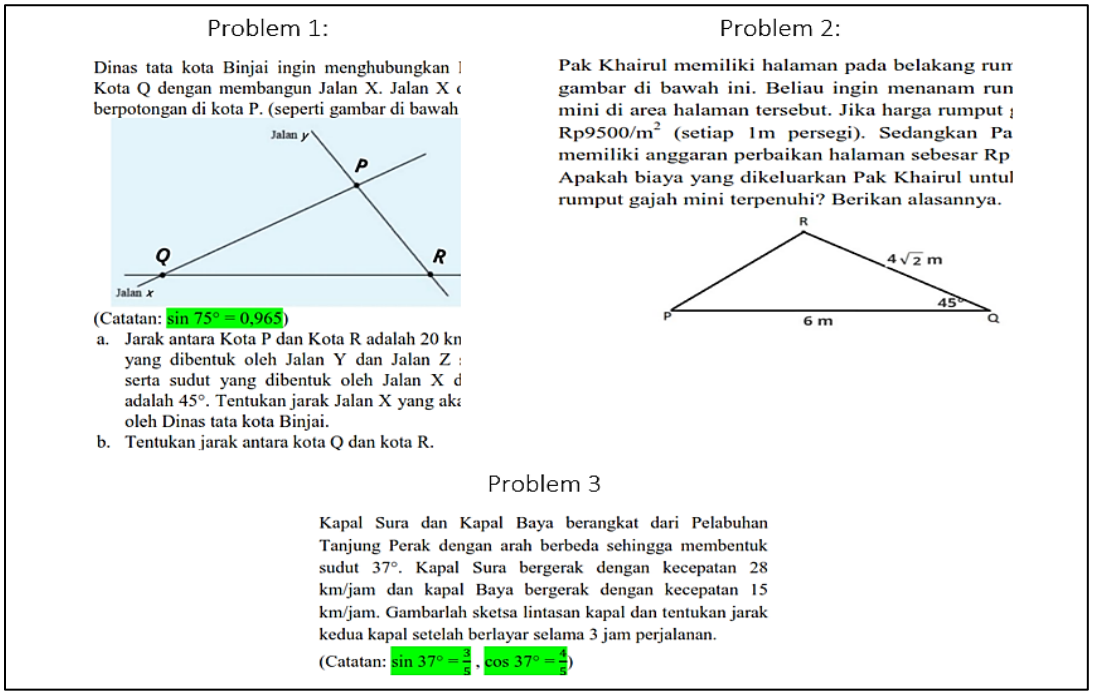

Figure 1. Instrument test on trigonometry

In the first question, students were given a road map and a question concerning calculating the distance. This problem can be solved with the law of sines. Then, in the second question, they were faced with a question to calculate the cost of grass planting, provided with the cost of grass as per m^2, this question

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can be solved by calculating the width of the map. Finally, in the last question, they were asked to calculate the distance between 2 ships going in different directions and velocity after 3 hours. To solve this problem, they have to understand the law of sines and cosines and be able to synthesize the calculation of distance, velocity, and time. All three questions above require the ability to comprehend, analyze, and evaluate logically to evaluate the students' critical mathematical thinking skills.

