Comparing Realistic Mathematics Education and Cyclical Learning: Effects on Mathematical Communication and Reasoning Skills

Anim[1], Andy Sapta[2*]

[1]Mathematics Education, Universitas Asahan
Jalan Jenderal Ahmad Yani, Asahan, North Sumatra 21216, Indonesia
[1]animfaqot30031991@gmail.com

[2*]Mathematics Education, Universitas Terbuka
Jalan Cabe Raya, Pondok Cabe, Pamulang, Tangerang Selatan, Banten 15437, Indonesia
[2*]andy.sapta@ecampus.ut.ac.id

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Abstrak

Penelitian kuasi-eksperimen ini bertujuan menguji perbedaan efektivitas model Realistic Mathematics Education (RME) dan Pembelajaran Bersiklus dalam meningkatkan kemampuan komunikasi dan penalaran matematis siswa. Melibatkan dua kelas eksperimen di SMP N 2 Kisaran (kelas VIII-4 dan VIII-5) yang dipilih melalui cluster random sampling, penelitian mengumpulkan data melalui instrumen tes yang kemudian dianalisis menggunakan ANOVA. Hasil penelitian menunjukkan bahwa kelompok RME mencapai performa yang secara signifikan lebih tinggi dalam kedua kemampuan dibandingkan kelompok Pembelajaran Bersiklus, dengan selisih 9,5% pada komunikasi matematis (F=4.627, p=0.03) dan 9,0% pada penalaran matematis (F=15.31, p=0.00). Temuan kunci mengungkap bahwa sementara kemampuan awal matematis tidak berinteraksi dengan model pembelajaran dalam memengaruhi komunikasi, interaksi signifikan terdeteksi pada kemampuan penalaran matematis (F=16.54, p=0.00). Kesimpulannya, model RME terbukti lebih unggul dalam mengembangkan kedua kemampuan matematis tersebut, menegaskan pentingnya pemilihan model pembelajaran yang sesuai untuk mencapai hasil belajar yang optimal.
Kata Kunci: Kemampuan Komunikasi Matematis; Kemampuan Penalaran Matematis; Model Pembelajaran Siklus; Pendidikan Matematika Realistik

Abstract

This quasi-experimental study aims to examine the difference in effectiveness between the Realistic Mathematics Education (RME) model and Cyclical Learning in improving students' mathematical communication and reasoning skills. Involving two experimental classes at SMP N 2 Kisaran (classes VIII-4 and VIII-5) selected through cluster random sampling, the study collected data through test instruments, which were then analyzed using ANOVA. The results showed that the RME group achieved significantly higher performance in both abilities compared to the Cyclical Learning group, with a difference of 9.5% in mathematical communication (F=4.627, p=0.03) and 9.0% in mathematical reasoning (F=15.31, p=0.00). Key findings revealed that while initial mathematical ability did not interact with the learning model in influencing communication, a significant interaction was detected in mathematical reasoning ability (F=16.54, p=0.00). In conclusion, the RME model proved to be superior in developing both mathematical abilities, emphasizing the importance of selecting an appropriate learning model to achieve optimal learning outcomes.
Keywords: Cyclical Learning Model; Mathematical Communication Skills; Mathematical Reasoning Skills; Realistic Mathematics Education

I. INTRODUCTION

Mathematics plays a fundamental role in developing students' logical thinking (Pattimukay, 2024) and reasoning abilities (Puspita, Muzdalipah, & Nurhayati, 2023). However, mathematics education in Indonesia still tends to emphasize procedural fluency and formula memorization, often overlooking the development of essential skills such as mathematical communication and reasoning. These skills are crucial not only for academic success but also for applying mathematics in real-world situations (Puspita, Herman, & Dahlan, 2023; Utari et al., 2024).

Despite their importance, students' mathematical communication and reasoning abilities remain low. Challenges include articulating mathematical ideas clearly, interpreting representations, modeling real-world problems mathematically, and making logical inferences. Such difficulties have been consistently reported across different educational levels, including junior high school (Anim et al., 2022; Chasanah et al., 2020; Afifah & Fatmawati, 2024; Syafrizal et al., 2020).

Mathematical communication involves expressing ideas using appropriate symbols and representations, while reasoning involves connecting concepts and justifying solutions. Strengthening these abilities requires instructional approaches that are both student-centered and contextually meaningful. Realistic Mathematics Education (RME) and the Cyclical Learning model are two approaches that aim to foster these competencies through active engagement and reflection. According to Sipakkar & Anim (2024), these skills are interrelated and are best developed through learning models that promote exploration, discussion, and reflection—characteristics present in both RME and cyclical approaches.

RME promotes learning through real-life contexts and encourages students to express, negotiate, and refine their mathematical thinking—thereby enhancing both communication and reasoning (Hamid & Afriansyah, 2024; Nuzula et al., 2023; Ramdhani & Rohaeti, 2018). The Cyclical Learning model similarly supports students in refining their understanding through iterative learning processes, which can reinforce both expressive and analytical thinking (Susanti et al., 2021). However, studies rarely compare the effects of these two models on both communication and reasoning skills within the same context.

This study addresses that gap by comparing the effects of RME and the Cyclical Learning model on junior high school students' mathematical communication and reasoning skills. Additionally, this research seeks to identify which is more effective in fostering critical mathematical competencies—thereby contributing to efforts in improving the quality of mathematics education.

The hypotheses proposed in this study are as follows:

H1: There is a difference in the average mathematical communication skill scores between the two test groups.

H2: There is an interaction between the learning model and students' prior mathematical ability on mathematical communication skills.

H3: There is a difference in the average mathematical reasoning skill scores between the two test groups.

