The Pedagogical Content Knowledge of Mathematics Pre-Service Teachers on Geometry Topic in Universitas Negeri Makassar

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Abstrak

Penelitian ini bertujuan untuk mendeskripsikan pengetahuan pedagogik konten (PCK) mahasiswa calon guru di jurusan Matematika, Universitas Negeri Makassar dengan memperhatikan beberapa indikator yakni pengetahuan materi, pengetahuan pedagogik, dan pengetahuan terkait peserta didik. Subjek penelitian ini adalah 45 orang mahasiswa yang telah memprogramkan mata kuliah-mata kuliah terkait materi matematika dan mata kuliah-mata kuliah pedagogik termasuk microteaching. Pendekatan pada penelitian ini mencakup pendekatan kuantitatif dan pendekatan kualitatif. Data pada penelitian ini dikumpulkan dengan cara memberikan beberapa pertanyaan kepada subyek yang mengukur pengetahuan pedagogik konten mereka pada topik geometri di tingkat sekolah menengah dan mengadakan wawancara yang bersifat semi-terstruktur. Dari hasil analisis kuantitatif deskriptif, dapat disimpulkan bahwa 77,78% subyek mempunyai PCK pada kategori sedang. Sedangkan 22,22% subyek berada pada tingkat PCK yang rendah. Secara eksplisit, kebanyakan subyek tidak memiliki pengetahuan yang baik pada topik-topik geometri dan hanya menerapkan teknik pedagogik yang sedikit. Selain itu, kebanyakan dari mereka tidak mempunyai pengetahuan tentang peserta didik yang bersifat komprehensif.
Kata Kunci: PCK; Pengetahuan Materi; Pengetahuan Pedagogik; Pengetahuan Peserta Didik.

Abstract

This study aims to describe the pedagogical content knowledge (PCK) of pre-service teacher in mathematics department, Universitas Negeri Makassar by taking into account of the indicators i.e., their content knowledge, pedagogical knowledge, and knowledge related to learners. The subjects of this study were forty five students who had already programmed mathematics subject matter courses and pedagogical courses including microteaching. The types of the research used are quantitative and qualitative approaches (Mix Method). The data were collected by giving the subjects questions measuring pedagogical content knowledge in the topic of geometry in secondary school and conducting semi-structured interviews to them. The results of the quantitative analysis suggest that 77.78% of subjects had PCK levels that are in mediocre category, and 22.22% of the subjects still have PCK levels that are in the low category. Explicitly, most subjects don't have good understanding of certain geometry topics as well as they simply use less pedagogical techniques. Moreover, most of them do not tend to have knowledge of learner comprehensively.
Keywords: PCK; Content Knowledge; Pedagogical Knowledge; Knowledge of Learner.

I. INTRODUCTION

Mr. Alfi is a mathematics teacher in elementary school. One day he is giving a problem to his students namely finding the perimeter of the plane figure formed by five identical squares whose side is 1 cm as illustrated in the following figure:
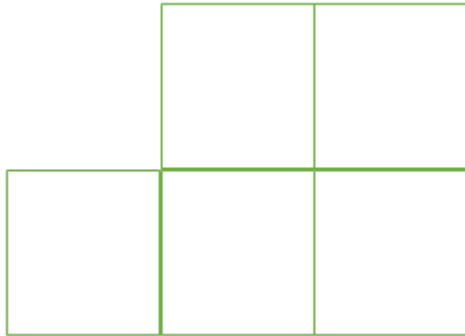

Figure 1. The Problem

Almost all the students answer 10 centimeters. Although few of them don't answer it correctly, Mr. Alfi feels comfortable. To create a discussion and to enlighten the students who are not able to solve it, he asks Agung, one of the students who answer the problem correctly, to present his answer in the whiteboard. Agung shows his method by counting the vertex of the squares in the circumstance of the figure. The method of Agung is also applied by some students in the class. Since the method is reasonable, Mr. Alfi is not sure whether he should consider it true since the method of Agung works for almost similar problems. Then, he cannot decide whether let his students use such method or explain it as a misconception.

The story of Mr. Alfi is fictional but it may reflect some teachers. It describes that Mr. Alfi may has sufficient content knowledge (the ability to know the answer and how to find it), pedagogical knowledge (the ability to explain the correct answer), but he may be lack of knowledge of learners.