Identifying

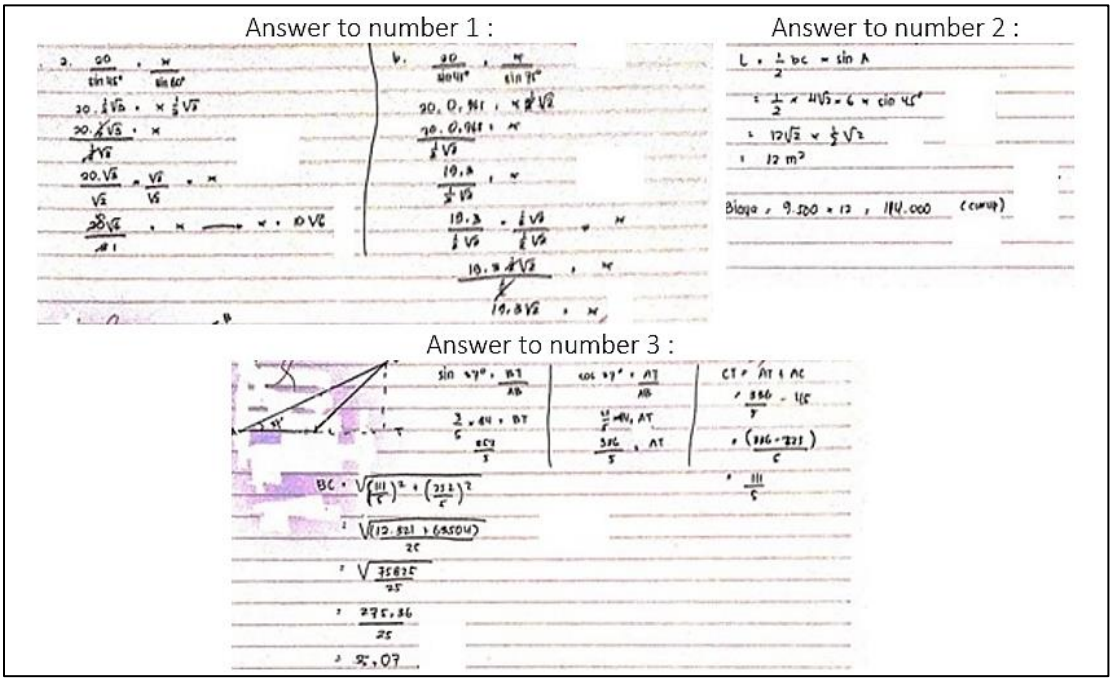

Figure 2. Answer sheet of subject 1

In the Figure 2, subject one did not seem to write the known and questioned data in the answer sheet. Subject one directly calculated the question to get the answer. This condition happened to all the questions given to subject 1, and all of them did not clearly identify the problems. Therefore, the researcher interviewed to identify the cause of this condition. Subject 1 stated that it is done with full awareness, and it is habitual behavior for subject 1 to solve the problem immediately after understanding the question. A direct and verbal question was given to subject one to evaluate the student's understanding of the problem mentioned before. Subject 1's answer was "First question, known information are the distance between city P and Q, which is 20 km, 60 degrees that were formed between the road Y and Z, and 45 degrees formed between road X and Z, the information questioned is the distance of road X, and the distance between city Q and R". Subject 1 continued "This problem used an information of scalene triangle map with 2

of the section's sides and one of the degrees already known, and calculate the amount of money needed to buy the grasses that are required to place onto the triangle." In the third question, subject 1 explained, "Information obtained are the degrees formed between 2 ships, the velocity of those ships, and traveling time of those two ships. Next, the question is the distance between 2 ships after 3 hours" Based on the interview, subject one can identify the problems, despite the absence of problem identification on the sheets (See Figure 3).

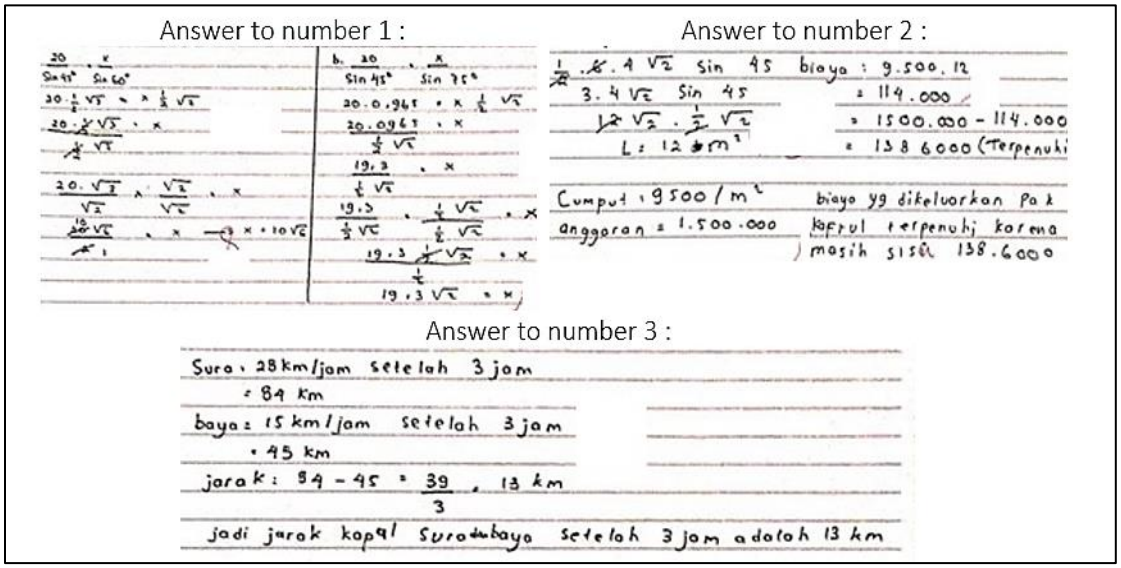

Figure 3. Answer sheet of subject 2

The same occurrence was done by subject two in problem identification. This subject did not write the known and questioned data/information on the answer sheet. However, subject 2 managed to explain theInformationn about the knInformation and question the verbal answer. Subject 2 explained, "The known information is two angles and one road distance, so the law of sines is used to find the other distance with comparison" In the second question, subject 2 said, "The known information is the length of 2 sides and one of the angle and the cost of grass per meter, and the question is how much money needed to buy the grasses". Onto the third question, subject 2 added, "2 ships traveling from 2 different directions with different velocities and angles, the question is the distance between those ships after 3 hours". Those questions indicated that subject two could comprehend the problem even though they were not written on the answer sheet. Regardless of that, subject 2 managed to do distance calculation for all of the questions (Figure 3).