H4: There is an interaction between the learning model and students' prior mathematical ability on mathematical reasoning skills.

II. Method

This study employs a quasi-experimental design with a quantitative approach. A quasi-experimental method was chosen instead of a true experimental design because the research was conducted in a natural school setting where random assignment of students to experimental groups was not feasible. The available classes had already been formed by the school, and ethical as well as administrative constraints prevented the reorganization of students into new groups. In such conditions, the researcher worked with intact, pre-existing classes, which is a key characteristic of quasi-experimental research. Furthermore, while this design does not allow full control over all external variables, it still enables the examination of causal relationships through the application of treatments and comparison of outcomes between groups.

The population consists of all students at SMP N 2 Kisaran in Asahan Regency academic year 2023/2024 even semester, North Sumatra Province. The sampling technique used was cluster random sampling. The selected samples were Class VIII-5 as Experiment Class 1 (35 students) and Class VIII-4 as Experiment Class 2 (38 students). The study was conducted over one month (August 2023), with a total of six sessions (12 teaching hours = 12 x 40 minutes) for each sample class. Experiment Class 1 received treatment using the RME model, while Experiment Class 2 received treatment using the Cyclical Learning model on the Pythagorean Theorem material.

This study uses test instruments designed to measure students' mathematical communication and reasoning abilities. In this study, mathematical communication skills are limited to written communication, which includes expressing mathematical ideas or situations from a given image or graph in their own words in written form, representing a situation in the form of a diagram or graph, and expressing situations using mathematical notations or models. Meanwhile, the indicators for mathematical reasoning skills in this study include drawing logical conclusions, using patterns and relationships to address mathematical situations, and examining the validity of an argument. Figure 1 presents an example of a question used to assess mathematical communication and reasoning skills.

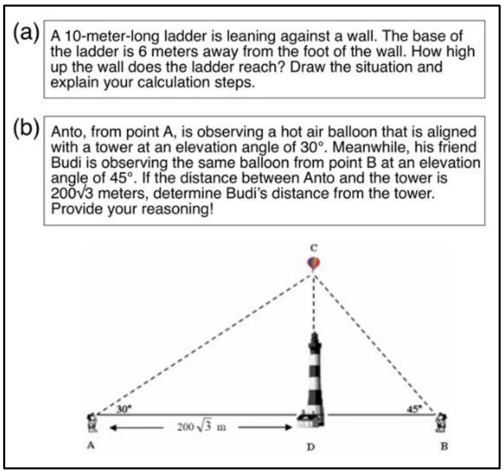

Figure 1. Example question for assessing communication (a) and reasoning skills (b)

The instrument was validated through expert judgment by three mathematics education experts. Based on the assessment, all items were deemed valid, with a Content Validity Index (CVI) score of 0.90, indicating a high level of content validity.

Before the learning process begins, the researcher administers a Mathematics Initial Ability Test (MIAT) consisting of 20 multiple-choice questions with reasoning, aimed at assessing the knowledge students possess prior to the learning process. Students' initial mathematical abilities are assessed through a set of test questions based on previously learned material. The test consists of essay questions and aims to determine whether students' abilities are high, medium, or low before the learning activities, as well as to observe whether there are any changes or improvements in their initial abilities.

The MIAT was not subjected to a try-out, as the questions were based on previously taught material and reviewed by mathematics education experts to ensure content validity. The test was used solely for categorizing students' initial abilities prior to the intervention. Based on expert judgment, the instrument obtained a CVI score of 0.92, indicating a high level of content validity

Students are divided into three categories—high, medium, and low achievers—based on their MIAT scores. The criteria for grouping students (Riyanto & Siroj, 2014) are determined using the mean ($\bar{x}$) and standard deviation (SD), as displayed in Table 1.

Table 1.
Criteria for Grouping Students' Abilities Based on MIAT

Ability	Criteria
High	Students with an MIAT score of $\geq \bar{x} + SD$
Medium	Students with an MIAT score between $< \bar{x} + SD$ and $> \bar{x} - SD$
Low	Students with an MIAT score of $\leq \bar{x} - SD$

Source: (Riyanto & Siroj, 2014)

After the learning process is completed, the researcher administers a posttest to evaluate students' mathematical communication and reasoning abilities, each consisting of four essay questions. These skills will be measured based on students' ability to solve problems that include indicators of mathematical communication and reasoning skills. The assessment instruments were validated by expert validators before being used in field testing.

The data on students' mathematical communication and reasoning skills were analyzed based on two independent variables: the learning approach (RME and Cyclical Learning) and students' prior mathematical ability levels (high, medium, low). Prior mathematical ability was not treated as a continuous covariate, but rather as a categorical variable, since students were grouped into three distinct levels based on their MIAT scores. Therefore, instead of using ANCOVA, which is more appropriate when controlling for a continuous covariate, this study used a two-way ANOVA to examine the main effects of the learning model and prior ability level, as well as their interaction effects on student outcomes. Before hypothesis testing, assumptions of normality and homogeneity were confirmed. All analyses were conducted using Microsoft Excel and SPSS version 26.

The normality test is aimed at determining whether the data from each learning group are normally distributed. The normality test is carried out using the Liliefors test. The testing criterion is that if L0 ≤ Ltable, then H0 is accepted, meaning Ha is rejected. In other words, if L0 ≤ Ltable, the data are normally distributed. The significance level used is 0.05 (Sudjana, 2005).

Meanwhile, the homogeneity test is conducted to decide whether the variances of the two groups are the same or different. The homogeneity testing criterion according to Sudjana (2005) state that H_0 is accepted if the calculated test statistic (F-ratio) falls within the range where the variances are considered equal, meaning there is no significant difference between the variances of the groups. This is determined when the p-value is greater than the significance threshold (0.05).

