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# Didactic Transposition from Scholarly Mathematics to School Mathematics: The Case of Function Concept

**Riki Andriatna[1*], Imam Sujadi[2]**

**Department of Mathematics Education, Universitas Sebelas Maret**
**Jl. Ir. Sutami No. 36, Surakarta, Jawa Tengah, Indonesia**
**[1*]*andriatna.riki@staff.uns.ac.id*, [2]*imamsujadi@staff.uns.ac.id***

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## Abstrak

Konsep fungsi menjadi jembatan beberapa konsep lain dalam matematika sehingga memberikan pandangan berbeda terhadap suatu permasalahan dalam matematika. Dengan demikian, proses transposisi pengetahuan menjadi sangat penting untuk diperhatikan. Penelitian ini bertujuan melihat transposisi konsep fungsi dari scholarly mathematics terhadap school mathematics. Penelitian menggunakan pendekatan fenomenologi dengan sumber data yaitu buku teks matematika universitas yaitu analisis real, struktur aljabar, dan kalkulus, serta buku teks sekolah yaitu matematika kelas X dan XI dari Kemendibudristek. Teknik analisis data yaitu deskriptif kualitatif. Hasil analisis menunjukkan transposisi terjadi pada konteks serta struktur dan bentuk. Pada konteks, konsep fungsi dapat dilihat pada bagaimana konsep fungsi tersebut dikonstruksi, dimana pada scholarly mathematics, konsep fungsi dibentuk dari konsep abstrak, sedangkan di school mathematics konsep fungsi didasarkan pada konsep konkret yang kemudian mengarah pada konsep abstrak. Pada aspek struktur dan bentuk, transposisi didaktis dapat dilihat dari penyajian konsep fungsi, dimana pada scholarly knowledge konsep fungsi disajikan dalam notasi abstrak, sedangkan pada school mathematics disajikan dalam beragam bentuk seperti diagram.

**Kata Kunci**: Fenomenologi; Scholarly Mathematics; School Mathematics; Transposisi Didaktis

## Abstract

The concept of function becomes a bridge for several other concepts in mathematics so as to provide a different view of a problem in mathematics. Thus, the process of knowledge transposition is very important to be considered. This study aims to look at the transposition of the concept of function from scholarly mathematics to school mathematics. This research used phenomenological approach. The data sources in this study are mathematics textbooks at the university level, namely real analysis textbooks, algebraic structures, and calculus, while at the school level, namely mathematics textbooks for grades X and XI published by the Ministry of Education, Culture, Research and Technology. The data analysis technique used is qualitative descriptive analysis. The results of the analysis show that transposition occurs in context and structure and form. In the context, the function concept can be seen in how the function concept is constructed, where in scholarly mathematics, the function concept is formed from abstract concepts, while in school mathematics the function concept is based on concrete concepts which then lead to abstract concepts. In the aspect of structure and form, didactic transposition can be seen from the presentation of the function concept, where in scholarly knowledge the function concept is presented in abstract notation, while in school mathematics it is presented in a variety forms such as diagrams.

**Keywords**: Didactic Transposition; Phenomenology; Scholarly Mathematics; School Mathematics

## I. INTRODUCTION

Paun stated that in the context of teaching, there are two main problems faced, namely problems regarding curriculum management and classroom management (Jamilah et al., 2021). Curriculum management problems are problems that focus on how to construct scholarly knowledge into classroom/school learning, namely knowledge to be taught and taught knowledge (Bosch & Gascon, 2006; Chevallard & Bosch, 2014), while classroom management is directly related to learning activities in the classroom. The existence of these problems has the potential to create difficulties for students in understanding concepts so that it can result in low student achievement. This condition has raised one of the difficulties caused by the learning process called didactical obstacles (Elia et al., 2016; Kuzniak & Rauscher, 2011; Rudi et al., 2020). Thus, it is necessary to ensure that mathematical knowledge can be conveyed as a whole in the learning process that is aligned with the curriculum and scholarly knowledge. Specifically, in the context of school mathematics, the knowledge in question is essentially the development of scholarly mathematics. Thus, the mathematical knowledge acquired by students is a process of transposition from scientific mathematical knowledge to mathematical knowledge learned by students. The process of transposing scholarly knowledge into knowledge at school or in the classroom is called didactic transposition (Bosch & Gascon, 2006; Chevallard & Bosch, 2014).

