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Learning Trajectories of Different Denominator Fractions with *Ote-ote*

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Abstrak
Pecahan berpenyebut berbeda adalah subtopik pecahan yang sangat berkontribusi dalam menjelaskan konsep dasar pecahan. Namun, bagi siswa sekolah dasar menantang karena didasarkan dari konsep yang membutuhkan representasi beragam. Penelitian ini bertujuan untuk mengembangkan learning trajectories (LT) pada pecahan berpenyebut berbeda dengan konteks Ote-ote. Ote-ote merupakan makanan khas Jawa Timur yang berbentuk mirip dengan lingkaran. Partisipan penelitian ini adalah 26 siswa kelas 5 Sekolah Dasar Negeri Jimbaran Wetan, Sidoarjo. Desain penelitian yang digunakan adalah desain research dengan tahapan: preparation, implementation, dan retropective analysis. Data dikumpulkan melalui lembar kerja siswa, observasi, dan tes. Teknis analisis data menggunakan triangulasi sumber. LT yang dikembangkan terdiri dari empat aktivitas, yaitu: (1) pengenalan pecahan, (2) penjumlahan pecahan berpenyebut sama (3) pecahan senilai, dan (4) penjumlahan pecahan berpenyebut berbeda. Aktivitas LT membangun pemahaman siswa mengenai pecahan berpenyebut berbeda, umumnya dalam kategori memuaskan.
Kata Kunci: pecahan berpenyebut berbeda; hypothetical learning trajectories; Ote-ote

Abstract
Different denominator fractions is a subtopic of fractions that greatly contributes to explaining the basic concept of fractions. However, it is challenging for primary students because it is based on a concept that requires multiple representations. This study aims to develop learning trajectories (LT) on different denominator fractions in the context of Ote-ote. Ote-ote is a typical East Java food shaped similarly to a circle. The participants of this study were 26 grade 5 students from Sekolah Dasar Negeri Jimbaran Wetan, Sidoarjo. The study design used was research design with stages: preparation, implementation, and retrospective analysis. Data were collected through student worksheets, observation, and tests—technical data analysis using source triangulation. The developed LT consists of four activities, namely: (1) introduction of fractions, (2) addition of common denominator fractions, (3) equivalent fractions, and (4) addition of different denominator fractions. The LT activities construct students' understanding of different denominator fractions, generally in the satisfactory category.
Keywords: different denominator fractions, hypothetical learning trajectories, Ote-ote

I. INTRODUCTION

Fraction is an important topic in primary school and at the advanced level. Understanding the topic of fractions in primary school is essential to support students' success in further learning about fractions, such as arithmetic, algebra, geometry and measurement, probability, and statistics (Torres-Peña et al., 2024). Relating to fractions with different denominator fractions can train primary students to apply knowledge about equality and relational thinking (Kalra et al., 2020). The solution requires procedural abilities and the manipulation of fractions to get the same denominator (Cabuquin & Abocejo, 2024). This ability involves a deep understanding of common denominator fractions and mathematical operations, which is often challenging for students due to the complexity of the calculation process and its representation (Copur-Gencturk, 2021).

Unfortunately, primary students' understanding of different denominator fractions is still inadequate (Magfirotin & Amir, 2024). Students need different denominator fractions to add basic fractions operations, such as adding different denominator fractions and the same (Wulandari & Amir, 2022). Students have a lower conceptual understanding than their procedural knowledge (Amuah & Davis, 2023). Concerning learning the addition of fractions, students experience difficulties (Wulandari & Amir, 2022) and experience misconceptions (Jarrah et al., 2022).

The difficulty of understanding fractions with different denominators has been identified for a long time, mainly due to the weak equivalent concept in students (Behr et al., 1983). Simon (1995) introduced hypothetical learning trajectory (HLT) as a teacher's prediction of students' learning process. On the other hand, visual representation is considered crucial to constructing a deeper meaning of fractions (Fuller, 1997). The HLT framework was further developed by Clements and Sarama (2014) in the form of systematic learning trajectories (LT) from concrete to abstract. However, Wijaya et al. (2014) showed a gap between HLT and student response in the context of mathematics tasks. To bridge this gap, culture-based approaches, such as the local food context, have improved students' understanding of fractions (Pramudiani et al., 2022; Afriansyah & Turmudi, 2022). In this case, the local food context developed in this study is *Ote-ote*.