The statistical model for this research experiment is (Sudjana, 1991):

$$Y_{ijk} = \tau + A_i + B_j + (AB)_{ij} + \epsilon_{k(ij)} \quad (1)$$

Where: Y_{ijk} = dependent variable (students' communication skills or mathematical reasoning skills), τ = true mean (a constant value), A_i = learning model (RME or cyclical learning), B_j = students' prior mathematical ability (high, medium, or low), $(AB)_{ij}$ = interaction between the learning model and students' prior ability, and $\epsilon_{k(ij)}$ = error term for the k-th observation in the treatment combination (ij).

If a significant main effect or interaction was found in the two-way ANOVA, Tukey's Honestly Significant Difference (HSD) test was used as a post hoc procedure to determine which group differences were statistically significant. This method was chosen to control for Type I error in multiple comparisons.

III. Result and Discussion

A. Result

Table 2 shows the MIAT measurement results from both experimental classes. The results indicate that the Experimental Class 2, taught using the Cyclical Learning model, achieved a higher MIAT score (39.18) compared to the Experimental Class 1, taught using the RME model (37.23). However, this difference needs to be statistically tested to determine its significance.

To conduct a difference test on the MIAT data from both classes, prerequisite tests such as normality and homogeneity tests are required. The results of the normality test for MIAT scores from both classes are presented in Table 3. These results indicate that both experimental classes show $L_0 < L_{table}$, with 0.098 < 0.150 for experimental class 1 and 0.079 < 0.144 for experimental class 2. Therefore, H_0 is supported, and it can be stated that the samples from both classes come from populations with a normal distribution.

Table 2.
Students' MIAT Results

Statistics	Experiment Class 1 (RME Model)	Experiment Class 2 (Cyclical Learning Model)
N	35	38
Mean ($\bar{x}$)	37.23	39.18

Statistics	Experiment Class 1 (RME Model)	Experiment Class 2 (Cyclical Learning Model)
Std. Dev.	10.06	11.91

Table 3.
Normality Test Results for Students' MIAT Scores

Groups	Liliefors Test		Decision
	L_0	L_{table}	
Experiment Class 1	0.098	0.150	Normal
Experiment Class 2	0.079	0.144	Normal

Note: α = 0.05

The results of the homogeneity test for the MIAT data are shown in Table 4. The results indicate that $F_{obs} < F_{table}$ (1.400 < 1.760), meaning H_0 is accepted, which suggests that the variances of the two groups are equal or homogeneous. Since both prerequisite tests have been satisfied, an independent samples t-test can be carried out to decide whether the two test groups have equivalent initial mathematical abilities.

Table 4.
Homogeneity Test Results for Students' MIAT Scores

Groups	Variance	F_{obs}	$F_{table(0.05,37,:}$	Decision
Experiment Class 1	101.18	1.400	1.760	Homogenous
Experiment Class 2	141.94			

Note: α = 0.05

Based on Table 5, it can be stated that there is no substantial difference in the MIAT scores between the two groups, as indicated by the Sig. (2-tailed) value of 0.453, which is greater than the significance threshold of 0.05. Therefore, it can be concluded that the two experimental groups have equivalent initial mathematical abilities and can be used as comparison classes for the implementation of the learning models.

Table 5.
Independent Samples t-test on Students' MIAT Scores

Statistics		t-test for Equality of Means						
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
							Lower	Upper
MIAT Score	Equal variances assumed	-.754	71	.453	-3.259	4.320	-11.873	5.355
	Equal variances not assumed	-.760	70.488	.450	-3.259	4.290	-11.815	5.296

Note: α = 0.05; df = degree of freedom

Table 6 presents the grouping of students based on their MIAT scores in the two experimental groups. Based on this data, the researcher formed heterogeneous student learning groups consisting of 7 groups, with each group comprising students of high, medium, and low abilities. This arrangement is intended to ensure that all groups can collaborate effectively with their members.

Table 6.
Student Grouping Based on MIAT Scores

Groups	Total Students' Based on MIAT		
	High	Medium	Low

Groups	Total Students' Based on MIAT		
	High	Medium	Low
Experiment Class 1	6	24	5
Experiment Class 2	6	25	7
Total	12	49	12

Table 7 shows the results of the mathematical communication skill measurements from both test groups. Descriptively, the class using the RME model (Experiment Class 1) had an average mathematical communication skill score 9.5% higher than the class using the cyclical learning model (Experiment Class 2).

Meanwhile, to test Hypotheses 1 (H1) and 2 (H2)—which concern the substantial difference in mathematical communication abilities between the two test groups and the interaction between MIAT and the learning model on mathematical communication abilities—prerequisite tests for normality and homogeneity were conducted. Tables 8 and 9 show the outputs of the normality and homogeneity tests on the mathematical communication skill data from both test groups. The normality test results for the mathematical communication data from both groups indicate that the data is normally distributed, as $L_0 < L_{table}$. Meanwhile, the homogeneity test results show that the mathematical communication skills data have homogeneous variance, as $F_{obs} < F_{table}$. Since both prerequisite tests have been met, an ANOVA test can be conducted.