Didactic transposition is a transposition that underlines the fact that taught knowledge in schools comes from other institutions, where taught knowledge comes from universities or other sources (scholarly institutions) (Chevallard & Bosch, 2014). Chevallard and Bosch (2014) describe the process of didactic transposition in the following Figure 1.

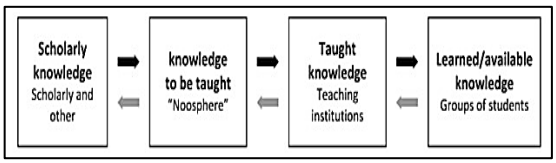


Figure 1. The Didactic Transposition Process

Bosch & Gascon (2006) state that the theory of didactic transposition consists of: (1) scholarly knowledge which is the result of scientists' thinking; (2) knowledge to be taught, as specified in the curriculum; (3) taught knowledge, which is actually taught by teachers in the classroom; and (4) learned/available knowledge as knowledge acquired by students. Figure 1 shows that the mathematical knowledge taught at school (taught knowledge) is generated from outside the school (scholarly institution) which is then transferred to the school with a series of adaptation processes. It is further explained that the adaptation process is a process of transposition of knowledge from scholarly knowledge into knowledge to be taught, so that it becomes taught knowledge which is then taught by teachers to students so that it becomes learnt knowledge.

In the context of education in Indonesia, scholarly knowledge can be the result of research/studies conducted by universities, then it becomes knowledge to be taught by the ministry in the form of curriculum. Furthermore, it becomes taught knowledge delivered by schools or teachers to students. Therefore, according to (Diana et

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al., 2020) that any modification of knowledge under instructional objectives is a didactic transposition. This shows that didactical transposition aims to produce scientific analyses of didactical systems based on the assumption that knowledge is determined as the object of learning has pre-existence as scientific knowledge (Diana et al., 2020). Thus, didactic transposition is one of the important processes in the administration of mathematics learning (Arzarello et al., 2014; Rudi et al., 2022).

The process of didactic transposition shows a series of changes in knowledge, academic environment, and school environment. Atalar & Ergun (2018) stated that the process of didactic transposition occurs in two stages, namely the external stage and the internal stage. The external transposition stage is the transition stage from scholarly knowledge to knowledge to be taught, which includes the exposure of scholarly knowledge to various changes during the transition, while the internal didactic transposition as the transition from knowledge to be taught to knowledge to be taught, includes all the internal effects and transpositions in the education system to transform knowledge to be taught into knowledge learned by students (Atalar & Ergun, 2018).

Chevallard stated that in the Didactic Anthropology Theory there are three things that are interconnected, namely objects, individuals, and institutions (Atalar & Ergun, 2018). Atalar & Ergun (2018) explain that in a school environment, knowledge is an object, students or teachers are individuals, and classrooms or schools are institutions. When an object begins to exist for an individual, the individual recognises the object and develops his or her personality to recognise the object, so that the individual recognises the object (Frejd & Bergsten, 2016). This shows that learning is a change in an individual's personal recognition of objects, so learning does not change the individual, but their knowledge.

Mathematics as one of the sciences will continue to develop. Erdogan stated that mathematical concepts are not considered as absolutes (Gök et al., 2019). It is further explained by Erdoğan that mathematical concepts emerge in the context of a problem, are then defined by mathematicians based on that context, but acquire new meanings in their development in different contexts (Gök et al., 2019). Thus, mathematical concepts continue to develop dynamically. One of the developing mathematical concepts is the concept of function.

The concept of function is one of the fundamental concepts in mathematics, so the concept of function is one of the most important concepts to learn in the mathematics curriculum (Denbel, 2015; Kleiner, 1993; Makonye, 2014; Trujillo et al., 2023a). The concept of function developed inseparably from the history of Babylon (Kleiner, 1989). Function definitions or function descriptions have varied based on time, context, and level of presentation, where a function can be viewed as a formula, a rule, a correspondence, a relation between variables, a table of values, a graph, a mapping, a transformation, an operation, and a set of ordered pairs (Kleiner, 1993). Siu (1994) categorised the notion of function concept into three forms,

namely: (1) the static notion, in which functions are expressed as tables of values, correspondences, ordered pairs, which developed based on set theory; (2) the kinematic or geometric notion, referring to the notion of function as a representation of a curve; and (3) the algebraic notion, which is based on developments in the context of continuous functions, rows, analytic formulations, and representations.