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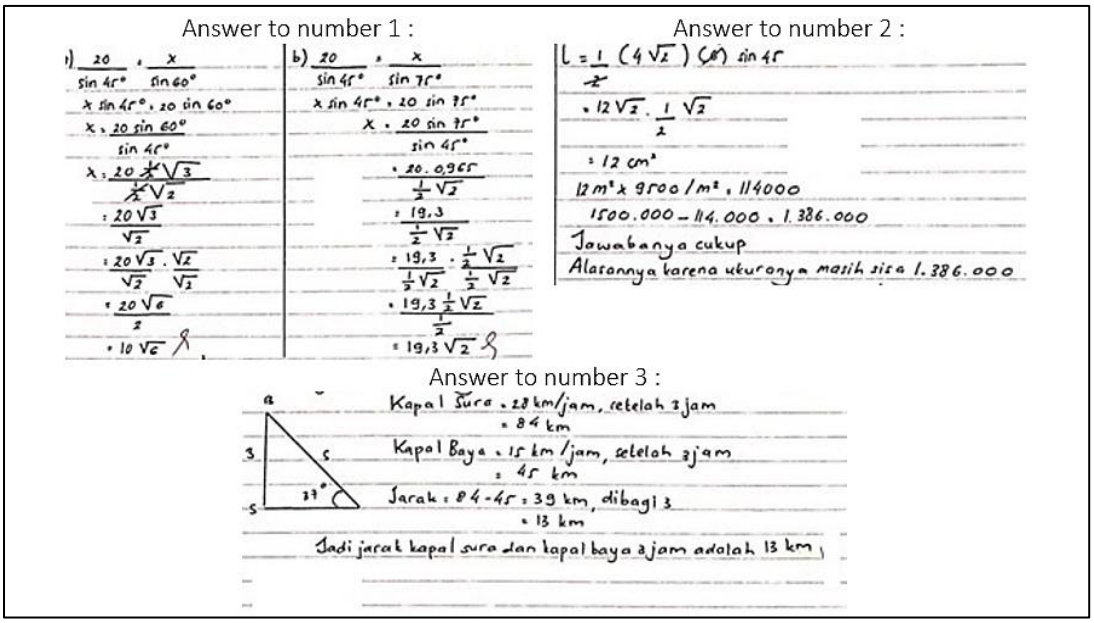

Figure 4. Answer sheet of subject 3

Based on the Figure 4, the same occurrence as the other subjects is also found relating to identifying. Starting from the first and second questions, problem identification could not be found. Nevertheless, in the third question, subject 3 attempted to reconstruct the image by making a triangle. This happened because the fast understanding of subject 3 made it possible for this subject to write the calculation to find the distance, and only brInformation was written to support the calculation. The interview was conducted to evaluate the level of understanding regarding the questions. Subject 3 explained, "Information that is provided in the first question is the distance between city P and R, an angle formed between road Y and Z, the angle formed by road X and, and that information can be used to find the distance of road X, and the distance between city Q to R", later on, subject three added, "In the second question, known information is the image and length of 2 triangle sides, one angle and the cost of grass per meter, and the problem questioned is the cost needed to buy the grasses for the whole garden which is shaped like a triangle." In the third question, the subject continued, "2 ships traveling from 2 different directions with different velocities and angles as well, asked to calculate the distance between ships if they made a 3-hour travel" Based on the conditions above, subject 3 managed to identify the problems by using interview test.

According to the result of research that can be obtained in this indicator, when the subjects showed no information or data known and questioned in their answer

sheets, it is concluded that the process of problem identification is not written on their answer sheet. Nonetheless, this identification can be done when the subjects were interviewed, they did not write it on the answer sheet because of their critical thinking, so they were very eager to write the answer on the sheets because that reason (Johanson, 2019). This phenomenon occurred because the subjects assumed this step or process could hinder their problem-solving. These findings are also supported by research results which emphasize that the majority of students do not write down problem information on their answer sheets because it hinders their time (Muhaimin & Kholid, 2023). The important thing is the understandingInformation known and questioned and being able to finish the problem effectively. This condition follows the research of Murawski (2022), which stated that people with good critical mathematical thinking skills would solve problems as effectively and fast as possible compared to others.

