p-ISSN: 2086-4280
e-ISSN: 2527-8827

Examining The Correlation Between Teachers' Interaction to Student Error and The Development of Students' Mathematical Identities

Irwani Zawawi[1], Sri Suryanti[2*], Lestariningsih[3]

Mathematics Education Program, Universitas Muhammadiyah Gresik
Jalan Sumatera No.101, Gn. Malang, Randuagung, Gresik, East Java 61121 Indonesia
[1]*irwani@umg.ac.id*

Mathematics Education Program, Universitas Negeri Surabaya
Jalan Lidah Wetan, Lidah Wetan, Kec. Lakarsantri, Surabaya, East Java 60213 Indonesia
[2*]*srisuryanti@unesa.ac.id*; [3]*lestariningsih@unesa.ac.id*

Article received: 03-05-2024, revision: 18-06-2024, published: 30-07-2024

Abstrak

Komponen penting dari pengajaran matematika yang efektif adalah secara aktif mengatasi kesalahan siswa. Pendekatan pedagogis ini memainkan peran penting dalam meningkatkan keterlibatan siswa. Studi ini menyelidiki cara guru di dua kelas mata pelajaran matematika sekolah menengah dalam memberikan kesempatan bagi siswa untuk mengembangkan identitas mereka, berdasarkan cara guru membahas dan terlibat dengan kesalahan siswa. Data dikumpulkan selama enam bulan melalui rekaman pembelajaran, catatan lapangan, dan wawancara dengan guru dan siswa. Data yang diperoleh kemudian dianalisis secara kualitatif. Penelitian ini mengeksplorasi dua perspektif tentang bagaimana guru menangani kesalahan: persepsi bahwa kesalahan dapat menyebar, dan persepsi bahwa kesalahan dapat berfungsi sebagai peluang belajar yang berharga. Pendekatan guru untuk membahas dan menangani kesalahan dipengaruhi oleh perspektif mereka tentang siswa dan pemikiran logis mereka dalam matematika, yang mempengaruhi identitas yang mereka tunjukkan kepada siswa. Kami menunjukkan bahwa persepsi siswa tentang metode pengolahan kesalahan oleh guru mereka mempengaruhi hubungan mereka dengan matematika. Namun, siswa itu sendiri juga berkontribusi pada pengembangan identitas matematika mereka.
Kata Kunci: identitas matematis siswa; kesalahan siswa; interaksi guru.

Abstract

Actively addressing student mistakes is an important component of effective mathematics instruction. This pedagogical approach plays an important role in increasing student involvement. The study investigates how teachers in two classes of high school mathematics subjects create opportunities for students to develop their identity as students based on how teaching discusses and engages with student mistakes. We collected the data over six months using learning recordings, field records, and interviews with both teachers and students. This research explores two perspectives on how teachers deal with mistakes: the view that mistakes can spread and the view that mistakes may serve as valuable learning opportunities. The perspectives of students and their logical thinking in mathematics influence the teacher's approach to discussing and dealing with errors, shaping the identity they portray to students. We showed that students' perceptions of the methods of processing their teacher's errors influenced their relationship with mathematics. However, the students themselves also contributed to the development of their mathematical identity.
Keywords: Mathematical identities of learners; students' errors; interactions of teacher.

I. INTRODUCTION

Making mistakes is a normal and essential part of learning mathematics (Radatz, 1980; Rushton, 2018). Even experienced mathematicians make mistakes, which can lead to the creation of new knowledge (Radatz, 1980). Errors that are systematic and consistently occur across contexts, indicating a deeper conceptual misunderstanding, are referred to as misconceptions (Brodie, 2014). Errors are classified into two types: sporadic errors, which do not indicate conceptual misunderstandings and are easily corrected (Dellantonio & Pastore, 2021), and systematic errors, which are rarely explicitly taught yet commonly made by students (Brodie, 2014).

Student engagement in mathematics learning can be influenced by how teachers respond to students' mistakes. The way teachers interact with students' errors shapes the criteria for success in the classroom (Brodie, 2014). In some classrooms, success is defined as not making mistakes, as errors are associated with labeling students as either capable or incapable (Heyd-Metzuyanim et al., 2015). In these settings, making mistakes may lead to feelings of shame, anxiety, and self-doubt, which can cause students to dislike mathematics because they perceive mistakes as failures (Heyd-Metzuyanim et al., 2015).