Ote-ote, as a typical Indonesian food, especially in East Java, including Sidoarjo, has a unique shape and is familiar to students; *Ote-ote* presents not only local context but also a visual structure that resembles a circle, thus supporting the representation of fractions in real terms. The shape of *Ote-ote* allows students to imagine different denominator fractions with a circle visualization, thus facilitating the transition from concrete to abstract understanding. This is in line with the opinion of Reinhold et al. (2020) that the use of concrete visual representation, such as local food, can help students understand fractions concepts. The relevance of this context is also in line with the view of Clements and Sarama (2020) that the learning trajectory needs to be supported by meaningful and contextual experiences.

Learning trajectory (LT) is a strategic solution in designing learning different denominator fractions because it allows systematic mapping of the development of students' understanding from concrete to abstract (Clements & Sarama, 2020; Salsabila et al., 2022). Integrating *Ote-ote* into the learning trajectory makes it more meaningful and connected to students' experiences. Thus, this can also help bridge the gap between the hypothetical learning trajectory (HLT) and the reality of students' thinking (Wijaya et al., 2014). Thus, the *Ote-ote*-based learning trajectory offers a solution that is not only systematic but also contextual and rooted in local culture.

Previous studies have developed many LTs in mathematics learning, especially on basic fractions and equivalent fractions Adelia et al. (2022); Febriani et al. (2023), but have not explicitly designed LT for different denominator fractions. Other studies discuss different denominator fractions without using local context (Ramadhan et al., 2022; Rizal et al., 2023). On the other hand, using local contexts, such as local food in mathematics learning, has been done by Ramadhani et al. (2024) and Tarida et al. (2023). However, the context has not been directed toward exploring LT in fractions material. Local context has also been explored in non-fraction materials, such as geometry or statistics Adha et al. (2024) and Sukasno et al. (2024), but did not target the different denominator aspects. Thus, no study has integrated designs for LT for different denominator fractions using the local Indonesian food context, especially *Ote-ote*.

Hence, using the *Ote-ote* model in learning different denominator fractions opens up long-term opportunities for more contextual and meaningful mathematics learning. Local contexts, such as typical foods, strengthen students' connection with the material and improve mathematical understanding. The *Ote-ote* context is flexible enough to be replicated in different regions and other cultural contexts. Thus, this study can contribute to developing sustainable and local culture-based learning.

II. Method

The method used was design research with three stages: preparation, implementation, and retrospective analysis (Bakker, 2004). This study designed an intervention to support students' understanding of the addition concept of different denominator fractions. The study was conducted at Sekolah Dasar Negeri Jimbaran Wetan, a public primary school located in a rural area of East Java, Indonesia. The participants were 26 fifth-grade students, with an average age of 11 years old, from lower-middle socioeconomic backgrounds. During regular mathematics lessons, data collection occurred from 21 October to 2 November 2024. Prior to the study, research approval was obtained from the school principal and classroom teacher, and written informed consent was collected from students' guardians. Students' identities were anonymized to ensure ethical standards, and participation was voluntary without any academic consequences.

The first stage of this study is preparation, which is done by conducting a literature review and taking the last exam data of participants. This literature review serves as the theoretical basis of the addition operation of different denominator fractions, which will be the basis for designing HLT to construct LT consisting of a learning goal, a set of learning tasks, and a hypothesized learning process (Lerman, 2020)

The data collection regarding students' mathematics ability was obtained from the last class exam. In this, mathematics ability (ma) is obtained from the average score of 2 students' last exam, then ma is categorized into three levels: low ($0 \leq ma < 60$), medium ($60 \leq ma < 80$), and high ($80 \leq ma < 100$) (Ayuningtyas et al., 2024). Results during the learning process are collected based on the students' worksheets. The study adjusted the HLT to conjecture students' thinking and designed learning activities and evaluations suitable for students. The activities and task items designed for the developed LT are depicted in Table 1.

