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A Systematic Literature Review on Algebraic Reasoning in Mathematics Instruction

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Abstrak

Penalaran aljabar menjadi topik hangat dalam dua dekade terakhir, tetapi siswa sekolah menengah mengalami kesulitan dalam mempelajari hal ini, sebab mengalami transformasi dari pemikiran aritmatika ke pemikiran aljabar yang abstrak. Penelitian ini menganalisis artikel penalaran aljabar dari basis data Scopus berdasarkan tahun penelitian, metode, dan sebaran geografis dalam satu dekade. Penelitian disajikan melalui tinjauan literatur sistematis dengan menggunakan protokol PRISMA meliputi Preferred Reporting Item for Systematic Reviews and Meta-Analytic. Dari proses seleksi, didapatkan 29 artikel yang memenuhi kriteria inklusi dan lolos tiga tahap penyaringan. Penelitian dengan topik ini cenderung mengalami penurunan, terutama dalam 4 tahun terakhir. Metode yang paling sering digunakan adalah metode penelitian kualitatif karena dianggap peneliti sangat cocok untuk meneliti kemampuan berpikir. Dari 8 wilayah yang melakukan penelitian penalaran aljabar, Indonesia merupakan negara yang paling banyak melakukannya. Hal ini dilatarbelakangi rendahnya penalaran aljabar siswa. Peluang penelitian pada topik ini masih terbuka lebar dengan menggunakan penelitian ini sebagai rujukan.
Kata Kunci: Matematika; Penalaran Aljabar; Sekolah Menengah

Abstract

Algebraic reasoning has become a hot topic in the last two decades, but high school students have difficulty in learning it, because they experience a transformation from arithmetic thinking to abstract algebraic thinking. This study analyzes algebraic reasoning articles from the Scopus database based on the year of research, methods, and geographical distribution in one decade. The research is presented through a systematic literature review using the PRISMA protocol including the Preferred Reporting Item for Systematic Reviews and Meta-Analytic. From the selection process, 29 articles were obtained that met the inclusion criteria and passed three stages of screening. Research on this topic tends to decline, especially in the last 4 years. The most frequently used method is the qualitative research method because researchers consider it very suitable for researching thinking skills. Of the 8 regions that conduct algebraic reasoning research, Indonesia is the country that does it the most. This is due to the low algebraic reasoning of students. Research opportunities on this topic are still wide open by using this research as a reference.
Keywords: Algebraic Reasoning; Mathematics; Middle School

I. INTRODUCTION

Mathematics is a study that must be owned and mastered by students as a provision in their lives, during their education and living in society. There are various contents in mathematics, one of which is algebra, a branch of mathematics that uses numbers, letters, and other systematic symbols to express and analyze the relationship between quantity concepts in the form of formulas, equations, and others (Kaliski, 1975). Algebra is the basis for studying various sciences, both in the field of mathematics itself and in other fields of science, such as physics, chemistry, statistics, and communication (NCTM, 2000; Valenti et al., 2024).

Although the concepts of arithmetic operations of addition, subtraction, multiplication, and division of numbers have been introduced at the elementary school level, algebra itself is only formally introduced at the beginning of junior high school (Andriani, 2015). The proportion of algebra at each level of education is presented in Figure 1.

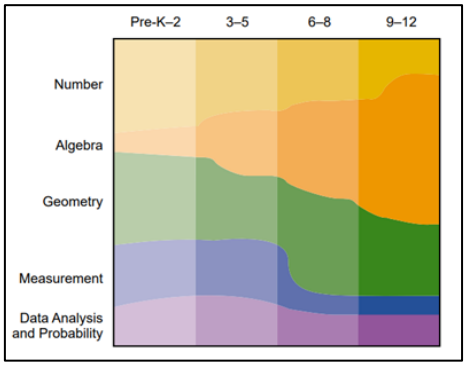

Figure 1. Distribution of Mathematics Content Standards at Each Level of Education (NCTM, 2000)

Based on the image, it can be seen that algebra content is content with an increasing proportion to be studied at every level of education. This can be an indication that algebra at a lower level is used as a basis so that students can more easily understand and master the algebra that will be studied at the next level of education. Therefore, it is very important for students to have a strong foundation in algebra to step into more complex algebra.