Table 7.
Results of Mathematical Communication Skills

Statistics	Experiment Class 1 (RME Model)	Experiment Class 2 (Cyclical Learning Model)
N	35	38
Mean ($\bar{x}$)	10.97	9.45
Std. Dev.	1.98	1.94

Table 8.
Normality Test Results for Students' Mathematical Communication Skills

Groups	Liliefors Test		Decision
	L_0	L_{table}	
Experiment Class 1	0.109	0.150	Normal
Experiment Class 2	0.087	0.144	Normal

Note: α = 0.05

Table 9.
Homogeneity Test Results for Students' Mathematical Communication Skills

Groups	Variance	F_{obs}	$F_{table(0.05,37,}$	Decision
Experiment Class 1	3.91	1.030	1.760	Homogenous
Experiment Class 2	3.77			

Note: α = 0.05

The two-way ANOVA statistical test is applied to examine whether there is a significant difference in mathematical communication abilities between students taught using the RME model and those taught using the Cyclical Learning model (H1), as well as whether there is an interaction between the learning model (RME and Cyclical Learning) and students' prior mathematical ability levels on their mathematical communication abilities (H2). The results of the two-way ANOVA calculation for students' mathematical communication abilities are summarized in Table 10.

Table 10.
ANOVA Test Results on Mathematical Communication Skills

Parameter	*df*	Sum of Squares	Mean Square	F	$F_{table\ (0.05)}$
Learning Model	1	31.51	31.510	4.627	3.990
MIAT	2	79.03	39.515	5.802	3.130
Learning Model*MIAT	2	16.96	8.475	1.240	3.130
Error	67	456.46	6.810	-	-
Total	72	302.03	-	-	-

Note: α = 0.05; df = degree of freedom

Based on Table 10, the ANOVA results suggest that there is a substantial difference in the average mathematical communication skill scores between the two test groups. This is evidenced by the F-value for the learning model (F = 4.627), which exceeds the critical value (F(table, 0.05) = 3.990). Therefore, the null hypothesis (H_0) is rejected, supporting the hypothesis (H1) that the learning model significantly influences mathematical communication skills. However, the interaction between the learning model and students' MIAT was not significant, as the F-value for the interaction term (F = 1.240) is less than the critical value (F (table, 0.05) = 3.130). Consequently, the null hypothesis for the interaction effect is accepted, indicating no significant interaction between the learning model and MIAT in influencing mathematical communication abilities. These findings suggest that while the learning model independently affects students' mathematical communication skills, its effectiveness is not contingent upon students' prior mathematical ability.

However, based on Figure 2, there appears to be an interaction between the learning model and students' MIAT on mathematical communication abilities. The crossing lines suggest that the effectiveness of the learning model varies depending on the MIAT level. Specifically, students with low MIAT tend to perform better using the Cyclical Learning model, while those with medium and high MIAT achieve higher scores with the RME model. This visual trend implies that the relationship between the learning model and mathematical communication abilities is not consistent across all levels of MIAT. Although the ANOVA results indicate that the interaction effect is not statistically significant, the observed pattern in the graph highlights the potential need for differentiated teaching strategies tailored to students' initial abilities. This descriptive insight suggests that further investigation with additional data or methodological refinement may be necessary to confirm this trend.

Although Figure 2 visually suggests a potential interaction between learning model and MIAT level, this effect was not statistically significant in the ANOVA. This discrepancy may be due to insufficient statistical power, small sample size, or high within-group variability, which can obscure interaction effects even when visual trends are present.

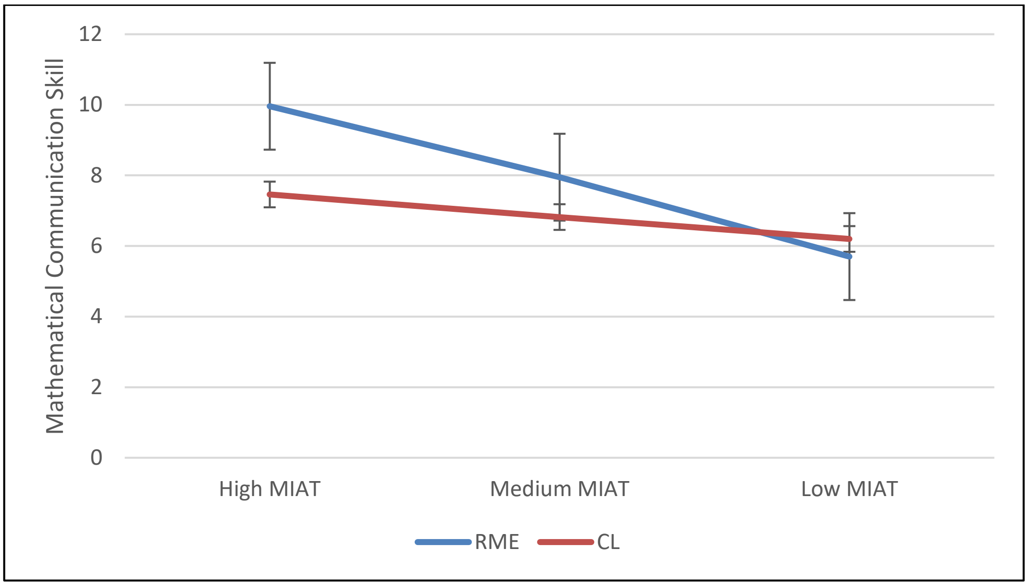

Figure 2. Interaction of learning models with MIAT on students' mathematical communication skills

Table 11 shows the results of the mathematical reasoning skill measurements from both test groups. Descriptively, the class using the RME model (Experiment Class 1) had an average mathematical reasoning skill score 9.0% higher than the class using the cyclical learning model (Experiment Class 2).