In other classrooms, success in learning mathematics involves learning from mistakes. In these environments, teachers use students' incorrect thinking as a resource to motivate them to learn from their errors and develop conceptual understanding (Radatz, 1980). Therefore, the way teachers handle mistakes can play a crucial role in shaping students' mathematical identities.

Over the past decade, the construction of identity has been increasingly studied in mathematics education (Radovic et al., 2019; Graven & Heyd-Metzuyanim, 2019). Focusing on identity in mathematics education is valuable in highlighting students' experiences of what it means to "do mathematics" and who is considered capable of doing mathematics (Graven & Heyd-Metzuyanim, 2019). Examining students' mathematical identities also allows for a deeper understanding of their engagement in mathematics learning (Graven & Heyd-Metzuyanim, 2019).

However, research exploring the relationship between teachers' approaches to student errors and students' mathematical identities remains limited. In this study, we investigate how two middle school teachers, who teach using a traditional approach, interact with student errors and how their approaches contribute to shaping students' identities. We aim to answer the following research questions: 1) How do teachers interact with student errors in mathematics learning? 2) How can teachers' approaches to errors shape students' mathematical identities?

The constructivist learning theory is commonly used in research on student errors in mathematics learning. A constructivist perspective provides insights into how students' thinking develops and how their errors emerge through engagement with mathematical tasks. Constructivist theory posits that knowledge is actively constructed based on prior

p-ISSN: 2086-4280
e-ISSN: 2527-8827

knowledge. The restructuring of schemas and the development of conceptual understanding occur through the processes of assimilation and accommodation (Dorko, 2019).

Assimilation involves integrating new knowledge into existing schemas, whereas accommodation occurs when a conflict between new knowledge and existing schemas leads to a reorganization of those schemas (Dorko, 2019).

Assimilation, where existing schemas are expanded to accommodate new knowledge, is generally easier for students compared to accommodation, where existing schemas must be modified to understand new concepts. When students assimilate new knowledge rather than accommodate it, overgeneralization from previously correct knowledge can occur in a new domain where it is no longer valid (Dorko, 2019).

This implies that correct knowledge from one domain is incorrectly applied in another (Dorko, 2019). For example, the following mistake made by a 7th-grade student, $\frac{1}{3}+\frac{1}{3}=\frac{2}{6}$, can be explained as an overgeneralization of previously correct knowledge about integer addition, which is incorrectly applied in the new domain of fractions.

Therefore, from a constructivist perspective, errors are reasonable and make sense to students.

While mistakes may make sense from a constructivist perspective, the importance of social context in shaping teachers' and students' views on mistakes is better understood through a situative perspective. From a situative perspective, student errors indicate either a failure to use the appropriate tools provided for learning activities or a lack of engagement in the learning process (Brodie, 2014). Mistakes can also result from students adapting to a new teacher whose instructional approach differs from their previous teacher (Brodie, 2014).

Using the same example as above, situative theorists explain that errors arise when newcomers engage with new tasks while still relying on previous tools and experiences, as they have spent years applying them correctly. Thus, newcomers will not fully develop their identity as full members of their new learning community until they acquire the necessary tools for meaningful participation. Discussions between newcomers and experienced members are essential to negotiate and create shared criteria for developing a mutual understanding of what it means to be a successful mathematics student (Brodie, 2014).

The relationship between learning and identity has been discussed by Paul (2022) and Sfard & Prusak (2005). According to Paul (2002), identity and learning are interconnected because learning transform's identity by changing who we are and how we participate in a community of practice. Learning is viewed as an experience of identity, as it involves becoming a member of a community.

For Sfard and Prusak (2005), identity consists of a collection of stories or narratives about a person, which define their identity and shape their actions. Although these stories may be told individually—either by the identity builder

or others—they are collectively constructed, meaning that narratives are co-constructed between society and social communities (Sfard & Prusak, 2005). Teachers and other significant individuals create narratives about students, which influence the opportunities provided to students for learning mathematics (Sfard & Prusak, 2005).

Based on the works of Paul (2002) and Sfard and Prusak (2005), we view students' mathematical identity as shaping their potential to learn and engage in mathematics through both practice and narrative. However, Paul's (2002) and Sfard and Prusak's (2005) definitions of identity tend to emphasize the social construction of identity while placing less emphasis on the individual's role in constructing their own identity.