In learning algebra, it is necessary to understand symbols, operations, and their rules, which requires students to interpret and manipulate these symbols based on their underlying rules and properties. Such abilities are contained in algebraic reasoning (Andriani, 2015; Ontario, 2013). Algebraic reasoning involves forming generalizations from experience with numbers and calculations, formalizing these ideas using symbols, and exploring the concepts of patterns and functions (Walle et al., 2009). Algebraic reasoning is the process where students generalize mathematical ideas in a given situation, build generalizations through argumentation, and express them formally according to age level (Blanton & Kaput, 2005). Carpenter & Levi, 2000 defines algebraic reasoning as the activity that a person does in building generalizations and using symbols to represent mathematical ideas and solve problems.

In recent decades, the development and use of reliable algebraic reasoning has been considered an essential component for success in mathematics (Kaput, 2000). Through algebraic reasoning, students can understand complex mathematical ideas (concepts or understanding of mathematics that are deep and can be used in many situations, not just memorizing formulas and performing procedures, but rather understanding why

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and how things work in mathematics) (Ontario, 2013). There are several forms of algebraic reasoning, including (a) the use of arithmetic as a domain to express and formulate generalizations (generalized arithmetic), (b) generalizing numerical patterns to describe functional relationships (functional thinking), (c) modeling as a domain for expressing and formulating generalizations; and (d) generalize mathematical systems drawn from calculations and relationships (Blanton & Kaput, 2005).

Although algebraic reasoning has an important role, unfortunately, many students still have difficulty understanding and applying algebra, especially middle school students who are new to algebra formally. This happens because algebra is a new material that has not been formally taught in elementary schools, so students have difficulty transforming their thinking from arithmetic with numerical calculations to abstract symbolic language in algebra. This is supported by Raharjo et al. (2020) research, which revealed that the algebraic reasoning of middle school students is low. The low algebraic reasoning of students is known from the number of students who have difficulty understanding problems, making generalizations, making general shapes, and solving number pattern problems. In the study, there are four levels of algebraic reasoning as a reference ranging from level 0 to level 3, but the results were obtained that students could only reach level 1 of algebraic reasoning. In addition, Hamdan Sugilar also explained the low mastery of algebra of students, which is characterized by students not mastering symbol sense and structure sense (Sugilar et al., 2019). The research results of Setiawan et al. (2018), also obtained the same results related to the low algebra ability of middle school students as seen from the students' answers in solving problems. This situation is unfortunate, given the wide use and application of algebraic content. Therefore, research that uses algebra content needs to continue to be carried out to become evaluation material to improve algebra learning in the classroom.

Previous research related to algebraic reasoning that focused on secondary schools has been conducted. Ellis et al. (2020) tried to develop better algebra teaching materials through teaching experiments, and found a positive impact on student understanding. In addition, Girit & Akyüz (2016) analyzed students' algebraic reasoning at each different level (6th grade, 7th grade, 8th grade) in generalizing patterns. The study found that the higher the level of education, the more algebraic reasoning ability in generalizing patterns also increases. In addition, there is also Rivera & Becker (2020) research which conducted a three-year study to find effective learning issues and steps for students' algebraic reasoning in understanding pattern generalizations. From the various studies on mathematics algebraic reasoning in middle school, it can be seen that research on this topic is still limited to experimentation, analysis, and development.

Based on previous studies, no research has specifically conducted a systematic literature review to explore algebraic

reasoning research, especially in middle school mathematics. Therefore, this research needs to be carried out. This study will review various previous studies to understand how the concept of algebraic reasoning develops and is applied in mathematics education, with a structured and systematic method using a systematic literature review through the following research questions: (1) What is the development trend of algebraic reasoning topics over the years?; (2) What research methods are used?; (3) What is the geographical distribution of studies related to algebraic reasoning?

II. METHOD

This study uses the Systematic Literature Review method with the PRISMA (Preferred Reporting Item for Systematic Reviews and Meta-Analytic) protocol. The stages of PRISMA consist of identification, screening, eligibility, and included (Pati & Lorusso, 2018). This research began with a literature search with Publish or Perish 8 using the keyword and title words "algebraic reasoning" and “algebraic thinking” in the Scopus database. The selection of Scopus as a database is motivated by good article quality.

The literature obtained is then managed using Mendeley software to determine the duplication of articles. In the process, it was found that one article had duplicates, which was then deleted to ensure the accuracy of the data. Furthermore, papers will be further selected based on inclusion and exclusion criteria. The inclusion and exclusion criteria used in this study are explained in Table 1.