Correlating

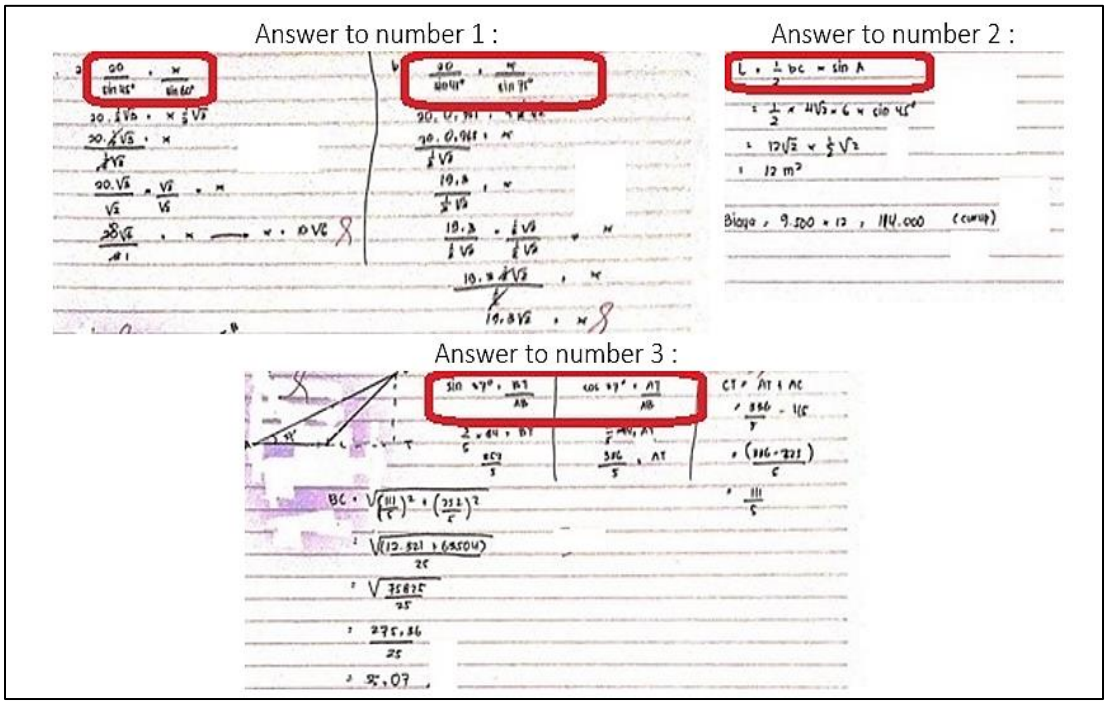

Figure 5. Answer sheet of the subject (After processing through correlating and analyzing indicators)

In the analysis of Subject 1's identification process, it can be seen that the subject managed to identify all problems, even though the known and questioInformation were not written on the answer sheet (Figure 2). Subject one can explain Information regarding known and questioInformation correctly. Next over, in this skill evaluation, it can be seen how the student correlates obtaining

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Information with the others. The real Information can be seen in the answer sheet (Figure 5) and the interview process. In problem number 1, subject 1 explained the correlation between the angle and the length in front of the angle "There is a connection between the length of the angle and the angle in front of it, to find one of their comparison with law of sines must be applied" And then on question number 2, subject one also explained "Correlation between angle and the length of 2 sides of the triangle result in width, so the calculation of this triangle can be easy to find, and more effective to finish" Number 3, subject 1 explained "For the correlation here I used the hint from the Information, which is 37 degrees and cos 37 degrees, concepts of the angle I used to find the length of BT and AT. And then the correlation between velocity and traveling time is to find the distance of travel of the ships". In the process of explanation, subject 1 seemed to explain calmly without any notes to make or write out, even though the subject occasionally looked at the question sheet to find Information. The answers follow what was written on the answer sheet (Figure 5). Subject 1 can correlate the question Information correctly according to the interview and answer sheet.

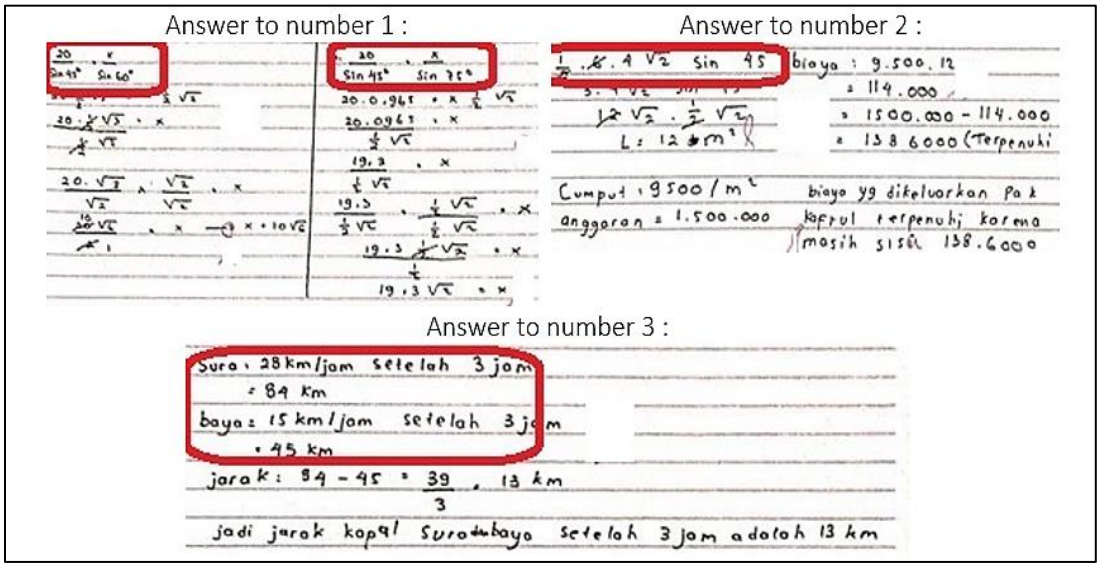

Figure 6. answer sheet of subject 2 (After processed through correlating and analyzing indicator)

Next, in subject 2, it can be seen that the subject identified all problems correctly, even though no information was written on the answer sheet (Figure 3). However, subject two can also explain Information very well. In identifying number 1's problem, subject 2 explained that the known data are two angles and one road distance. Afterward, the researcher asked again to evaluate the understanding of this subject relating to the correlation between those data.