Table 1.
Activities and Task Items for Learning Trajectories

Activities		Task items
Activity A: Identifying that fraction $\frac{a}{b}$ means fraction $\frac{1}{b}$ as many a times		
A.1	Cutting *Ote-ote* into $\frac{1}{b}$ with b=2, b=4, b=6, b=8	Cutting *Ote-ote* into $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$
A.2	Forming *Ote-ote* and drawing *Ote-ote* shape according to fraction value $\frac{a}{b}$	Forming *Ote-ote* and drawing *Ote-ote* shape according to fraction values $\frac{1}{2}, \frac{3}{6}, \frac{5}{8}$
Activity B: Identifying the concept of counting addition operation of common denominator fractions $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$		
B.1	Arranging *Ote-ote into* 2 plates with fraction $\frac{a}{b}+\frac{c}{b}$	Arranging *Ote-ote* into 2 plates – $\frac{1}{6}+\frac{3}{6}=$ – $\frac{3}{8}+\frac{2}{8}=$
B.2	Finding the concept of the number of pieces on 2 plates $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$	Finding how to add up the *Ote-ote* pieces on 2 plates – $\frac{1}{6}+\frac{3}{6}=\frac{1+3}{6}$ – $\frac{3}{8}+\frac{2}{8}=\frac{3+2}{8}$
Activity C: Understanding the concept of equivalent fraction $\frac{a}{b}=\frac{a\times c}{b\times c}$		
C.	Arranging the *Ote-ote* models onto 2 plates and comparing them $\frac{a}{b}=\frac{a\times c}{b\times c}$	Cutting the *Ote-ote* model according to the fraction value and then comparing the shape of the cut. – $\frac{4}{8}=\frac{1\times c}{2\times c}$ – $\frac{4}{6}=\frac{2\times c}{3\times c}$ – $\frac{1}{2}=\frac{1\times c}{6\times c}$
Activity D: Understanding the concept of counting addition operation of different denominator fractions through equivalent fraction $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$		
D.1	Finding addition rules of different denominator fractions through equivalent fractions $\frac{a}{b}+\frac{c}{d}=\frac{a\times e}{b\times e}=\frac{c}{d}$	Finding different denominator fractions with equivalent fractions. In this case, there is a cognitive conflict $\frac{1}{2}+\frac{1}{8}=\frac{5}{8}$ $\frac{1}{2}=\frac{4}{8} \rightarrow \frac{1}{8}+\frac{4}{8}=\frac{5}{8}$
D.2	Finding addition of different denominator fractions $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$	Finding different denominator fractions with denominator multiplied $\frac{1}{6}+\frac{1}{2}=\frac{2+6}{12}=\frac{8}{12}$
D.3	Finding addition of different denominator fractions using least common multiple $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{kpk\ bd}$	Counting different denominator fractions with – Least common multiple $\frac{1}{6}+\frac{1}{2}$ – The least common multiple of the denominators 6 and 2 is 6, then $\frac{1}{6}+\frac{1}{2}=\frac{1+3}{6}=\frac{4}{6}$

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In Table 1, there are four activities: Activity A, B, C, and D. Activities A, B, and D consist of sub-activities that are put together as activities to be applied as HLT designs. Each design describes an activity that becomes a tool for understanding the addition of fractions. Each activity has an LT conjecture that distinguishes students from high-ability, medium-ability, and low-ability, which will be presented in Table 2.