Table 11.
Results of Mathematical Reasoning Skills

Statistics	Experiment Class 1 (RME Model)	Experiment Class 2 (Cyclical Learning Model)
N	35	38
Mean ($\bar{x}$)	10.91	9.47
Std. Dev.	1.9	1.96

To test Hypotheses 3 (H3) and 4 (H4)—which examine the significant difference in mathematical reasoning abilities between the two test groups and the interaction between MIAT and the learning model on mathematical reasoning abilities—prerequisite tests for normality and homogeneity were conducted. The outputs of the normality and homogeneity tests for the mathematical reasoning skill data from both test groups are presented in Tables 12 and 13. The normality test results show that the data is normally distributed, as $L_0 < L_{table}$. The homogeneity test results indicate that the variance in mathematical reasoning skills is homogeneous, as $F_{obs} < F_{table}$. Since both prerequisite tests have been satisfied, an ANOVA test was performed.

Table 12.
Normality Test Results for Students' Mathematical Reasoning Skills

Groups	Liliefors Test		Decision
	L_0	L_{table}	
Experiment Class 1	0.141	0.150	Normal
Experiment Class 2	0.095	0.144	Normal

Note: α = 0.05

Table 13.
Homogeneity Test Results for Students' Mathematical Reasoning Skills

Groups	Variance	F_{obs}	$F_{table(0.05,37,}$	Decision
Experiment Class 1	3.61	1.060	1.760	Homogenous
Experiment Class 2	3.82			

Note: α = 0.05

The two-way ANOVA statistical test is used to examine whether there is a significant difference in mathematical reasoning skills between students taught using the RME model and those taught using the cyclical learning model (H3), as well as whether there is an interaction between the learning model (RME and cyclical learning) and students' initial mathematical ability levels (high, medium, and low) on their mathematical reasoning skills (H4). The results of the two-way ANOVA calculation for students' mathematical reasoning skills are shown in Table 14.

Table 14.
ANOVA Test Results on Mathematical Reasoning Skills

Parameter	*df*	Sum of Squares	Mean Square	F	$F_{table\ (0.05)}$
Learning Model	1	37.81	37.810	15.310	3.990
MIAT	2	81.69	40.845	16.540	3.130
Learning Model*MIAT	2	17.31	8.655	3.500	3.130
Error	67	165.22	2.470	-	-
Total	72	302.03	-	-	-

Note: α = 0.05; df = degree of freedom

Based on Table 14, the outputs of the ANOVA for mathematical reasoning skills show that there is a substantial difference between the two test groups. The F value for the Learning Model is 15.310, which exceeds the critical F value of 3.990, indicating that the learning model significantly affects mathematical reasoning skills. Additionally, the F value for the interaction between the learning model and MIAT is 16.540, which is greater than the critical F value of 3.130, suggesting a substantial interaction between the learning model and students' initial mathematical ability in affecting their mathematical reasoning skills. These findings support both hypotheses H3 and H4, indicating that there is a difference in mathematical reasoning abilities between the test groups and that the effect of the learning model on these skills is affected by the students' prior mathematical comprehension.

This result is supported by Figure 3, which shows a crossing between the lines representing the RME and CL models. The graph indicates that students with low MIAT scores perform better when taught using the cyclical learning model (CL), while students with medium and high MIAT scores show better performance when taught using the RME model. This interaction highlights how students' prior mathematical ability (MIAT) influences the effectiveness of the learning model in enhancing their mathematical reasoning skills. Thus, the graph visually supports the statistical findings that the learning model and students' prior mathematical ability have a combined effect on improving students' reasoning skills.

To further investigate the significant differences found in the two-way ANOVA, a Tukey HSD post hoc test was conducted. The results showed that for students with high MIAT scores, those taught using the RME model had significantly higher mathematical reasoning scores than those taught using the Cyclical Learning model, with a mean difference of 2.41 ($p < 0.05$). Similarly, for medium MIAT students, the RME group outperformed the CL group with a mean difference of 1.06, though the statistical significance should be interpreted cautiously and verified with the Tukey critical value. Conversely, among low MIAT students, the CL group slightly outperformed the RME group with a mean difference of −0.23, but this difference was not statistically significant ($p > 0.05$). These findings confirm that the effectiveness of the learning model is influenced by students' initial mathematical ability, supporting the interaction effect observed in the ANOVA and illustrated in Figure 3.

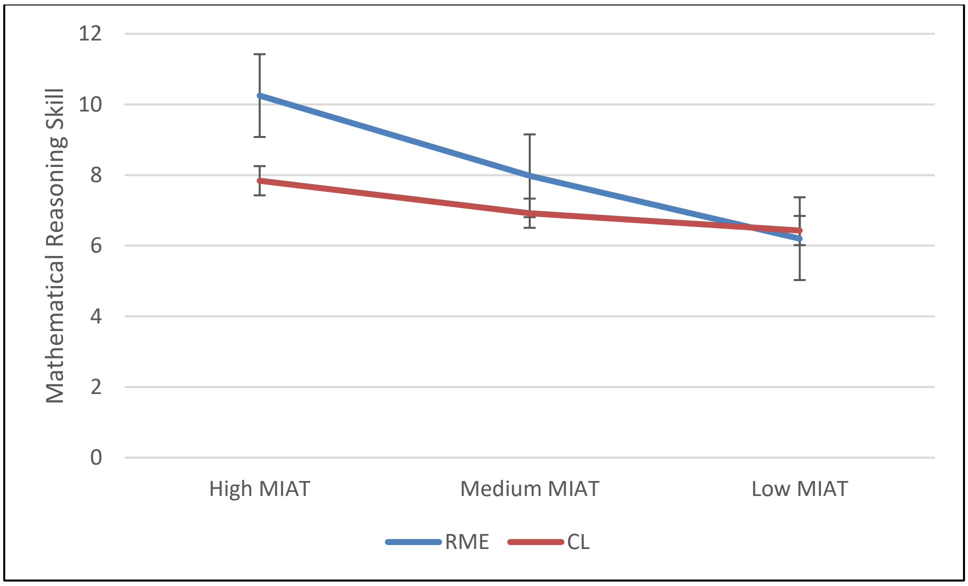

Figure 3. Interaction of learning models with MIAT on students' mathematical reasoning skills