Table 1.
Inclusion and Exclusion Criteria

Criteria	Inclusion	Exclusion
Year	2015 – 2024 (1 decade)	More than 1 decade
Reference	Scopus database	Other than the Scopus database
Language	English	Other than English
Research type	Article journal or proceeding	Other than article journal and proceeding
Subject	Middle school student	Other than a middle school student

Articles that meet the inclusion criteria and do not meet the exclusion criteria are then screened for articles that match the research objectives. Screening is carried out through three stages. In the first stage, the status of the article's access is considered. Articles that were not accessible were not used in this study. In the second stage, the screening is carried out, considering the title's and abstract's suitability. In the third stage, a check was carried out on the full-text article to see if all the content was suitable for the research. From those processes, articles suitable for analysis are obtained.

III. RESULT AND DISCUSSION

The selection of articles is carried out through a strict process to ensure that the research results are relevant to the topic to be studied. Figure 2 summarizes the literature selection flow, which includes the initial identification stage until the final results are obtained. This stage produces 29 articles, which will then be analyzed with reference to research questions.

As shown in Figure 2, the initial stage of identification resulted in as many as 177 articles from the Scopus database, which

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were then filtered using predetermined inclusion and exclusion criteria and obtained a total of 65 articles. Articles are filtered through three stages of screening, including access status, suitability of titles and abstracts, and suitability of full-text articles. In this stage, 36 articles were issued. Finally, 29 articles were selected for further analysis. This process ensures that only relevant, high-quality studies are used in this study. The article will be analyzed based on research questions.

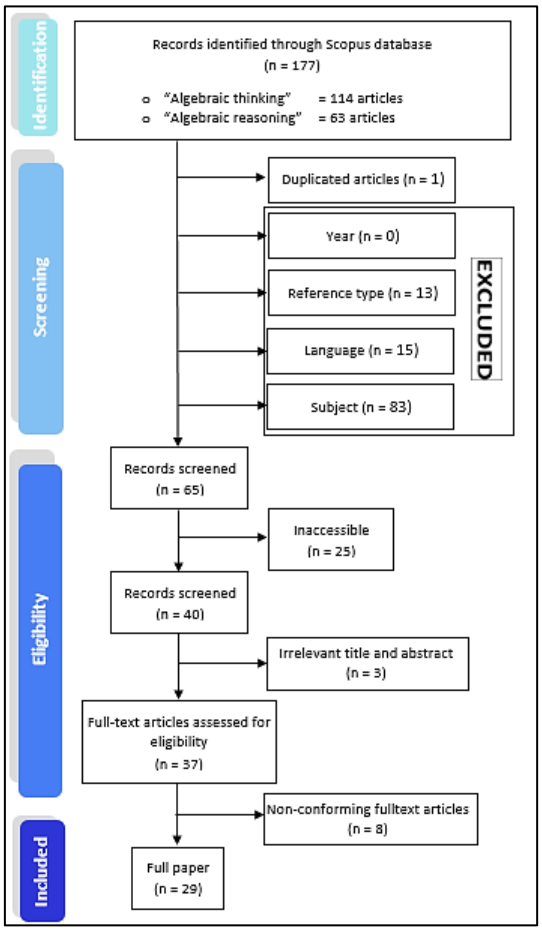

Figure 2. PRISMA Flowchart

A. Trends in Research Focus Over Time

Algebraic reasoning research in mathematics has been carried out by researchers over the years. This analysis is useful to find out the extent to which topics related to algebraic reasoning, especially in mathematics education, are developed and know what has been researched and has not been researched. It can be used as a reference to research similar themes that are broader and more in-depth. The year of this research refers to the year of publication. Various algebraic reasoning research in the field of mathematics that has been carried out is presented in Figure 3.