Table 2.
Conjecture for Learning Trajectories (LT)

Activity	Conjecture LT
A	– High-ability students can infer the fraction $\frac{a}{b}$ has the meaning of a fraction $\frac{1}{b}$ as many a time – Medium-ability students can deduce the fraction $\frac{a}{b}$ has the meaning of fraction $\frac{1}{b}$ as many a time – Low-ability students can conclude the fraction $\frac{a}{b}$ has the meaning of fraction $\frac{1}{b}$ as many a time
B	– High-ability students can arrange *Ote-ote* and find how to add common denominator fractions – Medium-ability students can arrange *Ote-ote* correctly but cannot find how to add common denominator fractions – Low ability students cannot arrange *Ote-ote* and find how to add common denominator fractions
C	– High-ability students can cut the *Ote-ote* model and compare the number and shape correctly – Medium-ability students can cut the *Ote-ote* model correctly but cannot compare the number and shape – Low-ability students cannot cut the *Ote-ote* model and compare the number and shape
D	– High-ability students can arrange *Ote-ote*, calculate the least common multiple of different denominators, and calculate the number of *Ote-ote* correctly – Medium-ability students can arrange *Ote-ote* and calculate the least common multiple of different denominators correctly but cannot calculate the number of *Ote-ote* – Medium-ability students can arrange *Ote-ote* correctly but cannot calculate the least common multiple of different denominators and cannot calculate the number of *Ote-ote* – Low-ability students cannot arrange the *Ote-ote*, calculate the least common multiple of different denominators and calculate the number of Ote-*ote* pieces not correctly

The second stage is implementation, which consists of two cycles. In the first cycle, a pilot experiment was conducted to identify the feasibility of the initial LT design and collect data needed to revise the LT. In the second cycle, a learning experiment was conducted to collect data needed for this study, specifically on how the developed LT helped students learn different denominator fractions. Data were obtained from students' worksheets, evaluation questions, and observation results during the learning process. In this, the results of the students' worksheets are explained on the students' interesting answers.

The last stage is the retrospective analysis, where the studies compare the HLT with the actual learning process in the learning experiment. The results of the students' worksheets were analyzed on the students' interesting answers. In this, each student was initialed according to their first name. Written results on students' evaluation problems were analyzed on one student at each level (low, medium, and high). This analysis explores the causes and improvements, aiming not for success but to understand how and why the intervention works.

The observation procedure followed five systematic steps: (1) Observing each LT session using a structured observation sheet by two researchers, covering four activities: introduction of fraction, addition of common denominator fractions, understanding equivalent fractions, and addition of different denominator fractions. (2) Directly recording students' problem-solving strategies, using the *Ote-ote* visual model, verbal reasoning, and interaction between students during the activity. (3) Focusing on the relationship between the *Ote-ote* context and mathematical concepts such as equivalent fractions and operations with different denominators. (4) Collaboratively reviewing and discussing the observations after each session to ensure consistency and depth of interpretation. (5) Analyzing the collected observation data to improve the LT design and support the retrospective analysis stage.

Data analysis was conducted qualitatively through source triangulation (Patton, 2002), namely sources from observation, written tests, photo documentation, and interviews. Each source was compared to test consistency and enrich the understanding of LT students. The analysis process followed data reduction, presentation, and verification (Miles et al., 2014). Each stage was compared with the HLT to identify gaps and actual patterns of LT activities produced in constructing an understanding of different denominator fractions in the context of *Ote-ote*.

III. RESULT AND DISCUSSION

In this study, we designed learning trajectories about adding fractions for fifth-grade primary school students, consisting of 4 activities (Activity A-D). Each activity uses a worksheet. In addition, students are provided with *Ote-ote*, *Ote-ote* model, knife, scissors, plate, and food paper. Each activity is explained in the following section.

A. Activity A: Introduction of fraction

Activity A introduces the fraction concept to students by arranging the *Ote-ote* according to the fraction value. This activity also introduces fraction shapes with the help of *Ote-ote*. In activity A.1, students cut Ote-ote with a knife into 2 parts, 4 parts, 6 parts, and 8 parts (Figure 1). In activity A.2, students arranged the *Ote-ote* according to the fraction value (Figure 2). The students gave different answers from activity A.2. The teacher asked them why they answered differently.

Figure 1. Students Cutting *Ote-ote*

Figure 2. Students Arranging *Ote-ote*

Teacher : "Who gave the argument why someone answered 2 and 1?"

Student D : "Because we did not arrange the *Ote-ote* according to the fraction value"

Teacher : "How many pieces of *Ote-ote* do we need to arrange *Ote-ote* $\frac{3}{6}$?"