B. Discussion

Based on the results, the RME model is more effective in optimizing students' mathematical communication compared to the Cyclical Learning model. This aligns with Nuzula et al. (2023), which asserts that RME encourages students to express their ideas in their own words, thereby improving their mathematical communication abilities. The principles underlying RME, such as guided rediscovery and progressive mathematization, promote a deeper understanding of mathematical concepts, which, in turn, enhances students' ability to articulate their thoughts and reasoning. Additionally, the interactive nature of RME encourages students to engage in discussions, share their reasoning, and collaborate with peers—key components of mathematical communication (Sibarani et al., 2022).

The RME model is likely more effective than the Cyclical Learning model in enhancing students' mathematical communication skills because it emphasizes the use of real-world contexts and practical applications, such as calculating distances or solving geometry-related problems. This approach makes the material more relevant and motivates students to express their understanding clearly. Additionally, RME adopts a constructivist approach that encourages students to actively explore and reflect on contextual problems, fostering discussions and explanations that naturally enhance communication skills. The collaborative learning aspect of RME later supports this by requiring students to articulate their reasoning and justify their solutions during group work. Moreover, RME focuses on conceptual understanding rather than rote memorization, enabling students to explain the Pythagorean theorem through geometric proofs or visual representations, thereby strengthening their ability to communicate abstract ideas. This approach also aligns with the cognitive development of junior high school students, as it transitions gradually from concrete, contextual problems to abstract concepts, helping students articulate their understanding at each stage.

The statistical testing of the interaction between the learning model and students' prior mathematical skills shows no significant interaction, even though the scores indicate that students with low initial mathematical skills achieve better mathematical communication with the cyclical learning model compared to those with the RME model. This lack of statistical significance can be attributed to several factors. Firstly, the sample size may not be large enough to detect a statistically significant interaction effect, as smaller sample sizes reduce the test's power. Secondly, high variability within the groups could dilute the observable differences between the learning models, particularly for students with low initial mathematical skills. Additionally, there may be an overlap in performance between the groups, potentially due to individual differences in learning styles or external factors such as classroom dynamics, which minimizes the apparent differences. Furthermore, while the cyclical learning model may descriptively benefit students with lower initial skills by providing iterative reinforcement, the effect size might not be strong enough to achieve statistical significance. The instrument used to measure mathematical communication skills may also lack the sensitivity needed to detect subtle differences caused by the interaction. Finally, the cyclical learning model might naturally cater to the needs of students with lower initial skills, but its benefits may not be pronounced enough when compared to the RME model, which is more effective for students with medium or high initial skills. Together, these factors contribute to the observed phenomenon.

Based on the measurements of the mathematical reasoning skill test, students taught with the RME model generally achieved higher scores than those taught with the cyclical learning model. Previous research indicates that RME encourages active participation, allowing students to construct their own knowledge through exploration and problem-solving, which is crucial for developing reasoning skills (Ardiniawan et al., 2023; Sari Umar & Zakaria, 2022). Furthermore, the use of manipulatives and real-world problems in RME promotes deeper cognitive engagement, enabling students to visualize and understand abstract mathematical ideas (Sari Umar & Zakaria, 2022; Ulandari et al., 2019).

The interaction analysis reveals that there is an interaction between the learning model and students' prior mathematical skills in relation to their mathematical reasoning skills. Specifically, students with low initial mathematical skills perform better in mathematical reasoning when taught using the Cyclical Learning model compared to the RME model. This phenomenon can be explained by several factors. Firstly, the cyclical learning model may provide a more structured and repetitive learning process, which benefits students with lower initial skills by reinforcing basic concepts through repeated exposure and practice. This iterative approach allows students to build a stronger foundation in mathematical reasoning, as they are gradually guided through different stages of learning. The cyclical nature of the model gives students more opportunities to reflect, revise, and improve their understanding, making it easier for them to grasp foundational reasoning skills. In contrast, the RME model focuses more on real-world contexts and constructivist learning, which may be more effective for students with higher initial mathematical abilities who can handle abstract and complex problem-solving. For students with low initial skills, however, the abstract nature of RME may pose challenges in understanding mathematical reasoning without a solid foundational grasp of basic concepts. Additionally, the cyclical model's emphasis on review and repetition can help students with low initial mathematical skills to strengthen their cognitive scaffolding, gradually improving their ability to reason mathematically. Therefore, the cyclical learning model might be more suited to the developmental needs of these students, allowing them to gradually improve their reasoning skills through structured practice and guided learning.

Although the statistical analysis showed that the interaction effect between the learning model and students' initial mathematical abilities on mathematical communication skills was not significant, Figure 2 visually suggests a possible interaction, as indicated by the crossing of lines between the RME and Cyclical Learning groups across MIAT levels. This apparent contradiction can be attributed to several factors. First, statistical non-significance does not necessarily mean no interaction exists—rather, it may indicate that the sample size was not large enough to detect a significant interaction, especially if group differences are small or variability within groups is high. Second, visual trends in graphs can sometimes reflect patterns that are suggestive but not strong enough to reach statistical significance, particularly when the interaction effect size is modest. Lastly, error bars and within-group variability play a key role; overlapping error bars in Figure 2 may visually diminish the reliability of the crossing pattern. Thus, while the graphical representation offers descriptive insights, these should be interpreted with caution and supported by larger samples or follow-up studies to determine whether the observed trend represents a meaningful interaction.