As one of the fundamental concepts, both in mathematics and in mathematics learning (Kleiner, 1993; Oehrtman et al., 2008; Sajka, 2003), in Indonesia in particular, the concept of function has been studied since primary and secondary education. The concept of function is part of the algebraic domain in the Kurikulum Merdeka in phase F in the Algebra and Function element for grade XI and XII high school students. Specifically in phase F, students can determine inverse functions, function composition, and function transfromation to model real-world situations using appropriate functions. In Kurikulum 2013, the basic competencies related to the concept of functions in high school are explaining and determining functions formally including notation, origin, result area, and symbolic expressions and graph sketches. In addition, in high school the basic competencies related to functions are the ability to explain composition operations on functions and inverse operations as well as their properties and existence, including solving relevant problems. Based on the two curriculum reviews, there are basically similarities where the concept of function, both in the form of function definitions, is the main concept that must be understood first by students.

Several studies on didactic transposition of mathematical concepts have been conducted (Jamilah et al., 2021, 2020; Junaeti et al., 2023; Putra, 2020; Rudi et al., 2022; Sulastri, 2023). However, some of these studies focus on the concepts of limits, sets, vectors and at the primary and secondary education levels. While on the concept of function, specifically on the definition of function and some properties of function have not been fully carried out. In this case, the concept of function is a Sscholarly knowledge produced by mathematicians as scholarly mathematics. Furthermore, the concept of function is reconstructed by curriculum designers and mathematics textbook writers into the form of knowledge to be taught in the form of curriculum and mathematics textbooks. The concept of function as a form of knowledge to be taught as a result of structuring the concept of function in scholarly mathematics into the context of mathematics education (school mathematics). The importance of mastering the concept of function by students can potentially lead to mastery of other concepts in mathematics. However, in reality there are still errors in the function concept owned by students (Perbowo & Anjarwati, 2017; Rahmi & Yulianti, 2022; Septyawan et al., 2019). Thus, there is still a gap between the concepts mastered by students and scientific concepts that have an impact on student errors. Based on the above explanation, in learning mathematics didactic transposition has a very important role. It is based on the impossibility of interpreting school mathematics without

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considering the phenomenon of reconstructing mathematical knowledge in school mathematics from scholarly knowledge (Bosch & Gascon, 2006). In addition, a good didactic transposition will have an impact on the comprehensive acquisition of a mathematics education curriculum that can produce appropriate learning situations (Jamilah et al., 2020).

## II. METHOD

This research is a qualitative with a phenomenological study, aiming to understand the hidden meaning and its essence so as to describe the experience related to a phenomenon described (Giorgi & Moustakas in Creswell & Poth, 2016). This research aims to describe the phenomenon related to the transposition of knowledge on the concept of function from scholarly mathematics to school mathematics that will be learnt by students. Chevallard & Bosch (2014) stated that the transposition process is a didactic transposition which is described as the process of transferring knowledge from scholarly knowledge produced by institutions/universities or mathematicians to knowledge to be taught by the education system or nosphere. Knowledge that has been adapted by the noosphere then becomes taught knowledge that will be learnt by students in the classroom so that it becomes learned knowledge. In this study, the didactic transposition phase is only focused on scientific knowledge about the concept of function produced by institutions/universities or mathematicians as scholarly mathematics and knowledge to be taught in schools (school mathematics).

The diagram of the didactic transposition process in this study uses the transposition diagram of (Jamilah et al., 2020b) which is adjusted to the context of the research conducted as shown in Figure 2 below.