Student G : "3 pieces"

Teacher : "If we have an *Ote-ote* and cut it to a size $\frac{a}{6}$, how many pieces are needed?"

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Student F : "a times"
Teacher : "So, where did that number come from?"
Students : "From the numerator"

Some students answered 2 pieces of *Ote-ote* for arranging $\frac{1}{2}$ *Ote-ote*, and some answered 1 piece of *Ote-ote*. In this, the students did not understand that $\frac{1}{2}$ *Ote-ote* is the fraction form of $\frac{2}{2}$ *Ote-ote* divided into 2 equal parts because they did not arrange the *Ote-ote* according to the fraction value.

B. Activity B: Identifying the Concept of Counting Addition Operation of Common Denominator Fractions

Activity B aims to find the concept of counting operations in addition to common denominator fractions with *Ote-ote*. Activity B.1, students arrange *Ote-ote* on plate 1 and plate 2. Activity B.2 counts the number of *Ote-ote* on both plates.

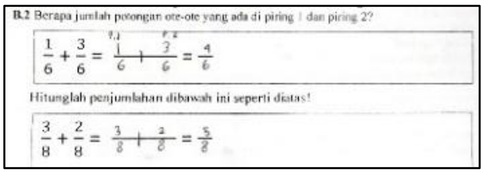

Figure 3. Students A' Answers

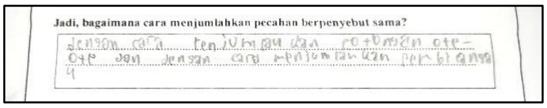

Figure 4. Students A' Conclusion

Figure 3 shows that the student's answers in their students' worksheets understand the concept of counting operations and the addition of common denominator fractions using *Ote-ote*. Next, the core of the HLT is to conclude from Activity B.1 and B.2 to identify the concept of counting addition operation of common denominator fractions using *Ote-ote* (Figure 4). The teacher asks the students to analyze the number of *Ote-ote* on each plate and then add up the number of *Ote-ote*. By adding up the number of *Ote-ote, students* will learn how to count addition operations of common denominator fractions with *Ote-ote*.

C. Activity C: Understand the Concept of Equivalent Fraction

Activity C aims to introduce the concept of equivalent fractions by comparing the shape of the *Ote-ote* model according to the fraction value. This activity also introduces examples of fraction shapes using the *Ote-ote* model. Students cut the *Ote-ote* model with scissors according to the fraction value. Students arrange the two fraction numbers to compare whether they are equivalent or non-equivalent fractions. Students gave different answers from activity C. The teacher asks the students why they answered the question differently.

Teacher : "Who wants to give an argument why there are equivalent fractions and non-equivalent fractions?"
Student D : "Because we did not arrange the *Ote-ote* model according to the fraction value"
Teacher : "Let, $\frac{4}{8}$ and $\frac{1}{2}$, what type of fractions are they?
Student G : "Equivalent fractions"
Teacher : "So, how do you know if the fractions are equivalent?"
Students : "From the shape of the *Ote-ote* model"

Some students answered that the fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions. Some answer that fractions $\frac{4}{6}$ and $\frac{2}{3}$ are

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non-equivalent fractions. Some students do not understand that fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions because we did not arrange the *Ote-ote* model according to the fraction value.

Hence, based on activity C, students understand the concept of equivalent fractions and non-equivalent fractions. This activity allows students to directly observe the concept of equivalent and non-equivalent fractions through real or concrete objects.

D. Activity D: Identifying the Concept of Counting Addition Operation of Different Denominator Fractions

Activity D aims to find the concept of counting addition operation of different denominator fractions through equivalent fractions with the *Ote-ote* model. In activity D.1, students form an *Ote-ote* model with fraction values of $\frac{1}{8}$ and $\frac{1}{2}$, then cut the *Ote-ote* pieces according to the smallest size and add up all the *Ote-ote* pieces (Figure 5).