This study applied heterogeneous grouping in the learning process. The researcher created groups of students consisting of those with low, medium, and high initial mathematical skills. One of the primary advantages of heterogeneous grouping is the promotion of differentiated learning. Zulaikha & Laeli (2023) emphasizes that differentiated learning aims to tailor the educational process to meet individual learners' needs, thereby enhancing their potential based on their readiness, interests, and profiles. This customization is crucial in adaptive learning models, where the goal is to provide personalized educational experiences. The diverse perspectives and skills that heterogeneous groups bring can lead to richer discussions and a deeper understanding, as students learn from each other's unique viewpoints and experiences.

The effectiveness of the RME approach in this study can be closely tied to constructivist learning theory (Armiati & Sari, 2022), which emphasizes that knowledge is actively constructed by learners through meaningful experiences. RME aligns with this perspective by engaging students in problem-solving activities rooted in real-world contexts, encouraging them to explore, represent, and discuss mathematical ideas collaboratively. Within heterogeneous groups, students construct understanding not only through individual reflection but also through social interaction and dialogue, which are central to constructivist principles. Higher-ability students can articulate their thinking and clarify concepts, while lower-ability students benefit from peer explanations and contextualized learning scenarios (Adhikary, 2023).

The results of this study offer practical guidance for teachers seeking to enhance students' mathematical communication and reasoning skills through differentiated instruction. For students with lower initial mathematical abilities, teachers can implement cyclical learning strategies, such as structured repetition, guided practice, and staged feedback loops to reinforce foundational concepts. These students benefit from frequent opportunities to review and gradually build their reasoning skills. Meanwhile, students with medium to high initial abilities may respond better to RME approaches that emphasize problem-solving in real-world contexts, collaborative learning, and abstract thinking. Teachers should be encouraged to use flexible grouping, tiered assignments, and scaffolded questioning to align tasks with students' readiness levels. Additionally, professional development programs should focus on equipping teachers with the skills to analyze students' prior knowledge, adjust instructional methods accordingly, and use formative assessments to guide instruction.

IV. Conclusion

This study examines how different learning models affect students' mathematical communication and reasoning skills, considering their prior mathematical abilities. The results support Hypothesis 1 (H1), which shows a substantial difference in mathematical communication abilities between the two learning models. However, Hypothesis 2 (H2) is not supported, as there was no substantial interaction between the learning model and students' prior mathematical abilities in influencing communication abilities, even though a visual trend suggested it. For Hypothesis 3 (H3), the study found a substantial difference in mathematical reasoning skills between the two groups, with the Cyclical Learning model being more effective for students with low initial mathematical skills, while the RME model worked better for those with medium to high skills. Hypothesis 4 (H4) is also supported, showing that the learning model's effect on reasoning skills is influenced by students' prior abilities. In conclusion, the study suggests that both learning models can enhance students' skills, but the choice of model should take students' prior abilities into account, particularly for enhancing reasoning skills.

While the findings provide important insights, this study is not without limitations. The relatively small sample size may limit the generalizability of the results, especially regarding the interaction effects observed visually but not supported statistically in the ANOVA. Additionally, potential confounding variables, such as differences in teacher implementation style, classroom environment, or student motivation, may have influenced the outcomes. To gain a deeper understanding of why certain interactions occur—particularly the differences in how students with varying initial abilities respond to different learning models—future research should consider using longitudinal designs to track learning growth over time. Incorporating mixed-methods approaches, such as classroom observations, student interviews, or reflective journals, could also provide richer insights into the mechanisms behind these interactions and how instructional strategies can be tailored more effectively to diverse learners.

References

Adhikary, R. P. (2023). An Analysis of the Challenges and Strategies of English Language Teachers in Teaching Mixed-Ability Classes: A Qualitative Inquiry. *Revista Multi-Ensayos*, *9*(18), 3–20. https://doi.org/10.5377/multiensayos.v9i18.16427

Afifah, D. S. N., & Fatmawati, N. I. (2024). Student Reasoning Based on Mathematical Ability in Solving HOTS Problems through Differentiated Learning. *Athena: Journal of Social, Culture and Society*, *2*(1), 314–324. https://doi.org/10.58905/athena.v2i1.232

Anim, A., Saragih, S., Napitupulu, E. E., & Sari, N. (2022). Analysis of Student Answer Process on Mathematical Communication Skills Through Process Oriented Guided Inquiry Learning (POGIL) Model. *Mathline: Jurnal*

Matematika dan Pendidikan Matematika, *7*(1), 66–76. https://doi.org/10.31943/mathline.v7i1.245

Ardiniawan, D. Y., Subiyantoro, S., & Kurniawan, S. B. (2023). Effectiveness of the RME (Realistic Mathematical Education) Approach to Learning Achievement in View of Students' Mathematic Reasoning. *QALAMUNA: Jurnal Pendidikan, Sosial, Dan Agama*, *14*(2), 783–800. https://doi.org/10.37680/qalamuna.v14i2.3520

Armiati, A., & Sari, R. P. (2022). Local instructional theory of derivative topics based on realistic mathematics education for grade xi senior high school students. *Jurnal Pendidikan Matematika (JUPITEK)*, *5*(2), 71–82. https://doi.org/10.30598/jupitekvol5iss2pp71-82

Chasanah, C., Riyadi, & Usodo, B. (2020). Analysis of Written Mathematical Communication Skills of Elementary School Students. *Proceedings of the 3rd International Conference on Learning Innovation and Quality Education (ICLIQE 2019)*. https://doi.org/10.2991/assehr.k.200129.082