Subject two stated, "To find one of the road distances, the law of sines can be used, and it uses two angles and one road distance to find the questioned distance," a researcher asked again, "What about number 2?" "In number 2, the known information is the length of 2 sides, and one of the angles, a correlation between these data is used to calculate the width of the map in the form of the triangle" is subject 2's answer. In question number 3, subject 2 explained, "The relation of the ship's velocity and the traveling time is used to measure the ship's distance, then the ship's distance and angle later will be used to find the distance of those ships by applying the law of sines". Being interviewed, subject 2 explained the relation smoothly between the questions by only reading the question sheet. The answers follow what was written on the answer sheet (Figure 6).

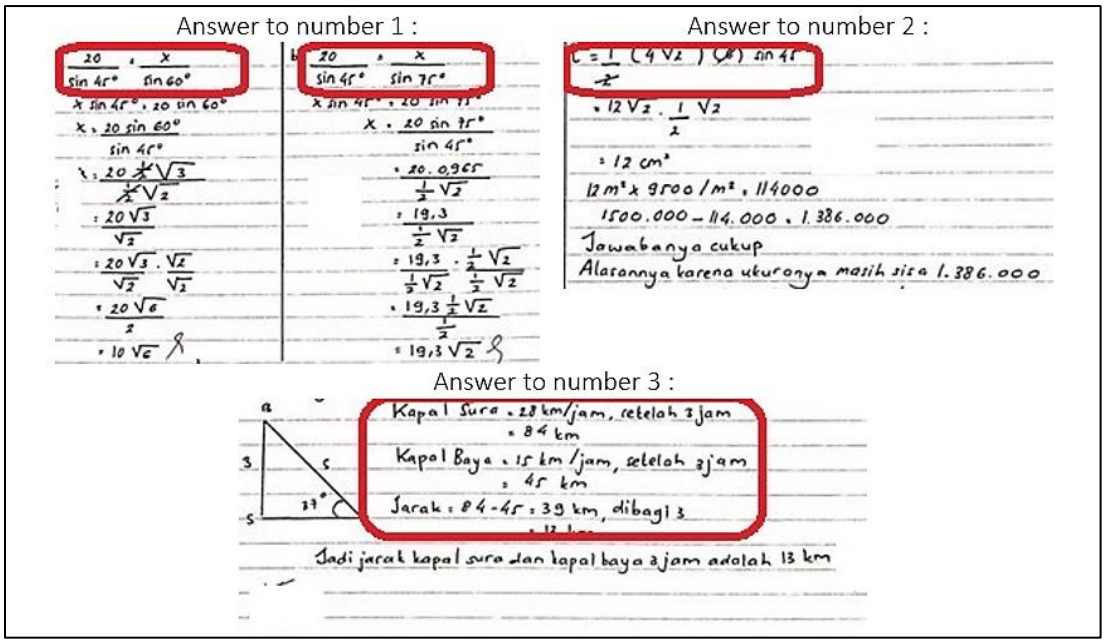

Figure 7. Answer sheet of subject 3 (After processing through correlating and analyzing indicators)

Correlating skill in subject three can be seen in depth by looking at the interview to obtain Information on this skill. The researcher inquired about the correlation between the Information of the questions from number 1 to number 3 that has already been mentioned by subject 3 in identifying problems. Subject 3 started to explain from number 1 "In this problem, the correlation between city P and R and the angle formed between road X and Z, and road Y and Z are used to find the distance of road X and distance between city Q and R by using the law of sines". In question number 2, subject explained "Correlation from the known information two lengths of the sided and one of the angles in the map of a triangle is the compositions used to find the triangle's width of the surface then the price of

grass is used to find the total cost that is needed to buy the grasses required for the garden". Another addition from subject three regarding the third question "Information related to velocity, time, and angle that can be obtained have a correlation and are used to find the distance between 2 ships by using the law of cosines that I have already written". Subject three only looked at the question sheet when giving these answers about the topic because subject three did not write the known and questioned data beforehand (Figure 4). The answers given follow what was written by subject three on the question sheet (Figure 7)

Students with critical thinking have their way of solving problems and they are also not fixated on formal problem-solving procedures (Basri et al., 2019; Muhaimin & Kholid, 2023). Just like the result that can be obtained in this research, in the identifying and correlating indicator, students did not write what they obtained regarding known and questioInformation on their answer sheet. The correlation betwInformation was directly written in the form of a formula in order to solve the problems as effectively as possible. Virtanti & Yuniastuti (2021) claimed that students' critical thinking is way faster to comprehend problems than most students. So following these findings, by only looking at Information on the questions, students explained the correlation between Information, even though they did not write the known and questioInformation on the answer sheets.