Indonesia government has issued regulation, i.e. The act No. 14/2005 that demands the teachers to have academic qualification at least in bachelor level and to have holistically integral competencies namely pedagogic competence, personal competence, professional competence, and social competence. In addition, examining the knowledge or the competencies of teachers which are accordance with the demand of the government has become attracted many researchers. The common competencies issued by most researchers are content knowledge, pedagogical knowledge, and knowledge of learners (Wilson et al., 2002). Shulman (1987) stated that the integration of these knowledge is theoretically named as pedagogical content knowledge (PCK). Moreover, Fennema & Franke (1992) considered these knowledge are important for teachers.

There are several findings related to the level of PCK, one of them, namely several teacher candidates who have low PCK in the topic of decimal numbers (Stacey et al., 2001). Specifically, the teacher candidates were asked to do a test that instructs them to identify problems that students would find difficult and why. Most of them couldn't give comprehensive and clear explanation. Furthermore, there are also the findings by (DANİŞMAN & TANIŞLI, 2018) who examined the knowledge of secondary school teachers' PCK in the topic of probability. It suggested that the PCK of the teachers is low. Regarding the content knowledge, these teachers felt that their

knowledge about the the topic was not enough. Moreover, related to knowledge of learners, they couldn't specifically explain the difficulties of students in learning probabilities.

Meanwhile, pre-service teachers who are studying in prospective teacher colleges, including in Indonesia, are obliged to do field teaching practice as the first experience before graduating and teaching in schools. They are required to have sufficient knowledge and good abilities in teaching since they serve as teachers who play important role for their students (Ingvarson et al., 2004). It is also supported by Sowder (2007) who argued that the key in increasing the knowledge of students is a highly knowledgeable teacher.

Based on the previous discussion related to competencies mandated for teachers, the lack of PCK of both prospective teachers and teachers, as well as the theories and findings that emphasize the importance of PCK, the authors perceive that there may be possible gaps that can be further examined whether pre-service teachers have low PCK. It is also supported by the statement of Maryati et al. (2019), that measuring the PCK of educators is important. Hence the purpose of this study is to describe the PCK level of pre-service teachers. Geometry was set as the topic in this study.

II. METHOD

Although the components of PCK are various according to many researchers (Cochran et al., 1993; Grossman, 1990; Shulman, 1987; Smith & Neale (1989), 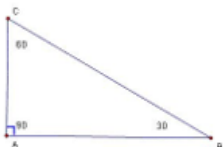Tamir, 1988), this study simply focused on three aspects, namely, content knowledge, pedagogical knowledge, and knowledge of learners as perceived by the authors, they are empirically and directly reflect the knowledge of teachers in classroom. In this study, the referred definition of each aspect, i.e., content knowledge referring to mathematical representation and knowledge of mathematics (Turnuklu & Yesildere, 2007), pedagogical knowledge which is defined as the ability of teacher to use appropriate activities in instruction and use real life examples and analogies in the instruction, meanwhile knowledge of learners including students' difficulties, misconceptions, errors, and conceptions (Kim, 2004).

This study uses a Mix Method which consists of quantitative and qualitative approaches. In the quantitative approach, it used descriptive analysis, of which data collection technique is a test given to 45 students majoring in Mathematics Education, Universitas Negeri Makassar. The university is located in the urban area of Makassar city, one of the big cities in Indonesia. The students have received many mathematical courses such as linear algebra, plane geometry, calculus, etc. of which school mathematics are extensively and conceptually learned and pedagogical courses such as instructional design, problem in mathematics education, strategies of mathematics learning, microteaching, etc. Having programmed microteaching subject itself is actually the requirement for these college students to be appointed as the subjects. The test itself as shown in the figure 1, including the content and the scoring, was adapted from

(Turnuklu & Yesildere, 2007) which has been validated by our some colleagues. It was aimed to categorize the PCK level of the subjects as shown in the table 1. Moreover, as shown in the figure 1, the test contains a situation of which certain students having a test and the subjects were asked to explain whether the students have incorrect answers, misconception, or correct answer. The subjects were also instructed to give a treatment to solve the students' problem related to the incorrect answer and misconception. Meanwhile, in the qualitative approach, it attempts to describe more deeply the PCK of two students taken from the subjects. They were interviewed using an interview guidelines previously validated by the colleagues. The interview was carried to seek for information why the participants chose certain answers of the given test (See Figure 2).
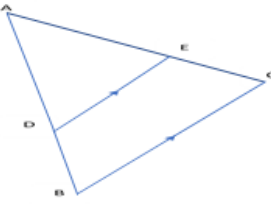