A strong focus on the social construction of identity limits our understanding of the individual's role and how self-concept is related to identity (Löwstedt & Räisänen, 2014; Hogg & Reid, 2006). To better understand how students' mathematical identity is constructed—considering both individual and social aspects—Marks & O'Mahoney's (2014) work on personal identity, social identity, and agency in relation to mathematics is illustrated in Figure 1.

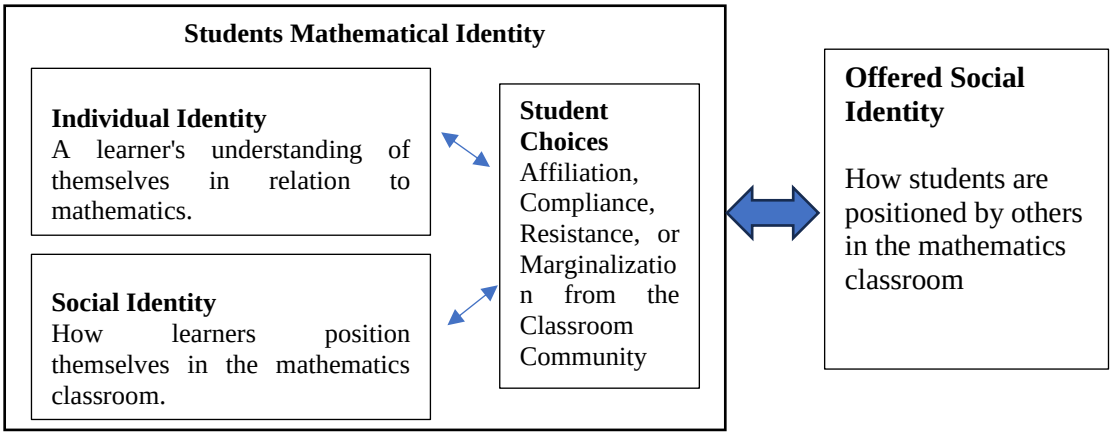

Figure 1. Students Mathematical Identity

Marks and O'Mahoney (2014) explain that personal identity emerges from the self, which is shaped through individual interests and actions. Personal identity involves an understanding of oneself, influenced by students' emotions—such as their enjoyment of learning mathematics, their motivation, and their future aspirations in mathematics—while also being informed by the social context in which individuals learn about themselves.

Social identity, according to Marks and O'Mahoney (2014), is the position that is formed between personal identity and the way individuals believe they should be perceived within their social environment. Thus, personal identity informs how learners develop their social identity by shaping how they position themselves and how they want to be seen by others as members of their classroom community. The personal and social identities that students construct is closely interconnected, and both are strongly linked to the social context.

p-ISSN: 2086-4280
e-ISSN: 2527-8827

Social identity is also offered to students by others, such as teachers, through the narratives they construct about students and the opportunities they provide for students to participate and become members of the classroom community, including making mistakes. While teachers offer students a social identity, students have the agency to construct their identities differently from what is presented to them.

This choice is determined by agency, which involves students actively deciding how to engage based on their social context (Marks & O'Mahoney, 2014). Student agency emerges from the relationship between the individual and their social environment (Matusov et al., 2016). A student's agency, in terms of their participation in the classroom, is influenced by the social identity offered by their teacher, as well as their personal and social identities. As students exercise their agency, they gain a deeper understanding of themselves, reinforce their personal identity, and assess the significance of their membership in the classroom community.

During classroom interactions, teachers may continuously offer students the same social identity or present them with a different social identity than previously offered or constructed. Similarly, students may maintain their existing mathematical identity or reconstruct and adopt a new identity based on the social identity presented to them.

Marks and O'Mahoney (2014) and Gardee (2021) developed a framework for students' mathematical identity that incorporates both individual and social dimensions. This framework illustrates how students can construct their mathematical identity when offered a social identity in the form of affiliation, partial affiliation, or marginalization by the teacher (Gardee, 2021).

A social identity of affiliation is offered when teachers provide students with full access to learning resources. A social identity of partial affiliation is offered when teachers grant students only partial access to learning resources. Marginalized social identity emerges when teachers restrict students' access to tools and learning resources. Depending on how students exercise agency and construct their personal and social identities, they may identify with, comply with, or reject the social identity offered to them by developing their mathematical identity through affiliation, marginalization, compliance, or resistance within their learning community (Gardee, 2021), as detailed in Table 1.