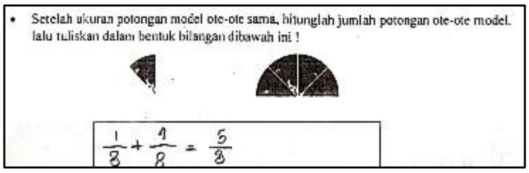

Figure 5. Students C' Answer

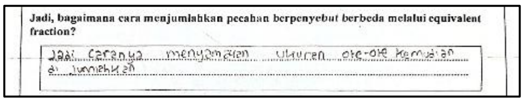

Figure 6. Students C' Conclusion

Figure 5 shows that the student's answers in their students' worksheets understood the concept of counting operations and the addition of different denominator fractions through equivalent fractions with the *Ote-ote* model. Furthermore, the core of the HLT is to conclude activity D.1, to identify the concept of counting addition operation of different denominator fractions through equivalent fractions with the *Ote-ote* model (Figure 6). The teacher asks the students to cut the *Ote-ote* model according to the fraction value. Then, students are asked to equalize the shape of the pieces on the smallest piece; after the size is the same, students count the number of pieces. Thus, students will learn the concept of counting operations in addition to different denominator fractions through equivalent fractions with the *Ote-ote* model.

This is followed by activity D.2, which aims to find the concept of counting addition operation of different denominator fractions with the *Ote-ote* model. Students are asked to form an *Ote-ote* model with fraction values $\frac{1}{6}$ and $\frac{1}{2}$, then equalize the denominators of the numbers, cut the *Ote-ote* model according to the new denominator, and add up the whole.

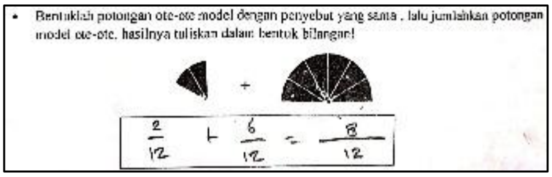

Figure 7. Students C' Answer

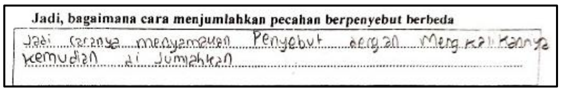

Figure 8. Students C' Conclusion

Figure 7 shows that the student's answers in their students' worksheets understood the concept of counting operations and the addition of different denominator fractions with the *Ote-ote* model. Furthermore, the core of the HLT is to conclude Activity D.2, which aims to identify the concept of counting addition operation of different denominator fractions with the *Ote-ote* model (Figure 8). The teacher asks the students to cut the *Ote-ote* model according to the fraction value. Then, students are asked to equalize the denominator, cut the *Ote-ote* model according to the new denominator, and count the number of pieces. Thus, students can learn the concept of counting addition operations of different denominator fractions with the *Ote-ote* model.

Activity D.3 aims to find the concept of counting addition operation of different denominator fractions using the least common multiple through the *Ote-ote* model. Activity D.3, students are asked to write the largest denominator, then cut the *Ote-ote* model according to the denominator.

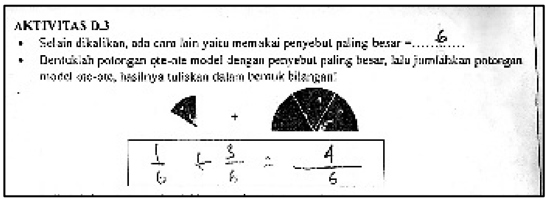

Figure 9 Students C' Answer

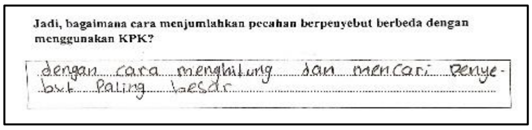

Figure 10. Students C' Conclusion

Figure 9 shows that the students' answers in their students' worksheets understood the concept of counting operations and the addition of different denominator fractions by using the least common multiple through the *Ote-ote* model. Furthermore, the core of the HLT is to conclude Activity D.3, to identify the concept of counting addition operation of different denominator fractions using least common multiple through the *Ote-ote* model (Figure 10). The teacher asks the students to use the largest denominator. Then, the students are asked to equalize the denominators. The students cut the *Ote-ote* model according to the new denominator and counted the number of pieces. Thus, the students will learn how to count operations and add different denominator fractions using least common multiples through the *Ote-ote* model.