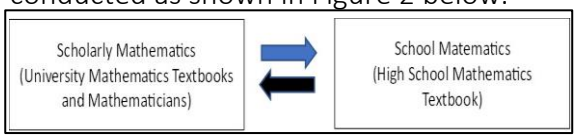


Figure 2. Modification of Didactic Transposition Process Diagram

The data sources used in the study are University mathematics textbooks as scholarly mathematics and mathematics textbooks used in Indonesian schools based on the Kurikulum 2013 and Kurikulum Merdeka. The identity of the mathematics textbooks described in Table 1. In addition, this study also uses data sourced from mathematicians regarding the definition of functions quoted from Cha in an article written by (Gök et al., 2019). The data were analysed descriptively.

Table 1.
Identity of the Mathematics Textbook

| Category of Books | Title | Authors (Year) | Publisher |
|---|---|---|---|
| University Mathematics Textbook | Introduction to Real Analysis Third Edition | Bartle & Sherbert (2020) | John Wiley & Sons, Inc. |
| | Modern Algebra: An Introduction Sixth Edition | Durbin (2009) | John Wiley & Sons, Inc. |
| | Calculus with Analytic Geometry, 5th Edition | Purcel & Varberg (1987) | Pretince-Hall, Inc. |

| Category of Books | Title | Authors (Year) | Publisher |
|---|---|---|---|
| High School Mathematics Textbook | Matematika untuk SMA/SMK Kelas XI | Susanto et al. (2021) | Pusat Perbukuan Badan Standar, Kurikulum, dan Asesmen Pendidikan Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi |
| | Matematika untuk SMA/MA/SMK/MAK Kelas X Edisi Revisi 2016 | Sinaga et al. (2016) | Pusat Kurikulum dan Perbukuan, Balitbang, Kemdikbud |

The data analysis procedure is based on the four basic stages in qualitative research according to (Creswell & Poth, 2016) which are described as follows Figure 3.

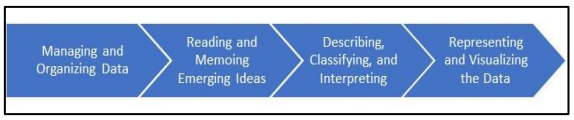


Figure 3. Data Analysis Technique

At the stage of managing and organizing data, data was obtained which is arranged or formed and then organized so that it is arranged correctly. Furthermore, at the stage of reading and memoing emerging ideas, activities are carried out to read data through the text, including making notes or initial coding. In the third stage, describing the essence of the phenomenon, developing significant statements including classifying these statements based on their groups. At this stage, textural and structural description is also carried out and ends with the representing and visualizing stage in the form of narrative presentation in the form of tables, figures, or discussions.

## III. Result and Discussion

### A. Function Concept in Scholarly Mathematics

In mathematics, the concept of function is a fundamental concept in mathematics so it has a very important role (Denbel, 2015; Kleiner, 1993; Makonye, 2014; Trujillo et al., 2023a). The concept of function was first formally proposed by Bernoulli, who stated that, "one calls here function of a variable a quantity composed in any manner whatever of this variable and of constants" (Gök et al., 2019; Jones, 2006). In the history of its development, many mathematicians have contributed to the development of the concept of function. Cha (Gök et al., 2019) summarises some mathematicians' definitions of the function concept as in the following Table 2.

Table 2.
Definition of Function by Mathematicians

| Year | Mathematicians | Definition of Function |
|---|---|---|
| 1665 | Newton | Any relationship between variables. |
| 1673 | Leibniz | Any quantity varying from point to point of curve. |
| 1697 | Bernoulli | Quantities formed using algebraic and transcendental expressions of variables and of constants. |
| 1718 | Bernoulli | Function of certain variable as a quantity that is composed in some way from that variable and constants. |
| 1748 | Euleur | Formula or analytic expression composed in any manner from that variable quantity and members or constant quantities representing the relation between variables. |

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| Year | Mathematicians | Definition of Function |
|---|---|---|
| 1755 | Euleur | If $x$ denotes a variable quantity then all the quantities, which depend on $x$ in any manner whatever or are determined by it. If some quantities depend on others such a way that if the latter are changed the former undergo changes themselves then the former quantities are called function of the letter quantities. |
| 1829 | Dirichlet | $y$ is a function of a variable $x$, defined on the interval $a < x < b$, if to every value of the variable $x$ in this interval there corresponds a definite value $y$. |
| 1917 | Caratherdory | A rule of correspondence from a set A to real numbers. |
| 1939 | Bourbaki | A rule of correspondence between two sets. |
| 1939 | Bourbaki | Let $E$ and $F$ be two sets, which may or may not be distinct. A relation between a variable element $x$ of $E$ and a variable element $y$ of $F$ is called a functional relation in $y$, if for all $x \in E$ there exist a unique $y \in F$ which is in the given relation with $x$. |
| 1950 | Dirichlet-Bourbaki | Any correspondence between two sets which assigns to every element in the domain exactly one element in the range. |