Problem No 1

In the figure, $AC = 4$. Falih, an eighth grade students considers that $AB = 8$.

a. *Is Falih's answer true?*
b. *What are the possible reasons why Falih has such an answer?*
c. *How to make Falih aware of his misconception? (this part c is given only if the subjects answer the part a correctly)*
d. *How should Falih find the length of AB?*

Problem No. 2

In the figure, Salfa is an eighth grader students asked by her teacher to find the value of $\frac{DE}{BC}$, if the $AD = 2$ and $DB = 6$. Then Salfa answers 1/3.

a. *What does Salfa think so that she gets 1/3?*
b. *Is Salfa's answers true? Why?*

If a subject answers the question part b correctly, then he will be given additional question, *c. what can you explain to solve Salfa's misconception?*

Problem No. 3

A quadrilateral with the same length edge of which its diagonals perpendicularly intersect and cut each other into two equal parts. If the length of the side is 10 cm, find the area of the quadrilateral?
Two eighth graders, Aktar and Gelya, have different answers. Aktar's answer is 100 cm² and Gelya argue that it can't be still determined

a. *Explain why these students have these kinds of answers!*
b. *which one is true? Explain!*
c. *What knowledge might the student who don't answer correctly not have?*
d. *How do you explain to the student in part c to solve his/her misconception?*

Problem No. 4

An eighth grader Filzah is asked by her teacher to find the area of a circumscribed circle of triangle with side lengths respectively 8, 15, and 17. Filzah argues that it can't be determined.
a. *Is Filzah's argument true?*

When the subject has the true answer for the problem, then he will be given two additional questions:

b. *What knowledge might Filzah not have?*
c. *What question the teacher should ask to make Filzah aware of her mistake?*

Figure 2. The Given Test

For each problem, every correct answer and clear reason or explanation is marked 3. For the answer which is true but it is not precise or not including true reason is marked 2. For instance, the problem 1 part c, i.e. how do you explain the length of AB? is explained using Pythagorean theorem is marked 2 since the eighth graders has not learned about the topic, instead, they will learn it when they are in the ninth grade. This situation reflects that the subject's knowledge of learners is not precise and still necessary to be improved. Especially, for the pedagogical knowledge problem, when the subjects take into account of some pedagogical aspects as Zhang (2015) suggests, such as scaffolding, cognitive conflict, etc., they can be given 3 points. When they simply explaining directly the true answer, they can be given 2 points.

Although each problem may contain more than one component of PCK, for instance the problem 1-b and 1-d cover content knowledge and knowledge of learners, it can be mainly categorized in one component as presented in the Table 1 and 2.

Table 1.
PCK Criteria of Each Problem

Problems	Criteria
Problem 1-a	Content Knowledge
Problem 1-b	Knowledge of Learners
Problem 1-c	Pedagogical Knowledge
Problem 1-d	Knowledge of Learners
Problem 2-a	Knowledge of Learners
Problem 2-b	Content Knowledge
Problem 2-c	Pedagogical Knowledge
Problem 3-a	Knowledge of Learners
Problem 3-b	Content Knowledge
Problem 3-c	Content Knowledge
Problem 3-d	Pedagogical Knowledge
Problem 4- a	Content Knowledge
Problem 4-b	Knowledge of Learners
Problem 4-c	Pedagogical Knowledge

Table 2.
Categorization of PCK Level

Score Range	Level	Description
14-21	Unsufficient	➤ Having difficulty to understand both students' difficulties and the reasons for students' difficulties ➤ Neither being able to understand students' thought process with questions nor having the ability to create solutions to students' learning difficulties ➤ Exhibits low conceptual and procedural understanding of a topic ➤ Being unable to solve problems
22-35	Mediocre	➤ Understanding students' difficulties and understanding the reasons for students' difficulties ➤ Failing to ask proper and meaningful questions to understand their thought process ➤ Having difficulty to create solutions to misconceptions ➤ Exhibits middle thorough conceptual and procedural understanding of a topic. ➤ Displaying moderate skills for solving problems
36-42	Excellent	➤ Understanding students' difficulties and understanding the reasons for students' difficulties ➤ Being able to ask proper and meaningful questions in order to understand their thought process. ➤ Having the ability to create solutions to overcome students' learning difficulties. ➤ Exhibits deep and thorough conceptual and procedural

understanding of a topic
➢ Displays good skills for solving problems

III. RESULT AND DISCUSSION

The result of the test for the 45 students is presented in the Table 3. Most of the students are in the mediocre category, and there is no students in the excellent category.