Hamid, H., & Afriansyah, E. A. (2024). Peningkatan kemampuan berpikir kreatif matematis dengan pendekatan realistic mathematics education berbantuan kahoot ditinjau dari gaya belajar honey-mumford. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *3*(3), 356-371. https://doi.org/10.31980/pme.v3i3.2661

Nuzula, L. H., Eko Nugroho, S., & W. (2023). Mathematical Communication Ability in Core Learning Realistic Mathematics Approach with the Assist of Quizizz. *International Journal of Research and Review*, *10*(2), 720–729. https://doi.org/10.52403/ijrr.20230288

Pattimukay, N. (2024). Behind the Numbers: Revealing the Logical-Mathematical Thinking Abilities of Border College Students. *Mosharafa: Jurnal Pendidikan Matematika*, *13*(4), 1035-1046. https://doi.org/10.31980/mosharafa.v13i4.486

Puspita, T., Muzdalipah, I., & Nurhayati, E. (2023). Kemampuan Penalaran Proporsional pada Materi Perbandingan. *Plusminus: Jurnal Pendidikan Matematika*, *3*(1), 107-116. https://doi.org/10.31980/plusminus.v3i1.1227

Puspita, W. R., Herman, T., & Dahlan, J. A. (2023). Students' Mathematical Literacy in Addressing Contextual Questions Through Adversity Quotient. *Mosharafa: Jurnal Pendidikan Matematika*, *12*(4), 771-782. https://doi.org/10.31980/mosharafa.v12i4.1190

Ramdhani, F. A., & Rohaeti, E. E. (2018). Comparison of mathematical students of smp students between learning using a realistic approach with using open ended approach. *(JIML) Journal of Innovative Mathematics Learning*, *1*(3), 329. https://doi.org/10.22460/jiml.v1i3.p329-334

Riyanto, B., & Siroj, R. A. (2014). Improving Reasoning Ability and Mathematics Achievement Using a Constructivist Approach in High School Students. *Journal: Jurnal Pendidikan Matematika*.

Sibarani, G., Simanjorang, M. M., & Mukhtar, M. (2022). Analysis of Mathematic Communication Difficulties and Student Mathematics Problem Solving in the Application of Realistic Mathematics Education Approach in 10th Grade Sultan Iskandar Muda Senior High School. *Proceedings of the 7th Annual International Seminar on Transformative Education and Educational Leadership, AISTEEL 2022, 20 September 2022, Medan, North Sumatera Province, Indonesia*. https://doi.org/10.4108/eai.20-9-2022.2324677

Sipakkar, T. R. B., & Anim, A. (2024). Analysis of Mathematical Reasoning Ability Based on Problem Based Learning Value Comparison Material. *Holistic Science*, *4*(1), 87–91. https://doi.org/10.56495/hs.v4i1.509

Sudjana. (1991). *Desain dan Analisis Eksperimen*. Tarsito.

Sudjana. (2005). *Metoda Statistika* (5th ed.). Tarsito.

Susanti, P. A., Hutapea, N. M., & Saragih, S. (2021). The Effect of the Application of the 5E Learning Cycle Model on Mathematical Communication Skills in Junior High School Students. *Journal of Educational Sciences*, *5*(2), 257. https://doi.org/10.31258/jes.5.2.p.257-266

Syafrizal, A., Syahputra, E., & Irvan, I. (2020). Differences in Increasing The Ability of Reasoning in Problem Based Learning Model and Computer-Based Group Investigation. *Malikussaleh Journal of Mathematics Learning (MJML)*, *3*(2), 51. https://doi.org/10.29103/mjml.v3i2.2422

Ulandari, L., Amry, Z., & Saragih, S. (2019). Development of Learning Materials Based on Realistic Mathematics Education Approach to Improve Students' Mathematical Problem-Solving Ability and Self-Efficacy. *International Electronic Journal of Mathematics Education*, *14*(2). https://doi.org/10.29333/iejme/5721

Umar, U. P. S., & Zakaria, Z. (2022). The effectiveness of the realistic math education (RME) learning method based on manipulative media in improving the problem-solving abilities of elementary school students. *Ekspose: Jurnal Penelitian Hukum Dan Pendidikan*, *21*(1), 1369–1376. https://doi.org/10.30863/ekspose.v21i1.3405

Utari, R. S., Putri, R. I. I., Zulkardi, Z., & Hapizah, H. (2024). Designing Learning Trajectories to Support Prospective Teachers' Statistical Literacy Skills. *Mosharafa: Jurnal Pendidikan Matematika*, *13*(4), 877-894. https://doi.org/10.31980/mosharafa.v13i4.2546

Zulaikha, I. P. N., & Laeli, A. F. (2023). Differentiated Learning in Diverse Students to Meet Curriculum Targets. *International Social Sciences and*

Humanities, *2*(2), 436–441. https://doi.org/10.32528/issh.v2i2.259

AUTHOR'S BIOGRAPHY

Anim, S.Si, M.Pd.

She is a lecture on mathematics education in Universitas Asahan. Her research interests focus on educational methodologies, particularly in implementing innovative teaching strategies to enhance student engagement and learning outcomes. She is also interested in assessing the effectiveness of these approaches in improving students' understanding, mathematical communication skills, reasoning abilities, and overall academic performance.

Dr. Andy Sapta, M.Pd., M.Si.

He is a lecturer in Educational Technology at Universitas Terbuka. His research interests include mathematics education and error analysis, educational technology with a focus on augmented reality for enhancing learning, and instructional design and evaluation for improving teaching methods and learning materials.