Figures 1 and 2 show that *Ote-ote* visuals help students understand fraction concepts when cut evenly but can be hindered if the cut does not match the fraction value. Figures 3 and 4 support understanding the addition of common denominator fractions through symmetrical and consistent cuts. In contrast, Figures 5-10 show a new challenge: disproportionate cuts make it difficult for students to see fraction equivalence. The interview shows that the non-uniform size of the pieces creates a false perception of equivalent fractions. This confirms the importance of proper visual representation in fractions learning.

The teacher gave evaluation problems to evaluate students' understanding of counting addition operations of different denominator fractions. The following figure presents the answers of three students with various abilities, namely high-ability students (Figure 11), medium-ability students (Figure 12), and low-ability students (Figure 13).

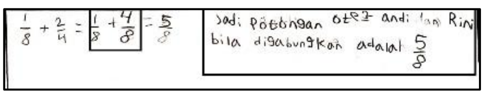

Figure 11. High-ability students

In Figure 11, the high-ability student shows complete understanding. He equated the denominator using least common multiple (LCM) from 8 and 4 to 8, equated the numerator correctly ($\frac{1}{8} = \frac{1}{8}, \frac{2}{4} = \frac{4}{8}$) and did the addition to produce the correct final answer, which is $\frac{5}{8}$.

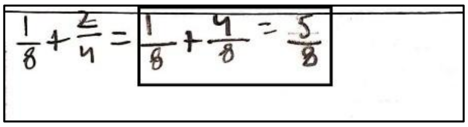

Figure 12. Medium-ability students

In Figure 12, the medium-ability students also correctly equated the denominator and numerator but did not continue the solution to the final summation stage. The main error lies in not concluding the final result even though it is procedurally correct.

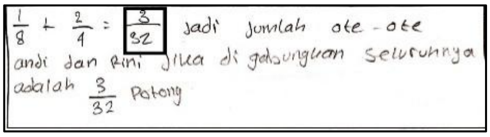

Figure 13. Low-ability students

In Figure 13, the low-ability students directly multiplied the denominator (8×4=32) without finding the LCM and did not equate the numerator to the new denominator. The underlying error is conceptual, namely, not understanding the relationship between different denominators and being unable to represent equivalent fractions.

Figure 11-13 analyze students' errors based on ability level. In Figure 11, high-ability students equated the denominator with LCM, adjusted the numerator, and summed correctly, showing conceptual and procedural understanding. Figure 12 shows medium-ability students who understand the initial step but do not complete the summation, indicating limitations in integrating procedures. Meanwhile, Figure 13 shows low-ability students directly multiplying the denominator without equalizing the numerator, which indicates misconceptions of equivalent fractions.

Student strategies in solving the addition of different denominator fractions vary by ability level (Figures 11-13). The high-ability students' strategy (Figure 11) equated the denominator with LCM, adjusted the numerator, and solved until the correct answer. Meanwhile, the medium-ability students' strategy (Figure 12) follows the correct procedure but does not complete the addition operation. Meanwhile, low-ability students (Figure 13) directly multiply the denominator without equalizing the numerator, showing misconceptions about equivalence. This difference confirms that the student's ability level strongly influences the completeness and accuracy of the strategy.

Although the HLT was systematically designed with *Ote-ote* context to construct an understanding fraction, the results showed a gap between prediction and student response. In activity C, some students failed to cut the *Ote-ote* proportionally, resulting in an inaccurate representation fraction (Figure 5-10). This technical barrier interfered with the achievement of the conceptual goal. In the follow-up activity, low-ability students were still wrong in equalizing the denominator without adjusting the numerator (Figure 13). This finding confirms that HLT implementation does not always run linearly, as it is influenced by students' characteristics and complex classroom dynamics.