Table 3.
The Pre-Service Teacher's PCK Level

Levels	F	Percentage (%)
Level 1 (Low)	10	22,22
Level 2 (Mediocre)	35	77,78
Level 3 (Excellent)	0	0

In detail, the work of the students according to the criteria of content knowledge is presented in the Table 4.

Table 4.
The Percentages of the subjects' responses according to Content Knowledge

Problems	Points					
	3		2		1	
	F	Percentage	F	Percentage	F	Percentage
Problem 1-a	27	60%	4	8%	14	32%
Problem 2-b	16	35,6%	19	42,2%	10	22,2%
Problem 3-b	2	4%	8	17,8%	35	77,8%
Problem 3-c	2	4%	5	11,1%	38	84,4%
Problem 4- a	5	11,1%	21	46,7%	19	42,2%

In the table 4, the problem which can be well tackle by most subjects is the problem 1.a., which is the relationship between the angle and the side of a triangle. Conversely, the problem which is the most difficult to solve satisfactorily is the problem 3-B, i.e. the properties of a plane figure. It shows that the subjects' knowledge of the contents is not well distributed. The answer of one of the subjects is shown in the Figure 3.
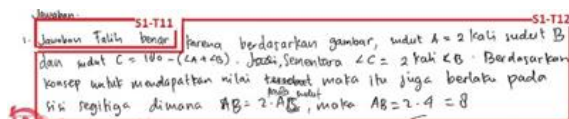

Figure 3. One of the Subjects' Responses to the Problem 1.a

Based on the Figure 3, the subject considers that if $m\angle C = 2m\angle B$, then $AB = 2\,AC$. In this problem, the subject's score is marked 1 since it is completely false, i.e., the ratio of the sides of a triangle is not always determined by the ratio of their opposite angles. The process of the subject's thought is shown in the interview fragment 1.

The Interview Fragment 1.

Since I agree with Falih. This is my answer I have explained. For example, angle C is two times angle B, this is angle A is two times angle B, however, I now realize that I made an error. Actually it is angle C (not angle A). Why I thought like that, since if angle C is 60^0 and angle B is 30^0, automatically angle C is two times angle B. Then based on a concept, to find that value (AB), it also satisfies for the side length where AB = 2 AC. Then AB = 2 x 4 = 8.

The next interview fragment shows how the subject explain the length of AB.

The Interview Fragment 2

Interviewer *Do you know how to find the length of AB when the angles are known and the length of AC is 4?*

Subject *As far as I know, we should use sine rule or cosine rule, but I forget the formula*

In the interview fragment 2, the subject argued that he can apply sine rule or cosine rule. Here, the subject shows his insufficient knowledge of the content, namely, he forgets the formula and he doesn't realize that Falih is an eighth grader which has not had acquaintance with the sine rule or cosine rule. In addition, some responds of the subjects for the other problems are listed as follows:

Problem 2-b:

- Salfa's answer is false since the side BC doesn't correspond to the side BD in the two similar triangles (3 points)
- Salfa's answer is true since $DE/BC = AD/DB$ (2 points)
- Salfa's answer is false (without reason) (2 points)
- Unable to give an answer (1 point)

Problem 3-b:

- Gelya is true since it is also likely that it is a rhombus, besides a square (3 points)
- Gelya is true since the diagonals of a square do not perpendicularly intercept and equally bisect each other (2 points)
- Aktar is true since it is a square (1 point)
- Unable to give an answer (1 point)

Problem 3.c:

- Knowledge of which a parallelogram with four equal sides besides square is rhombus (3 points)
- Knowledge of which the diagonals of a square are not perpendicular (2 points)

- Unable to give an answer (1 point)

Problem 4.a:

- Filzah is not true, since a triangle with side lengths 8, 15, and 17 is a right-angle triangle. The right angle is opposite to the diameter which is the longest side, i.e., 17. Then the diameter can be identified (3 points)
- Filzah is not true, since an inscribed triangle is possible to find its area (2 points)
- Filzah is true (1 point)

Moreover, the pedagogical knowledge indicators of the subjects are presented in the table 5.