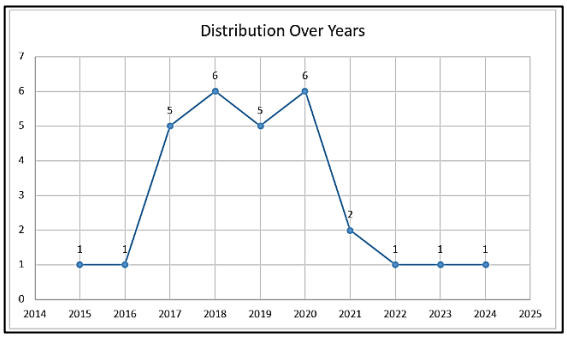

Figure 3. Distribution Over Years Diagram

Based on the data presented, it can be seen that, in general, the research of algebraic reasoning in mathematics in middle school students has changed from year to year. Most research occurred in 2018 and 2020, with the most significant increase occurring in 2016-2017, and a significant decrease occurred in the range of 2020-2022 until now. Selanjutnya akan dianalisis apa saja topik yang dibahas dari tahun ke tahun serta topik apa yang banyak dibahas dalam penelitian penalaran aljabar.

Research in 2015 was conducted by (Dougherty et al., 2015). The research developed a concept that can help students think algebraically, namely by using questions related to reversibility, flexibility, and generalization. In this concept, teachers must ask questions related to reversibility, flexibility, and generalization on number material so that

students can have an understanding that leads to algebraic thinking.

Furthermore, research in 2016 was conducted by (Hitt et al., 2016). This study conducted an experiment that attempted to connect arithmetic and algebra using triangular numbers using the ACODESA activity theory. In addition to using a paper-pencil-based test, this study also uses a technology application in the form of excel to present problems.

Research in 2017 was conducted by Hitt et al., (2017), Ayber (2017), Chimoni & Pitta-Pantazi (2017), Nurhayati et al. (2017), dan Muthmainnah et al. (2017). Meskipun penelitian tersebut ada pada tahun yang sama, namun topik penelitian yang diangkat berbeda-beda. Hitt et al. (2017) proposed a theoretical and practical framework for conducting high school arithmetic-algebra learning in order to bridge the existing leap using polygonal numbers using the ACODESA teaching method. Ayber (2017) analyzes whether students' textbooks contain activities that support the generalization of students as one of the things that are a concern in the Turkish mathematics curriculum. The generalization used as the perspective of the analysis is the core of algebraic reasoning (Carraher et al., 2006). Based on the research findings, both 7th and 8th-grade textbooks use number patterns to lead to algebra. The textbook assessment refers to the aspect of algebraic reasoning by (Kaput et al., 2008), where core aspect A is related to expressing generalizations and core aspect B is related to using representation to express generalizations. Next, Chimoni & Pitta-Pantazi (2017) wanted to find out if there was a relationship between algebraic thinking skills and other basic cognitive abilities. (Nurhayati et al., 2017) identifies students' algebraic reasoning abilities and uses the math-talk learning community to overcome them. And research by (Muthmainnah et al., 2017) identified student errors in completing algebraic thinking tests.

Research conducted in 2018 focused on problem-solving strategies and algebraic representations performed by students (Lepak et al., 2018), describe students' algebraic reasoning in solving math problems based on learning style (Indraswari et al., 2018), improve students' algebraic thinking skills through a variety of representation strategies using a realistic approach (Kusumaningsih et al., 2018a), know students' algebraic reasoning in solving mathematical problems reviewed from gender differences (Kusumaningsih et al., 2018b), characteristics of Algebraic Thinking of Junior High School Students (Rahmawati, 2018), and integrating algebraic thinking into PBL (Problem Based Learning) methods (Mustaffa, 2018).

In 2019, Agoestanto, Sukestiyarno, Isnarto, Rochmad, et al., (2019) conducted a study describe the location and causes of students' mistakes in thinking algebra based on cognitive style. They analyzes students' algebraic thinking skills as seen at the generational, transformational, and global meta level aspects, (Wilujeng et al., 2019) uses Merrill's First Principles of Instruction (MFPI) in developing students' algebraic thinking, Pratiwi et al. (2019) make learning design to facilitate the transformation of arithmetic and algebra, and Rahmawati et al. (2019) identifies the

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characteristics of students' algebraic thinking based on the SOLO model.

Research conducted in 2020 includes Ellis et al. (2020) research which proposes a scaling-continuous reasoning model to support students' algebraic reasoning. (DİNÇER & CANTÜRK GÜNHAN, 2020) made innovations to develop students' algebraic reasoning, namely by coding on robot software. Through coding the robot, students can learn about the process of generalizing and performing representations. (Pitta-Pantazi et al., 2020) menganalisis kemampuan berpikir aljabar siswa berdasarkan generalized arithmetic, functional thinking, modeling languages, and algebraic proof. (Agoestanto & Rinachyuan, 2020) analyze algebraic thinking student error on the global aspect of meta-level. Nada et al. (2020) conducted a study to analyze the characteristics of students' mathematical representations in solving algebraic thinking problems. And research by Suhaedi (2020) uses realistic mathematic education to improve students' algebraic thinking skills.