Using the *Ote-ote* context in this study helps students understand the addition of different denominator fractions. Visualizing proportional pieces can facilitate the concrete construction of equivalent fractions. High-ability and medium-ability students developed conceptual strategies for equalizing denominators and adjusting numerators. Even low-ability students tried to imitate the visual structure but made procedural errors. This finding confirms that local contexts such as *Ote-ote* support visualization and encourage cognitive engagement and student's emotional state.

This study shows that using the *Ote-ote* context in the learning trajectory can help some students understand the addition of different denominator fractions, especially in the concrete *Ote-ote* model representation stage. However, the results of this study still have limitations, which lie in the limited number of participants and the specific local context, so the results cannot be generalized widely. In addition, the differences in individual responses to the *Ote-ote* context also show that non-instructional factors can affect understanding. On the other hand, Table 3 students' evaluation test on counting addition operation of different denominator fractions.

Table 3.
Students' Achievement

The Number of Students	
Pass (satisfactory)	Fail (unsatisfactory)
22	4

Table 3 shows that most students (22 out of 26) achieved the "satisfactory" category in learning to add common denominator fractions. In this, only 4 students were classified as "unsatisfactory". This shows that the success rate is relatively high. This achievement indicates that designing learning trajectories with *Ote-ote* context can facilitate students' understanding.

The study findings show that the developed LT consists of four activities: introduction of fractions, addition of common denominator fractions, equivalent fractions, and addition of different denominator fractions. Unlike previous studies that predominantly used number lines or symbolic representations Flores et al. (2020); Saili et al. (2023), this LT highlights the local context of *Ote-ote* to bridge concrete experience to mathematical abstraction (Reinhold et al., 2020). Activity equivalent fractions are designed as a crucial step, which is often overlooked in previous LT (Martin & Hunt, 2022), to overcome common misconceptions such as equating denominators without adjusting numerators (Hearne, 2021). *Ote-ote*-based visual models can also be said to offer more meaningful representation flexibility than static models (Pedersen & Bjerre, 2021).

Activities in LT gradually construct students' understanding of adding different denominator fractions with satisfactory average achievement. This finding aligns with studies that show that real context can strengthen understanding of mathematical concepts (Rosmayasari et al., 2023). LT stages that explicitly transition from equivalent fractions to complex sums can effectively address common misconceptions, such as numerator equalization errors (Wulandari & Amir, 2022). This study complements previous research that has not provided concrete stages (Hearne, 2021). Visual *Ote-ote* strengthens conceptual representation through concrete-symbolic linkages Amo-Asante and Bonyah (2023); Flores et al. (2020), while addressing the lack of exploration of local context in fractions learning (Kohen & Orenstein, 2021).

This study has two main implications. Theoretically, the study results reinforce the importance of cultural context-based learning trajectory in mathematics education, especially in understanding the concept of different denominator fractions. The findings extend constructivist learning theory by showing that local contexts such as *Ote-ote* can bridge students' understanding from concrete to symbolic representation. Practically, the learning trajectory developed can be a reference for teachers in developing gradual and meaningful activities to teach fractions operation, especially in contexts close to student life.

IV. Conclusion

This study aims to develop a Learning Trajectory (LT) to support understanding the concept of addition of different denominator fractions in primary students. The LT includes four main activities: introduction of fractions, addition of common denominator fractions, understanding equivalent fractions, and addition of different denominator fractions. Results show that LT activities gradually and systematically construct students' understanding. The solution strategies varied by ability, ranging from using the least common multiple by high-ability students to fundamental misconceptions in low-ability students.

The contribution of this study lies in the development of mathematics learning design based on local context, especially for the addition of different denominator fractions. Practically, the results of this study can be an initial reference for teachers in designing contextual learning relevant to student's experiences. This study enriches the learning trajectory approach by gradually integrating cultural context to construct conceptual understanding. This study does not claim overall effectiveness but presents an initial picture of the potential use of local context in supporting students' understanding. Suggestions for future research include testing this design across a range of student backgrounds and exploring other local contexts in different mathematics topics.

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