Table 1 Offered and Constructed Identity

	Offered Social Identity	Constructed Mathematical Identity
Students can identify	Affiliation or Partial Affiliation	Affiliation, Marginalization
Students can comply	Affiliation or Partial Affiliation	Compliance
Students can persist	Affiliation or Partial Affiliation	Resistance

Students who receive a social identity of affiliation from their teacher tend to actively engage with learning resources and fully participate in the classroom community. They exhibit strong intrinsic motivation and perceive learning

mathematics as an essential part of their future. Conversely, students who experience marginalization tend to have low engagement in learning, with little or no emotional motivation to develop their mathematical skills. This illustrates how the social identity offered by teachers can influence the construction of students' personal identity and their engagement in learning mathematics.

Students who conform to the social identity of affiliation, partial affiliation, or marginalization tend to exercise their agency in similar ways by engaging with learning tools and resources in a limited manner. Regardless of the type of social identity offered, compliant learners typically restrict their interactions with the teacher while engaging more with their peers, allowing them to continue learning mathematics. These students may or may not be emotionally invested in mathematics learning. For instance, some compliant students are emotionally engaged in learning but feel uncomfortable actively participating, while others lack emotional interest in mathematics and may prefer to participate in a more limited capacity.

Some students may exercise their agency by rejecting or resisting the social identity offered to them. Students who reject the social identity of affiliation or partial affiliation provided by the teacher may choose not to participate, even when granted full access to learning tools and resources. These students reject the offered social identity by distancing themselves from the group. They may also lack emotional interest in learning mathematics.

II. METHOD

This study involved eighth-grade students at a well-resourced secondary school in Gresik, East Java, Indonesia. A total of twenty-nine (29) students and two (2) mathematics teachers participated. The two teachers, referred to as Mr. Z and Mr. SM (using initials for anonymity), along with three selected students from each of their classes, were chosen for in-depth discussion in this study. Both teachers have over 15 years of teaching experience, hold a bachelor's degree in mathematics education, and have obtained professional certification as mathematics teachers.

They demonstrated different ways of interacting with student errors and adopted distinct approaches to handling mistakes in their classrooms. The three students from each class were selected based on teacher recommendations, using the following criteria: one high-achieving student, one average-achieving student, and one low-achieving student. In Mr. Z's class, the selected students were Iqbal, Dimas, and Raka, while in Mr. SM's class, the selected students were Syauqi, Azmi, and Lintang.

This study employed a case study approach focusing on the relationship between three components: teacher practices, student errors, and students' mathematical identities. Data collection was conducted over six months, or one semester, in 2023. The collected data included video recordings of classroom instruction, field notes, and video-recorded interviews with both teachers and students.

Interviews with teachers aimed to understand how they discussed student

p-ISSN: 2086-4280
e-ISSN: 2527-8827

errors and why they assigned specific social identities to students. Meanwhile, interviews with students sought to explore how and why they constructed their identities in particular ways and how they perceived the significance of their mistakes.

Next, the data were analyzed using a qualitative approach. We coded the recorded data by analyzing teacher-student interactions to identify the types of identities offered and constructed by students.

We documented the teachers' instructional approaches, including how they designed their lessons and the learning activities they provided to create opportunities for student reasoning. The recorded classroom sessions were also analyzed to examine how each student participated in class.

Additionally, we documented instances of errors occurring in each lesson, noting which students made errors and how teachers responded to them. The teachers' interactions with student errors were analyzed using Brodie's (2014) framework, which categorizes responses into correcting, probing, and embracing errors.

We analyzed the interviews with a focus on teachers' pedagogy, students' errors, and students' mathematical identities.

Data from teacher interviews were used to understand their pedagogical approaches, their perceptions of student errors, their strategies for addressing errors, and the social identities they offered to students. Data from student interviews were analyzed to understand their mathematical identities, their perceptions of errors, and their experiences with their teachers' approaches to handling mistakes.

We used the criteria in Table 2 to code the identities that were offered to and constructed by students.