The research in 2021 was conducted by Faradillah et al. (2021) and Hidayanto & Lathifa (2021). Faradillah et al. (2021) conducted an analysis related to the validity and reliability of eight items of algebraic reasoning test instruments. And Hidayanto & Lathifa (2021) wanted to know the level of algebraic thinking of students from the process of arithmetic thinking to algebraic thinking using 3 levels, level I (reproduction). level II (connection), and level III (analysis).

Research from 2022 to 2024 has the same number of articles, namely one article. Research in 2022 conducted by Levin & Walkoe (2022) proposes an alternative perspective of learning by viewing algebraic learning as the process of restructuring existing ideas in a new way and more in accordance with the context through Knowledge in Pieces. Furthermore, in 2023, Ünal et al. (2023) will analyze students' algebraic reasoning skills, especially in visual representations (arithmetic and algebra) and symbolic representations. And the research conducted by da Silva Melo & Bisognin (2024) develops students' algebraic thinking skills using a practical design.

The various studies can be classified into seven research topics in table.

Table 2.
Research Topics Reviewed from the Research Year

Research Topic	Quantity	Year
Analysis of algebraic thinking skills	11	2015, 2017, 2018, 2019, 2020, 2023
Analysis of students' errors in solving algebraic problems	3	2017, 2019, 2020
Development of students' algebraic reasoning using a specific method or strategy or framework	9	2018, 2019, 2020, 2022, 2024
Experimentation using specific methods	2	2016, 2018
Analysis of learning resources related to algebraic reasoning	1	2017
Algebraic thinking level	1	2021
Development of instruments for measuring algebraic reasoning	1	2021

Based on these data, it can be seen that the most widely conducted algebraic reasoning research raises the topic of analyzing students' algebraic thinking ability or students' algebraic thinking

characteristics. This is in line with the results of the analysis of the VOSviewer application in figure 4 below.

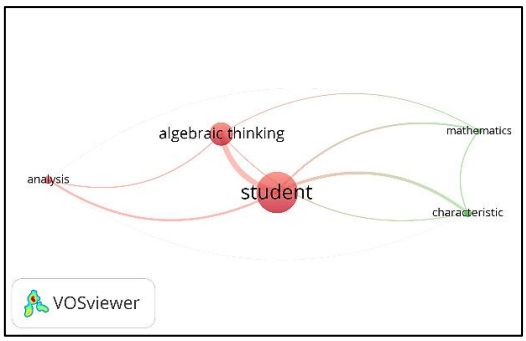

Figure 4. VOSviewer Algebraic Reasoning Research Analysis

The least discussed topics are related to the analysis of learning sources, algebraic thinking levels, and the development of instruments to measure algebraic reasoning. Based on these findings, it can be seen that there are still many opportunities for algebraic reasoning research at the high school level. Further research needs to pay attention to anothet topics, such as the analysis of learning resources, the level of algebraic reasoning, and the development of instruments to measure algebraic reasoning, as well as topics that have not been raised before, such as the use of media to develop students' algebraic reasoning.

B. Research Method

The results of the literature show that the research of algebraic reasoning in mathematics has fluctuated over the past decade. Various ideas and discussions related to this topic have been presented and developed by researchers. The research method chosen by the researchers is one of the things behind this diversity. In this study, research methods are divided into 3, namely qualitative, quantitative, and mixed methods. However, some studies do not explicitly explain the methods used. The number of articles regarding the research method used is presented in Figure 5 below.

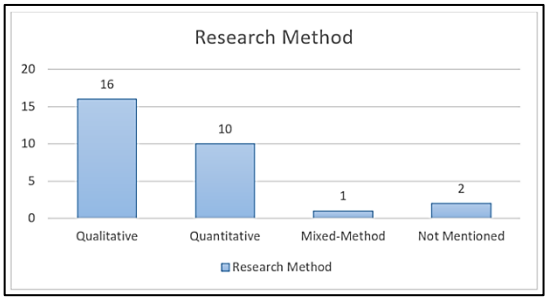

Figure 5. Research Method Diagram

The data in the figure 5 show that the research methods used vary. The qualitative research method is the most widely conducted, with as many as 16 articles. It was followed by a quantitative research method, which amounted to 10 articles, a mixed-method of 1 article, and two articles that did not explicitly explain the research method used.