Analyzing

After finding the correlation of the data obtained from the question, it is time to see the subjects' skill in analyzing, which is the skill to synthesize the formula to solve problems. The fifth image is the answer sheet of subject 3, from numbers 1 to 3, respectively. The first answer sheet shows that the subject wrote the law of sines to find the length of road X and the distance between city P and Q. That formula is indeed correct under the assumption of finding one of the side's length if the Information is the other side's length and angles positioned in front of the known side's length. Subject 1 stated, "This law of sines can be used because it applies a system of proportion/comparison between the side and the confronting angle.". Then in question 2, subject 1 used the formula "L=1/2 BC.sin A" to find the width of the surface in the garden map that is triangle shaped. That formula can be applied and is valid to find the width as long as the compositions are fulfilled. Subject 1 explained, "Width of the area can be obtained from the use of another triangle surface area formula that was explained by a mathematics teacher, and in my opinion it can be applied because of the angle and one of the sides is known". Last problem in question 3, subject 1 wrote sin 37 degrees and cos 37 degrees to find the length of BT and AT from the visualization in Image 5. This formula is

correct and is a good answer to find the length of BC or the distance of 2 ships after 3 hours of traveling time, which later the theorem of Pythagorean will be used (Figure 5). The subject said, “I made the map look like a right triangle on purpose so I can apply the Pythagorean concept to calculate the distance between those two ships.”

For subject two regarding question number 1, the subject applied the law of sines formula to find the length of road X and the distance between city Q and R. According to subject 2, “By comparing angle and length of the road, the law of sines can be applied, but the angle and width must be facing each other”, is visualized clearly in the Image 6. Then in question number 2, m subject used the same formula as subject 1, which is “L=1/2 BC.sin∠A” in calculating the width of the triangle’s surface, according to subject 2, "There is a lot of triangle’s surface area formula, it depends on the known data, and I used this particular formula because all the conditions are met”. In the last question, subject 2 used a different procedure and formula than subject one by calculating the distance of each ship with S= V.t, then finding the distance between 2 ships by subtracting the two’s distance. This formula and procedure are the right way to find the distance of those ships under the assumption that they are going in the same direction. The problem is that those ships are in a different direction with an angle of 37 degrees. This condition was confirmed by conducting an interview. Subject 2 stated, “Because there was no image of the triangle, I thought the ships are going in the same direction. I also did not see the 37 degrees hint, so I calculated it like that” (Figure 6).

Subject 3 used the same formula and procedure as the other subjects on the first question by applying the law of sines to find the length of road X and the distance between city Q and R. Subject 3 assumed that “To find one of the triangle’s sides if the known information is two angles and one of the lengths of the other sides with the comparison of the angles and sides of the triangle, under the condition the side and angle are facing each other”, that is the reason why the law of sines formula is applied in this problem. Then the formula used in question number 2 is no different than subjects 1 and 2, by using the formula “L=1/2 BC.sin∠A” to calculate the triangle’s width of the surface. In question 3, subject 3 also made the same misconception as subject 2, by calculating the ship's travel distance firsthand and subtracting it. According to subject 3, “I used this method because I thought distance is the difference of distance traveled by those two ships”, Subject 3 did not understand that the visualization of ships’ velocity and distance is shaped like a triangle, so sin and cos formula must be applied towards the question’s hints.

The critical reasoning of sharp students makes it possible to remember things

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faster, relating to what they thought (Ellerton & Dewey, 2022). Following this research, all subjects managed to synthesize the problem-solving formula and explain the reasoning behind the related use of a particular formula. It is also a part of their logical thinking (Andriani et al., 2019). This is proven by the subjects' choice regarding the formula for the triangle's width of surface; there are many formulas to calculate a surface area, but because of their logical thinking, subjects can choose the correct formula to solve the problems on the questions.