Table 5.
The Percentages of the subjects' responses according to Pedagogical Knowledge

	Points					
	3		2		1	
	F	Percentage	F	Percentage	F	Percentage
Problem 1-c	1	2%	22	48,9%	22	48,9%
Problem 2-c	0	0%	28	62,3%	17	37,8%
Problem 3-d	0	0%	7	15,6%	38	84,4%
Problem 4-c	3	6,67%	23	51,11%	19	42,22%

The table 5 shows that the pedagogical knowledge by the subjects is generally low. It is suggested that it is caused by their low content knowledge. For instance, they cannot give true explanation to the student in the problem when they themselves are not able to decide correctly whether the student's answer is true or false. Moreover, when some subjects have successfully answered the problem related to the

content knowledge, they seem to carelessly give the treatment to the students in the problem 2-c as shown in the Figure 4.
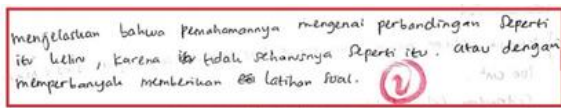

Figure 4. One of the Subjects' Responses to the Problem 2-c

As suggested by the Figure 4, the subject simply informs Salfa that her conception is wrong and asks her to frequently do an exercise. The subject does not stimulate her to do something, for instance, to draw two similar triangles, triangle ABC and triangle ADE and identifying the known lengths. By looking at the triangles, Salfa is likely to realize her mistake since the length of AB is 8. Moreover, there is also a respond by the subject in the problem 1-c which only suggests Falih to apply certain formula as described in the interview fragment 3.

The Interview Fragment 3

Interviewer   *If Falih did a mistake and when you are countering him, how to make him aware of his misconception?*

Subject   *I will explain him a formula related to angle*

In the interview fragment 3, the subject does not try to put emphasis on making Falih aware of his misconception by showing him how his concept does not work, instead, the subject initiates to tell the right formula straightforwardly. Moreover, some responds of the subjects for the other problems are listed as follows:

Problem 1-c:

- We can show Falih by drawing two couple of different rays in length which form angles of which the magnitude of one of the angles is two times larger than the other. Then, by connecting the end points of each ray which forms two sides and measuring their lengths using ruler, the side opposite to the larger angle is not absolutely two times longer than the other side (3 points)
- Explaining to Falih that angles are not absolutely proportional to sides (2 points)
- Unable to answer (1 point)

Problem 2-c:

- Explaining directly to Salfa the true answer (2 points)
- Considering that Salfa's answer is true

Problem 3-d:

- Explaining directly the true answer (2 points)
- Considering that Akhtar's answer is true (1 point)

Problem 4-c:

- Giving scaffolding to Filzah to remind triple Pythagorean or telling her that 8, 15, and 17 is a triple Pythagorean. When Filzah realizes them as a triple Pythagorean, we can remind her about the relationship between central angle and inscribed angle concept (3 points)
- Directly explaining Filzah the true answer (2 points)
- Considering that Filzah's answer is true (1 point)
- Unable to answer (1 point)

The subjects' responds to the problems are various. The subject which gets 3 points apply Piaget's cognitive conflict (in Lee et al., 2003). When being interviewed, although the subject was not acquaintance with the theory, he is aware of Falih's thought and misconception. So, he knows

how to show a way to make Falih aware of his misconception using scaffolding.

Furthermore, the subjects' knowledge of learners is presented in the Table 6.