Qualitative research is used through various research objectives, including to find out how students are capable (Hidayanto & Lathifa, 2021; Indraswari et al., 2018; Kusumaningsih et al., 2018b; Lepak et al., 2018; Nada et al., 2020; Rahmawati, 2018; Rahmawati et al., 2019), conduct experimentation studies using developed designs or frameworks (da Silva Melo & Bisognin, 2024; dinçer & cantürk günhan, 2020; Ellis et al., 2020; Hitt et al., 2016, 2017; Pratiwi et al., 2019), identify Students' Errors (Agoestanto, Sukestiyarno, Isnarto, Rochmad, et al., 2019; Muthmainnah et al., 2017) and analyze learning resources (Ayber, 2017). This researchers consider this method very relevant because they want to understand the phenomenon in depth in a specific

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context. In addition, it also allows researchers to interpret the results of the study in detail without trying to make broader generalizations so that the data obtained provides a rich and contextual picture. The many uses of this qualitative research method are in line with the results of systematic literature review research conducted by (Sibgatullin et al., 2022) which found that the qualitative research method is the most widely conducted research method to see this topic. In addition, Olivia et al. (2024) and Irfiani et al. (2023) also found that the qualitative research method is the most widely conducted research method to see students' thinking skills.

The quantitative research that has been carried out has different objectives, including finding correlations (Chimoni & Pitta-Pantazi, 2017; Ünal et al., 2023), identifying (Nurhayati et al., 2017), seeing the influence (Agoestanto, Sukestiyarno, Isnarto, & Rochmad, 2019; Mustaffa, 2018; Wilujeng et al., 2019), seeing effectiveness (Kusumaningsih et al., 2018a; Suhaedi, 2020), and test the validity and reliability of the instrument (Faradillah et al., 2021; Pitta-Pantazi et al., 2020). Quantitative methods analyze phenomena that can be measured and expressed in numerical format (Doucette, 2017). This expression is in line with the research article above, which generally wants to analyze data from the phenomenon more objectively and systematically. In addition, this method was chosen because it allows researchers to obtain valid results and can be generalized in various research contexts.

There is an algebraic reasoning study in mathematics that uses a mixed-method approach. This research is conducted by (Agoestanto & Rinachyuan, 2020). In the study, a qualitative approach was used to describe students' mistakes in thinking algebra and find out the causes of these errors. Data was obtained through observation and semi-structured interviews and then analyzed using steps such as data reduction, data presentation, and verification. Meanwhile, a quantitative approach is used to measure students' ability to think algebraically at the global meta-level, evaluate the achievement of minimum completeness criteria, and test hypotheses related to data distribution and completeness achievement. Quantitative data analysis involves a normality test using Shapiro-Wilk, an average test (t-test), and a proportion test (z-test).

Of all the articles obtained, there are two articles that do not straightforwardly explain what research method is used. The research was conducted by (Dougherty et al., 2015) and (Levin & Walkoe, 2022). The research by (Dougherty et al., 2015) is a theoretical study that develops a framework of questions to improve algebraic thinking skills in students with learning disabilities (LD). And research by (Levin & Walkoe, 2022) develops algebraic thinking from the perspective of Knowledge in Pieces (KiP).

To find out students' algebraic reasoning, the researchers developed instruments that can be used to measure algebraic reasoning. The design of the tasks used in the above study has many similarities, including those related to

representations (symbols, diagrams, graphs, etc.), giving meaning to variables, performing operations on variables, understanding the equals sign as an equality sign and building generalizations. These various problems have been proven to encourage students to reason algebra.

The articles that are most widely used to date use qualitative research methods. This is certainly understandable considering that students' algebraic reasoning can be known more carefully by directly digging into data sources (students), especially since the research wants to see what algebraic reasoning is like by students. How he thinks, what he does, his decisions and why will be clearly revealed. However, in the following years, research on this topic may be widely discussed using other research methods with a more diverse approach. In addition, researchers need to pay more attention to articles compiled by listing research methods. This ensures the data's validity, provides readers with information, and can be a reference in future studies.