Solving Mathematical Problems

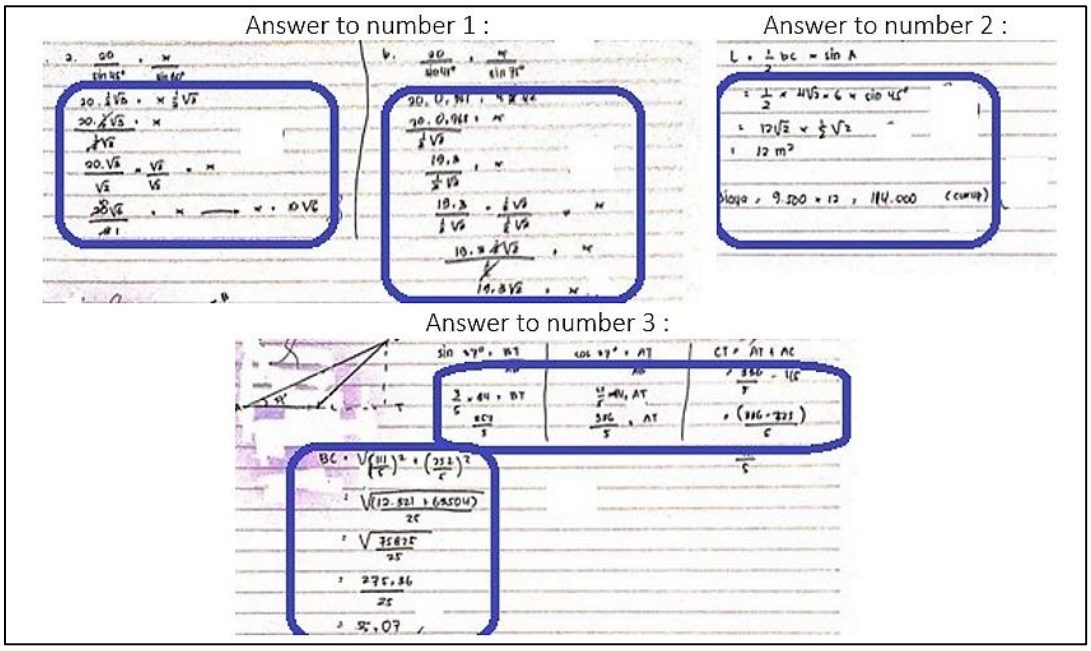

Figure 8. Answer sheet of subject 1 (After processing through problem-solving indicator)

Based on Figure 8, the answer sheet of subject 1 in the first question shows that the subject attempted the problem-solving correctly by using the calculation of direct comparison/proportion between the angle and the length of the road, resulting in the length of the road X and distance between city P and Q correctly. And so on with the number 2, subject one managed to solve the problem correctly. Computation was done by first calculating the value of sin 45 degrees and multiplying it. The conclusion was correct that the amount of money is "Sufficient", but there is an absence of reasoning behind it. The third question the subject attempted to solve by calculating the length of BT and AT with the law of sines and cosines angle and then applying the Pythagorean theorem to find the value of BC or the distance between 2 ships, looking at the calculation of the subject 2 (Figure 8), there are no traces of miscalculation, so the answer is correct. While solving these problems, subject two did not use paper to scribble. This subject calculated it directly on the answer sheet,

and in the process, this subject remembered all the formulas correctly.

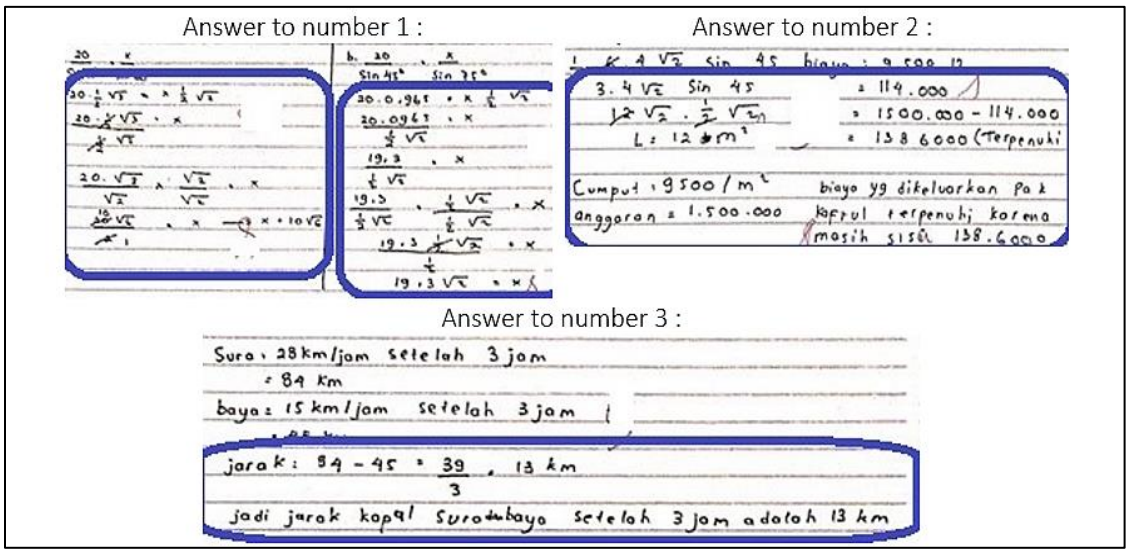

Figure 9. Answer sheet of subject 2 (After processing through problem-solving indicator)

Figure 9 shows the problem-solving process of subject two from number one to 3. In the first question, subject two calculated using direct proportion/comparison to earn the length of road X and distance between city P and Q. From what can be seen in the paper, the subject managed to do the calculation correctly and obtained the right answer. The subject stated that the calculation was done without the help of a calculator or another paper to scribble, and it was directly written on the answer sheet, and in the observation process, subject two did not show confusion in solving the problem. In the next question, the calculation was done just like how subject 1 did, by multiplying it to get the triangle's surface area, but the answer given by subject 2 is more comprehensive; comparing the expense with the remaining money, this is a strong reason to evaluate it as "sufficient". Lastly, in question 3, there is a difference in method compared to subject 1. Even the procedure and formula applied are also different. This also caused a different outcome as well. Subject 2 calculated the travel distance between 2 ships and subtracted it to get the difference in distance intended. This method would be correct if the ships were going in the same direction, but as stated in the question, their directions are different, resulting in a misconception. When the interview occurred, subject 2 explained that to find the distance of those two ships, all the subject knew was how to find the distance of them, and there was no clear visualization, just like in numbers 2 and 2 regarding the ship's direction.