Based on Table 2, it can be seen how the development of function definitions from year to year proposed by several mathematicians. Even (Gök et al., 2019) stated that the development of function definitions in each period is based on the development of science including the field of expertise of each mathematician. In addition, in its development, some mathematicians revised the definition of function that had been previously expressed, thus showing a dynamic development of the function concept. Euler hinted that the concept of function can be interpreted as an analytical expression, one of which is the use of the notation $f(x)$ (Septyawan et al., 2019). Furthermore, Nicolas Bourbaki (Jones, 2006) defined function as follows Table 2.

Specifically, in 1950, Dirichlet-Bourbaki introduced the terms domain and range in the definition of function as presented in the table above, where function is defined as a correspondence between two sets that pairs each element in the domain with exactly one element in the range. This definition is the definition of function that is widely used in mathematics, especially in learning functions. Thus, in the early period, the definition of function emphasises on quantity, formula, expression, or relationship and in the later period, the definition of function leads to correspondence. By paying attention to the definition presented above, many mathematicians accept the definition to understand the concept of function. The definition is considered as a more modern formal definition of function.

Furthermore, at the university level, the concept of function is taught in several courses including analysis (including calculus) and algebra. The concept of function presented is not only related to the definition, but also related to the concepts of domain, codomain, and range, notation, types/properties of functions, and algebraic

operations on functions. Table 3 presented the results of the analysis of data sources in the form of books used at the University as Scholarly Mathematics, namely the book Introduction to Real Analysis Third Edition (Bartle & Sherbert, 2000), the book Modern Algebra: An Introduction Sixth Edition (Durbin, 2009), and the book Calculus with Analytic Geometry, 5th Edition (Purcel & Varberg, 1987).

Table 3.
Summary of Function Concept Analysis in Scholarly Mathematics

| The Concept of Function | Book | Descriptions |
|---|---|---|
| Definition | Modern Algebra : An Introduction Sixth Edition and Calculus with Analytic Geometry, 5th Edition | A mapping from a set $S$ to a set $T$ is a relationship (rule, correspondence) that assigns to each element of $S$ a uniquely determined element of $T$. In book Calculus, term mapping is replaced with the term function. |
| | Introduction to Real Analysis Third Edition | The definition of function as above, only emphasising the use of the word set of ordered pairs. Let $A$ and $B$ be sets. Then a function from $A$ to $B$ is a set $f$ of ordered pairs in $A \times B$ such that for each $a \in A$,there exists a unique $b \in B$ with $(a, b) \in f$. |
| Domain and Range | Calculus with Analytic Geometry, 5th Edition | Domain is the set of members for which a function takes values. Range is the set of values obtained from $f$. |
| | Introduction to Real Analysis Third Edition | By referring to the above definition. The set $A$ or the first set is called the Domain of the function $f$ with notation $D(f) = A$, the second set is Range (notation $R(f)$) with $R(f) \subseteq B$. Thus $D(f) = A$ and $R(f) \subseteq B$. |
| Notation | Calculus with Analytic Geometry, 5th Edition | Functions are expressed with a single letter (e.g. $f$ or $g$ or $F$ etc.). The notation $f(x)$ pronounced $f$ of $x$ or $f$ at $x$ expresses the value given by $f$ at $x$. |
| | Introduction to Real Analysis Third Edition | $f: A \to B$ |
| | Modern Algebra : An Introduction Sixth Edition | Function notation uses Greek letters. Suppose $\alpha$ is a mapping from $S$ to $T$, denoted $\alpha: S \to T$. If $x$ is a member of $S$, then $\alpha(x)$ is expressed as the unique member of $T$ as the pair of $x$. The member $\alpha(x)$ is called the image of $x$ under the mapping $\alpha$. Alternatively, there is a formula for $\alpha(x)$, expressed by $f(x)$. |
| Properties | Introduction to Real Analysis Third Edition | Let $f: A \to B$ is a function $A$ to $B$.<br>1. The function $f$ is injective (or to be one-one) if $x_1 \neq x_2$, maka $f(x_1) \neq f(x_2)$, for $x_1, x_2 \in A$.<br>2. The function $f$ is surjective (or to map A onto B) if $f(A) = B$; that is, if $R(f) = B$. |