C. Geographic Distribution

The research area is used to find out which areas are conducting algebraic reasoning research. Information about this location is important because it provides an understanding of where the research was conducted, which can affect the context and results of the research. For example, if the research is conducted in a particular region, the geographical, cultural, or social factors of that region can affect the results of the research. The data on the distribution of regions in the article on algebraic reasoning in mathematics education is presented in Figure 6 below.

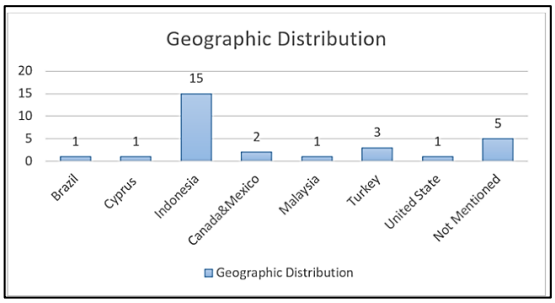

Figure 6. Geographic Distribution Diagram

The region that conducts the most algebraic reasoning research is Indonesia, with fifteen articles by (Hidayanto & Lathifa, 2021), (Nada et al., 2020), (Agoestanto, Sukestiyarno, Isnarto, & Rochmad, 2019), (Kusumaningsih et al., 2018[a]), (Suhaedi, 2020), (Faradillah et al., 2021), (Agoestanto & Rinachyuan, 2020), (Rahmawati et al., 2019), (Kusumaningsih et al., 2018b), (Muthmainnah et al., 2017), (Agoestanto, Sukestiyarno, Isnarto, Rochmad, et al., 2019), (Pratiwi et al., 2019), (Rahmawati, 2018), (Nurhayati et al., 2017), and (Wilujeng et al., 2019). The number of these studies is motivated by the fact that the algebra ability of Indonesian junior high school students is still low. This fact can be seen from the results of the Trend in International Mathematics and Science Study (TIMSS) survey. Algebra is one of the mathematical domains that is tested with a weight of 30%. In this domain, students get the lowest score compared to other material content with a score of 22% (TIMSS, 2012). In Indonesia, although actually at the elementary school level, the concept of arithmetic operations of addition, subtraction, multiplication, and division of numbers has been introduced, but formally, students only get algebra material

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at the middle grade level, so this material is fairly new to students (Andriani, 2015). Unlike arithmetic, the main purpose of algebra is not to perform numerical calculations but to provide an operating language for representing, analyzing, and manipulating the relationships contained in numbers and letters (alphanumeric) (Agoestanto, Sukestiyarno, Isnarto, & Rochmad, 2019). Therefore, many students have difficulty transforming from arithmetic thinking at the elementary school level to algebraic thinking, which tends to be abstract (Susac et al., 2014).

Students' abilities are diverse, one of which can be clearly seen through Rahmawati (2018) research. The research wanted to see students' ability to form models, use mathematical language, representation, arithmetic generalization, problem-solving, and quantitative reasoning. Researchers divide subjects based on their mathematical abilities. Based on the results of the analysis, it can be seen that students with high, medium, and low mathematical abilities all have the characteristics of algebraic thinking as modelling, mathematical language, and arithmetic generalization. However, it turns out that not all students can have the characteristics of algebraic thinking as problem solving and quantitative reasoning, such as students with low and medium math skills. Students with moderate and lower mathematical abilities emphasize the characteristics of algebraic thinking as modeling and arithmetic generalization, but have not yet achieved enough algebraic thinking as problem-solving and quantitative reasoning.

Turkey, as the second-most prominent position in researching algebraic reasoning, conducts research that focuses on its curriculum, such as generalizations in textbooks (Ayber, 2017) and representation skills (Ünal et al., 2023), as well as related to things that are in high demand, such as robotics (DİNÇER & CANTÜRK GÜNHAN, 2020). All of these topics are associated with algebraic reasoning as one of the skills that are considered important for students to have as a basis for mastering and developing mathematics, other sciences, or even solving problems in daily life. In addition, there is a difference in focus in the algebraic reasoning used. The research by (Ünal et al., 2023) and (DİNÇER & CANTÜRK GÜNHAN, 2020) has similarities in representation, whereas research by DİNÇER & CANTÜRK GÜNHAN (2020) uses representation to express the situation that occurs from the results of coding and observation of robot movements and research (Ünal et al., 2023) uses representation to solve problems in the material of equations and inequalities. In contrast to the other two studies, research (Ayber, 2017) focuses more on generalizations. Although the focus of algebraic reasoning used is different, both representation and generalization are all things that students must master. This is in accordance with (NCTM, 2000), which states that understanding patterns and making representations using algebraic symbols is the goal of learning algebra.