Table 2. Criteria

Offered Social Identity	Constructed Mathematical Identity	Constructed Individual Identity	Constructed Social Identity	Activity
Affiliation (A): The teacher facilitates participation and fully supports the students.	Affiliation (A)	Students enjoy learning mathematics and recognize its benefits.	Students feel comfortable being full members of the classroom community.	Students fully engage with tools and learning resources.
Partial Affiliation (PA): The teacher facilitates participation and supports the students to a certain extent.	Compliance (K)	Students may or may not enjoy learning mathematics or see its usefulness.	Students want to maintain their position to some extent within the classroom community.	Students participate in a limited way, usually because they feel uncomfortable engaging with the teacher.
	Rejection (P)	Students do not enjoy learning mathematics or see its usefulness.	Students do not want to engage in the community.	Students participate in a limited way or not at all with tools and learning resources.
Marginalization (M): The teacher	Marginalization (M)	Students do not enjoy learning	Students do not want to engage in	Students participate in a limited way or

Offered Social Identity	Constructed Mathematical Identity	Constructed Individual Identity	Constructed Social Identity	Activity
limits students' opportunities to participate and learn mathematics.		mathematics or see its usefulness.	the community.	not at all with tools and learning resources.
	Compliance (K)	Students may or may not enjoy learning mathematics or see its usefulness.	Students want to maintain their position to some extent within the classroom community.	Students participate in a limited way, usually because they feel uncomfortable engaging with the teacher.
	Rejection (P)	Students enjoy learning mathematics and see its usefulness.	Students aspire to be full members of the classroom community.	Students make an effort to fully engage with tools and learning resources.

Next, we coded how learners talked about errors, particularly in relation to whether they considered errors important for learning. We then compared their understanding of errors with their teachers' approaches to handling mistakes.

After that, we examined the relationship between students' mathematical identities and their teachers' interactions with errors.

Teachers, through their pedagogical approaches and interactions with students, convey messages about the nature of mathematics and the most effective ways to learn it. Table 3 illustrates the differences in teachers' pedagogical approaches, students' reasoning approaches, and their understanding and responses to errors.

III. RESULT AND DISCUSSION

Table 3. Teacher Practices and Approaches to Student Errors

	1st Teacher	2nd Teacher
Pedagogy	Mr. Z did not encourage collaboration and spent most of his lessons giving lengthy explanations and numerous examples, with some portions of the material allocated for class assignments. Mr. Z explained: "I teach by providing examples..."	Mr. SM asked a series of low-level questions to students, followed by giving them the opportunity to work with their peers on class assignments. The students discussed their assignments in groups while Mr. SM walked around and assisted them. Mr. SM explained: "When I give them class assignments, I allow them time to discuss the questions before attempting them, and then they engage in discussion. If they have no idea about something, they will call for help. I will go over there and assist them."
Approach to Student Reasoning	Mr. Z told students: *"If you follow instructions, you will be better than a calculator."* He did not encourage students to use methods other than the ones he taught: *"I don't want... there are only two ways, this way or that way. There is no other way."*	Mr. SM told students: *"I want you to be thorough with the answers you get. If the answer doesn't make sense, discuss it..., check it again."* He provided students with opportunities to find their own solutions.
Understanding of	*Errors are contagious.* He told students that he needed to	*Errors are lessons.* He told students: *"Don't feel bad about being corrected. It is part of learning."*

p-ISSN: 2086-4280
e-ISSN: 2527-8827

	1st Teacher	2nd Teacher
Student Errors	*"correct their thinking"* if he saw mistakes.	
Approach to Student Errors	Mr. Z typically corrected errors by re-explaining the concept; however, he did not accommodate mistakes. If students made an error, they had to correct it as soon as possible.	Mr. SM usually addressed errors after investigating them. He accommodated mistakes, stating: *"They need to learn from their mistakes. That's why we call them learners. If they are corrected, it becomes easier for them to remember what they have been corrected on because it's impossible for them to fix everything. They need to learn from their own mistakes."*

The pedagogical approaches of both teachers were based on what they believed would best support students in learning mathematics. For Mr. Z, lengthy explanations and numerous examples were essential for improving learning, and he did not encourage students to use methods other than the ones he taught. In contrast, Mr. SM preferred group discussions where students designed their own solutions.

These pedagogical approaches were shaped by the teachers' perspectives on students' reasoning. For Mr. Z, mathematics was something already discovered, and the goal of learning mathematics was to "understand" what was already known. For Mr. SM, mathematics was about exploring solutions.