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| The Concept of Function | Book | Descriptions |
|---|---|---|
| | | 3. If $f$ is both injective and surjective, then $f$ is said to be bijective. |
| Algebraic Operations on Functions | Calculus with Analytic Geometry, 5th Edition | Let function $f$ and $g$ with $D_f \cap D_g \neq \emptyset$.<br>1. Addition $(f+g)(x) = f(x) + g(x)$<br>2. Substruction $(f-g)(x) = f(x) - g(x)$<br>3. Multiplication $(f.g)(x) = f(x).g(x)$<br>4. Division $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ |

Referring to Table 3, the concept of function used in university-level books emphasises two things, namely correspondence and the set of ordered pairs. The definition in the book Modern Algebra (Durbin, 2009) states the definition of function as a correspondence, so the definition is categorised as static meaning as expressed by (Siu, 1994). The definition in the static context shows function as a correspondence where this definition developed in the 20th century based on set theory. In particular, the definition of function in the book Modern Algebra uses the Dirichlet-Bourbaki definition, where function is defined as a correspondence between two sets that pairs each element of the first set (domain) with exactly one element in the second set (range). In line with the book Modern Algebra, the book Introduction to Real Analysis Third Edition (Bartle & Sherbert, 2000) states the definition of a function as a set of ordered pairs, so that the definition is the definition of a function in a static context (Static Meaning). Thus, the definition of function presented in Scholarly Mathematics refers to the definition of function in a static context (Static Meaning) influenced by set theory.

Furthermore, the book also presents the notions of domain, range, types of functions, and algebraic operations of functions. By paying attention to the definition of function in the static sense, especially based on Bourbaki's definition, the concept of function related to notation, types of functions/ properties of functions, and algebraic operations of functions is defined. In particular, the function notation used refers to the definition of function based on Bourbaki. Function notation is expressed using letters such as f or F.

## B. Function Concept in School Mathematics

The concept of function is one of the fundamental concepts that must be understood by students. Referring to the document for the Kurikulum Merdeka, namely the Decree of the Head of the Education Standards, Curriculum and Assessment Agency of the Ministry of Education, Culture, Research and Technology Number 008/H/KR/2022, the concept of function is part of Algebraic elements where the learning outcomes are phase F on Algebraic elements and Functions for students in grades XI and XII of high school. In addition, in the Kurikulum 2013, the concept of function is also presented at the high school level starting from the definition of function-to-function operations including function composition

and inverse function. For this reason, the data sources in the form of school mathematics textbooks used are book with title Matematika untuk SMA/SMK Kelas XI with authors Susanto et al. (2021) based on the Kurikulum Merdeka and book with title Matematika untuk SMA/MA/SMK/MAK Kelas X Edisi revisi 2016 with authors Sinaga et al. (2016) based on the Kurikulum 2013. The following presents the results of the analysis of data sources in the form of books used in Schools as School Mathematics in Table 4 and Table 5.

Table 4.
Summary of Function Concept Analysis in Kurikulum 2013

| The Concept of Function | Descriptions |
|---|---|
| Definition | The definition is presented through an illustration of a function in the workings of a machine by introducing the terms input and output and students are asked to recall the definition of a function in junior high school, but it is not explicitly stated. |
| Domain and Range | Domain and range are presented in some problems. The definition of domain is expressed as the set of all real numbers $x$ that make the function $f$ definable, while the definition of range is not explicitly stated as domain. |
| Notation | Function notation is explained through an example/illustration, then presented in the form $f(x) = y$ where $y$ is a function of $x$. |
| Algebraic Operations on Functions | If $f$ is a function with domain $D_f$ and $g$ is a function with domain $D_g$, then the algebraic operations of addition, subtraction, multiplication, and division are expressed as follows.<br>1. Addition $(f+g)(x) = f(x) + g(x)$ with domain<br>2. Substruction $(f-g)(x) = f(x) - g(x)$<br>3. Multiplication $(f.g)(x) = f(x).g(x)$<br>4. Division $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ |

Table 5.
Summary of Function Concept Analysis in Kurikulum Merdeka

| The Concept of Function | Descriptions |
|---|---|
| Definition | A function is a relation that connects one member of a set to exactly one member in another set. A function is a more specific relation. |
| Domain and Range | The definitions of domain and range are not stated explicitly, only through the presentation of some problems and illustrations about domain and range. |
| Notation | Functions are explicitly expressed in the form $f(x) = y$, where $f$ is a function, $x$ is an input variable and y is an output variable. |
| Properties | This book does not explicitly present the definitions of injective, surjective, and bijective functions. Students are asked to examine three function diagrams that show injective, surjective, and bijective functions, respectively. |
| Algebraic Operations on Functions | If $f$ is a function with domain $D_f$ and $g$ is a function with domain $D_g$, then the algebraic operations of addition, subtraction, multiplication, and division are expressed as follows.<br>5. Addition $(f+g)(x) = f(x) + g(x)$ with domain<br>6. Substruction $(f-g)(x) = f(x) - g(x)$<br>7. Multiplication $(f.g)(x) = f(x).g(x)$<br>8. Division $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ |

The concept of function presented in both school mathematics books does not directly convey the formal definition of function. The presentation of the definition of function in school mathematics books is done inductively, starting with contextual problems related to the definition of function. Here is one of the concepts of

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functionality delivered to students on the book in Kurikulum 2013.


Figure 4. Function Concept Illustration in book of Kurikulum 2013

By referring to the problems given, in the book published for the Kurikulum 2013, students are invited to understand the definition of function as a relation that pairs one member of a set with exactly one member of another set. However, the book does not emphasise that the relationship must pair every member of a domain, so it has the potential to cause misunderstanding to students. Meanwhile, in the book in the Kurikulum Merdeka, the definition of function is only presented through an illustration of input and output on a machine, but in the next part of the book, the definition of the function is not explicitly presented. In addition to the definition of function, the concepts of domain and range in both books are also presented through a problem that leads to the understanding of domain and range.

Chevallard stated that school mathematics is the result of the development of scholarly mathematics produced by mathematicians (Jamilah et al., 2020). Further explained by (Jamilah et al., 2020) that the result of this development is a form of adjustment based on a series of adaptations through structuring scientific mathematics into mathematics taught at school (school mathematics), in the form of changes in sequence, structure, and form and context. The process of change or adaptation occurs in a transposition process that (Chevallard & Bosch, 2014) stated the noosphere, which is at the level of curriculum compilers to then be adjusted to the order, structure, or form presented in school mathematics books that will be taught to students.

Table 3, Table 4, and Table 5 above present the results of the analysis of the two sources, where Table 3 is the result of the analysis of the concept of function in scholarly mathematics and Table 4 and Table 5 are the results of the analysis in school mathematics. Based on the analysis of Table 3, Table 4, and Table 5, it is found that there is a transposition of knowledge on the concept of function from scholarly mathematics to school mathematics, which occurs in the context and structure and form. In the context, the process of transferring knowledge of the function concept from scholarly mathematics to school mathematics can be seen in how the function concept is constructed or learnt by students. If in scholarly mathematics, the concept of function is presented or built abstractly by using formal definitions, then in school mathematics, the concept of function is constructed through contextual problems, such as the definition of function and domain and codomain. Thus, the concept of function in school mathematics is formed inductively, starting from some specific examples/terms towards the formal definition. The preparation of the concept is related to the achievement of learning

objectives of the function concept by adjusting the level of cognitive development of students.