Table 6.
The Percentages of the subjects' responses according to Knowledge of Learner

	Points					
	3		2		1	
	F	Percentage	F	Percentage	F	Percentage
Problem 1-b	0	0%	28	62,3%	17	37,8%
Problem 1-d	0	0%	21	46,7%	24	53,3%
Problem 2-a	19	42,2%	26	57,8%	0	0%
Problem 3-a	0	0%	13	29,9%	32	71,1%
Problem 4-b	0	0%	15	33,3%	30	66,7%

Based on the Table 6, the percentage of subjects who get 3 points is very less. This is because the focus of the subject analysis of students is not comprehensive and has not covered the point of the conception. One of the subjects' answers of the problem 1-a is presented in the Figure 5.
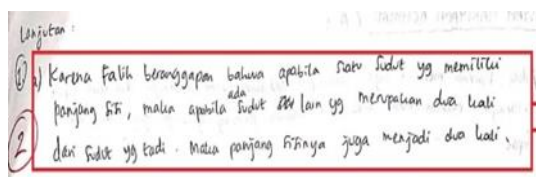

Figure 5. One of the Subjects' answers on the problem 1-b

As shown in the figure 3, most subjects merely explain the relationship between angles and their opposite sides. When being interviewed, most subjects assert that, it is the only thought that Falih has and there is no other possibility. In fact, there is also possibility that Falih can think the relationship between the central angle of a circle and its chord. In the problem 1-d, no one of them explains using the concept of equilateral triangle. In fact, Falih can realize his misconception if the triangle is doubled, using the side AB as the axis of

symmetry, then it will form an equilateral triangle of which the length of the side BC is 8. Besides that, there are some incorrect responds from the subjects since they have insufficient content knowledge, for instance, some of them are not able to give answer about what Salfa think in the problem 2-a, since they don't know how to tackle with the problem.

Overall, by seeing the results, the author assumes a trend that the content knowledge is a crucial thing the subjects should know to have good pedagogical knowledge although it is likely for the subjects to fail to get the maximum score in tackling pedagogical knowledge problems. In this study, what makes several subjects fail is that they don't try to apply some pedagogical techniques, instead, they directly tell the students in the problems the true answer. When being interviewed to know whether they know other techniques to make students aware of their misconceptions, some of them admit that they know what scaffolding is, however, they don't have any idea how to apply it because of their less content knowledge.

Leinhardt & Greeno (1986) proposed the knowledge of lesson structure can assist teachers to indicate the thought of the students. In a small-scale lesson like guiding the students to cope with their misconceptions, when they know the structure of the content, they are likely to know how to bring the right concept to student by giving scaffolding precisely. The content knowledge of the subjects also plays important roles to identify the knowledge of learners. When they know whether an answer is true or wrong, it may come up in the subjects' mind what's wrong in the students' thought.

The findings of this study clarify the level of PCK of pre-service teachers in several previous research (Cochran et al., 1993; Carpenter et al., 1988); Turnuklu & Yesildere, 2007). The subjects in this study are still inexperience and less required to be professional than teachers who have joined in teacher profession education. Based on the National Qualification Framework, the subjects are simply demanded to apply the knowledge they receive in college. Specifically, the portions of the pedagogical knowledge the subjects mostly cover the theories of designing learning, assessment, and learning instruction strategies with less application. Conversely, the teachers with couple of year experiences including studying in teacher profession education equally derive theories and applications equally by also taking account of pedagogical content knowledge.

IV. CONCLUSION

From the obtained results, it can be concluded that, in the topic of geometry,

the PCK of most of the pre-service teachers in Universitas Negeri Makassar are categorized in the mediocre level. Most of them should increase their knowledge of the content as well as the knowledge how to tackle some situations of which there are misconceptions and how to fix the misconceptions. It is assumed that the pre-service teachers never receive any discourse about PCK. Some educational researchers such as Lim et al. (2011) and (MacPhail et al. 2013) suggest that, there should be a program to introduce PCK to pre-service teachers since most of them have weakness in PCK. In an additional interview done by the authors, some of the pre-service teachers in this study realize the importance of PCK. According to them knowledge of teaching mathematics in general is not sufficient, instead, they should also know how to teach a specific topic.

What can be also found in this study is that the content knowledge really influences the pedagogical knowledge and knowledge of learners of the pre-service teachers. When they don't know a concept, it is hard for them to explain or to teach it and they find difficulty to think of what learner know about the concept. It can be a suggestion for further research to analyse the relationship within the components of PCK.

The number of the geometry topics covered in this study are relatively less. In addition, based on the Bloom taxonomy, based on the review from our colleagues, they are all categorized in, at least, analysis level. It can be a suggestion for further research to apply a test that covers many

topics and all levels in the Bloom Taxonomy.

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