Two studies were conducted simultaneously in two regions, namely Canada and Mexico. The two studies were

conducted by the same researcher and included subjects from first-grade secondary schools in Canada and third-grade secondary schools in Mexico. Although it was carried out by the same researcher with the same subject, the focus of the research was different. (Hitt et al., 2016) focuses more on developing cognitive structures through spontaneous and visual representations to unify arithmetic and algebraic thinking using the example of triangular numbers. Meanwhile, research (Hitt et al., 2017) is broader, not only talking about triangular numbers but also discussing in general whether learning algebra must have a "big leap" (from arithmetic to algebra) or be able to run continuously (continuity).

One study from Brazil focused on analyzing the development of algebraic thought in students in the 8th and 9th years of primary education (da Silva Melo & Bisognin, 2024). Basic education in Brazil is taken for 9 years, starting from grade 1 to grade 9, aged between 6 and 14 years. Although there are differences in policies that have an impact on the age of children at the secondary school level, the researcher refers to the age of middle school students so that this research can be used. Forms of algebraic reasoning such as understanding equivalence, building relationships/comparisons based on the skill of understanding and operating ordinary symbols in other forms of representation (such as tables), operating the unknowns, constructing first-degree polynomial equations, identifying patterns, and constructing generalizations can be mastered well by students. However, students' understanding of functions is still low. This can be seen when students are not able to relate changes between one variable and another, make representations in coordinate fields, and determine ordered pairs.

Research from the United States focuses on students' problem-solving strategies and algebraic representations (Lepak et al., 2018). Furthermore, research in Cyprus by Pantazzi validated the theoretical framework of algebraic thinking and the relationship between students' cognitive abilities and algebraic thinking (Pitta-Pantazi et al., 2020). Research related to algebraic reasoning was also conducted in Malaysia by Mustaffa (Mustaffa, 2018). The study wanted to find out whether there was an effect of integrating algebraic reasoning on problem-based learning (PBL) by comparing it with conventional learning and learning with algebraic thinking. From the study, the results were obtained that the PBL approach with the integration of algebraic thinking was able to improve algebraic thinking among students at the junior high school level.

In some of the previously identified articles, the location of the study is clearly mentioned. However, five articles in this study do not include information about the location of the study, making it difficult to understand the geographical background of the study. This research was conducted by (Ellis et al., 2020), (Indraswari et al., 2018), (Chimoni & Pitta-Pantazi, 2017), (Dougherty et al., 2015), and (Levin & Walkoe, 2022). The information provided only shows that the research was conducted in middle schools.

The analysis's results show that the diverse research locations add to the

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diversity of research studies. Many things can influence this diversity, such as a region's policies, interests, and culture. This explains that topics related to algebraic reasoning can be combined and matched with the various diversity.

IV. CONCLUSION

This study aims to explore the development of middle school algebraic reasoning research, especially in mathematics, through the year the research was conducted, the research method, nd the research area was carried out for a decade (2015-2024). These points were obtained from the results of the analysis of 29 articles that meet the inclusion criteria and pass the three stages of screening. Algebraic reasoning research from year to year tends to decline due to several things, one of which is the change in the focus of education research. In addition, the most widely used research method is the qualitative research method because it is considered appropriate to know how students' algebraic reasoning is through a series of assignments and interviews. This method is also considered appropriate for researching students' thinking skills. Dari 8 wilayah yang melakukan penelitian penalaran aljabar, Indonesia merupakan negara yang paling banyak meneliti topik ini. This is motivated by the low algebraic skills of Indonesian students and the many views on the importance of transitioning from arithmetic thinking in elementary school to algebraic thinking in middle school.

The importance of algebraic thinking in daily life and every level of education requires students to be able to reason algebraically. Therefore, it is very important to conduct research on algebraic reasoning, especially in high schools that are experiencing a transition from arithmetic thinking to algebra.

This research is used as a basis for determining the next step of algebraic reasoning research in middle shool. Future research needs to pay attention to another topics, such as the analysis of learning resources, the level of algebraic reasoning, and the development of instruments to measure algebraic reasoning, as well as topics that have not been raised before, such as the use of media to develop students' algebraic reasoning.

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