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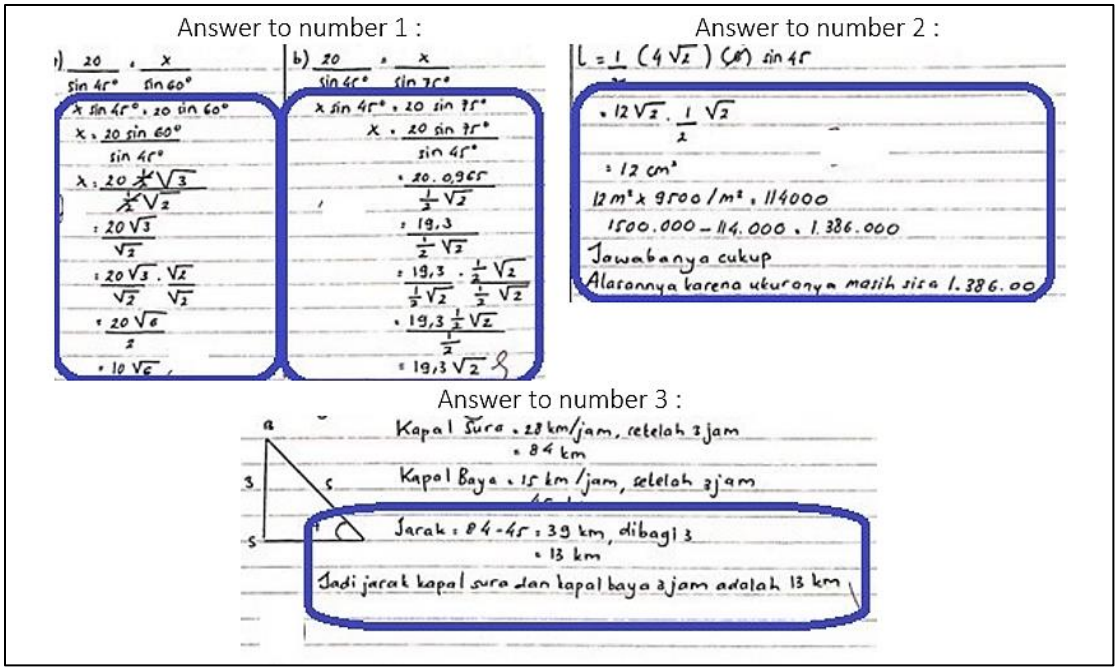

Figure 10. Answer sheet of subject 3 (After processing through problem-solving indicator)

Figure 10 shows the answer sheet of subject 3 in an attempt to calculate the answer to numbers 1 to 3. In the first question, just like the other subjects, direct proportion/comparison was done to earn one of the sides of this map. In this order, by applying the law of sines. The calculation of subject 3 is correct, resulting in a good conclusion. In the problem-solving process, subject three smoothly calculated it, and this subject did not require any scribbles to calculate the road's length. In the second question, the calculation is again; the same as the others, by multiplying the angle to get the triangle's surface area. However, then, the answer to subject 3 is more comprehensive by comparing the expense and remaining money, and this is valid and strong proof to evaluate this answer as "sufficient" because the remaining money is RP. 1.386.000. In the last question, subject 3 did the same calculation as subject 2, and they both had a misconception. Subject 3's misconception is the lack of visualization compared to questions 1 and 2.

Based on the result found in this indicator showed that 2 out of 3 subjects attempted the wrong method in question 3. The key factor is the lack of focus in calculation and the limitation of the student's skill. Pertiwi (2018) research explained that students with good critical mathematical thinking would solve problems quickly but are often not concentrated enough to calculate them. However, students with good critical mathematical thinking are creative, dare to be different in solving problems, and are not fixated on the procedure of problem-solving (Bora, 2020). This

condition was found in subject 1 in the attempt to solve number 3's problem, that by applying the Pythagorean concept, the subject managed to solve the problem correctly, despite the different approach compared to subjects 2 and 3.

IV. CONCLUSION

Based on the findings and discussions above, it can be concluded that students with critical thoughts tend to solve problems effectively and fast, so they develop their way to solve them, and their thoughts focus on getting the answer from the problems. In comparison, students with critical mathematical thinking can faster comprehend and correlate the connection between Information to the others in the question. The analysis indicator showed that critical mathematical thinking in students resulted in logical thinking and good memories. So, it makes it easier for them to determine the correct formula to solve the problems. Because of this criticality, some students are not focused on the problem-solving indicator, resulting in the answers they obtain. But students with good critical mathematical thinking tend to have a good creativity.

This research is only limited to knowing the characteristics of mathematical reasoning in solving trigonometry problems so that future research can provide an analysis of students' obstacles or difficulties in solving trigonometry problems, and then also this research is limited to certain areas or schools, further research can provide an expansion of how students' abilities students' mathematical reasoning, especially in the regions of Indonesia.

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