The construction of function concepts through specific problems is in line with students' cognitive development. High school students in Indonesia have an average age of over 15 years, so that according to the theory of cognitive development put forward by Piaget, they are at the formal operational stage (Ahmad et al., 2016; Blake & Pope, 2008; Hamilton & Ghatala, 1994; Huang, 2021). It is further explained that this stage is the final stage of cognitive development according to Piaget, students have carried out a logical and systematic thinking process through the ability to think hypothetically-deductively, including the ability to think scientifically through the ability to generalize and predict (Huang, 2021). The concept of function as something abstract at the level of scholarly mathematics is transferred into school mathematics by the noosphere through a series of activities to be constructed by students. However, in the book of Matematika untuk SMA/MA/SMK/MAK Kelas X Edisi Revisi 2016, the formal definition of the concept of function is not explicitly stated so that during learning, care is needed so that students get the concept of function, especially the definition of function, correctly.

Another form of didactic transposition on the concept of function from scholarly mathematics to school mathematics is seen in the structure and form of presentation of material in school mathematics books. In scholarly mathematics, the concept of function is presented abstractly, either in the form of definitions, theorems, including some examples presented. While in school mathematics, some concepts of functions are presented through contextual problems or examples of concrete problems related to functions. Based on these contextual problems, students are then invited to construct the concept of function into an abstract form according to scientific mathematics in scholarly mathematics (generalization). However, in some concepts, school mathematics does not explicitly state the concept in question. In addition, in the concept of function, such as the properties or types of functions and emphasis on the requirements for function operations, such as function composition and function inverse that are contained in scholarly mathematics are not contained in school mathematics.

As one of the fundamental concepts in mathematics, the concept of function must be mastered by students. The gap in the knowledge transposition process from teachers to students can have a negative impact on students. In the process of learning functions, the transposition of function concepts from scholarly mathematics to school mathematics through textbooks should at least pay attention to two fundamental aspects of function concepts according to Jibao, namely the basic concepts of functions that must be clear and the application of function concepts (Shao et al., 2021). In line with that, Trujillo et al. (2023) stated that one of the learning difficulties in the concept of function is related to the definition and interpretation/visualization of functions. In the learning process, the concept of function is often given a rule that is determined in the form of algebraic

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expressions (Trujillo et al., 2023) so that some students perceive the concept of function as something abstract and difficult. With the transposition of the function concept, Pettersson (2012) states that the function concept can be viewed as an input-output machine as presented in school mathematics. Trujillo et al. (2023) stated that the function concept can be expressed as a transformation, not just an association between domain and codomain. Thus, the process of transposing the function concept becomes one of the important processes in learning mathematics so that students' mastery of the function concept becomes correct.

## IV. CONCLUSION

The concept of function is one of the fundamental concepts in learning mathematics at school, so it is necessary to pay attention to how the transposition of knowledge about the concept of function from scholarly mathematics to school mathematics. The results of the analysis show that there is a didactic transposition of the concept of function from scholarly knowledge to mathematical knowledge taught in schools in the form of context and structure and form. In context, the function concept is transferred from scholarly mathematics to school mathematics through how the function concept is constructed. While in structure and form, didactic transposition is seen in the presentation of the function concept where the function concept is presented through contextual problems first in accordance with the stage of cognitive development of students. The transposition process as a form of knowledge change becomes one of the important processes in learning. For this reason, in the learning process, it is hoped that the results obtained can be a consideration for designing didactical situations that will be carried out so that the concept of function as one of the fundamental concepts in mathematics can be mastered correctly by students without eliminating the truth of the scientific concept.

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## AUTHOR'S BIOGRAPHY

**Riki Andriatna, S.Pd., M.Pd.**

Born in Cibeureum, August 14, 1988. Teaching staff at Mathematics Education Study Programme Universitas Sebelas Maret. Undergraduate studies in Department of Mathematics Education Universitas Pendidikan Indonesia, graduating in 2012; completed postgraduate studies in Department of Mathematics Education Universitas Pendidikan Indonesia, graduating in 2016.

**Dr. Imam Sujadi, M.Si.**

Born in Semarang, September 15, 1967. Teaching staff at Mathematics Education Study Programme Universitas Sebelas Maret. Undergraduate studies in Mathematics Education Universitas Negeri Semarang, graduating in 1990; completed postgraduate studies in Mathematics Universitas Gadjah Mada, graduating in 2000; and Doctor's Degree in Mathematics Education Universitas Negeri Surabaya, graduating in 2010.