The beliefs and knowledge of these teachers shaped how they understood and handled students' mistakes. For Mr. Z, mistakes had a negative connotation, capable of "growing" and "spreading." He structured his lessons by limiting students' opportunities to explore and develop their own methods and solutions to problems to prevent mistakes from occurring. When mistakes did happen, he typically corrected them with lengthy explanations.

In contrast, Mr. SM viewed mistakes positively as learning opportunities and used them as a tool to encourage students to learn from their errors. He designed his lessons to support students' reasoning and foster deeper mathematical thinking.

Mr. Z offered social identities to students based on how he perceived them as learners in his classroom. Iqbal was given a social affiliation identity because of his high academic ability. Dimas was offered a partial affiliation identity due to his moderate academic ability, while Raka, who had low academic ability, was assigned a marginalized social identity.

Mr. Z had minimal interaction with Raka and did not provide much assistance when he struggled or made mistakes, thus limiting Raka's opportunities to understand why his reasoning was incorrect. Table 4 outlines the social identities offered to students and how they constructed their own identities in response.

Table 4. Offered and Developed Identities

Student	Offered Social Identity	Developed Mathematical Identity	Developed Individual Identity	Developed Social Identity	Activity	Understanding of Errors	Teacher's Approach to Errors

Student	Offered Social Identity	Developed Mathematical Identity	Developed Individual Identity	Developed Social Identity	Activity	Understanding of Errors	Teacher's Approach to Errors
Iqbal	A	A	He enjoys mathematics and is motivated to pursue it in the future.	He is a full member of the classroom community.	He frequently participates in class but occasionally hesitates.	Making mistakes is important for learning.	Mr. Z is sometimes surprised, which makes him feel uneasy; however, he corrects his mistakes.
Dimas	M	K	He likes mathematics, wants to pass the subject, and pursue it in the future.	He feels uncomfortable being a full member.	He does not participate with Mr. Z; instead, he asks Iqbal for help.	He does not "feel guilty" when making mistakes.	Mr. Z is not always willing to help him or answer his questions.
Raka	M	K	He considers mathematics "difficult," but he wants to pass and pursue it in the future.	Although he does not participate with Mr. Z, he remains a member of the class.	He does not engage with Mr. Z but usually asks a friend for help.	Mistakes are a "bad thing," and he "feels completely uncomfortable" about them.	Mr. Z prohibits students from making mistakes.

Among the three students in Mr. Z's class, Iqbal was the only one who developed a mathematical affiliation identity, while the other students adopted a compliance identity. Iqbal was motivated to learn mathematics, reflecting his personal identity, and aspired to establish his social identity as one of the top mathematics learners in his class.

Although Iqbal developed an affiliation identity, Mr. Z's approach to errors influenced his mathematical identity, particularly in how he exercised his right to participate in class. He felt uncomfortable when making mistakes, even though he believed that errors were important for learning mathematics. As a result, he sometimes hesitated to participate. However, since he was offered a social affiliation identity and received support from Mr. Z when he made mistakes, Iqbal was able to develop his identity through affiliation and learn from his errors.

Mr. Z's approach to errors also influenced Dimas's mathematical identity. Although Dimas did not feel "bad" about making mistakes, he had limited interaction with Mr. Z because he felt uncomfortable with his errors and sensed that Mr. Z was not always willing to assist him.

Despite his limited participation with Mr. Z, Dimas developed his social identity

by seeking help from other students. His personal identity, which recognized the usefulness of mathematics for his life and future, motivated him to engage with his peers rather than relying solely on the teacher.

Raka was the only student who fully experienced Mr. Z's approach. He explained that Mr. Z's approach to his mistakes made him feel "good" and "confident" about engaging in mathematics learning.

Raka stated that his participation in the classroom community was not driven by a desire to "excel" or "fail" in mathematics but rather by his nature as someone who cares about others. His motivation to pass mathematics was rooted in his aspirations for the future.

Mr. SM provided all students in his class with opportunities to participate and build their mathematical identities in affiliation with the classroom community. He encouraged every student to engage, believing that everyone could do mathematics and that "effort" and "practice" were essential for success in learning mathematics.

Table 5 illustrates the social identities offered to students, how they constructed their identities, and their perspectives on the importance of making mistakes in class.

Table 5. Offered and Constructed Identity

Student	Offered Social Identity	Constructed Mathematical Identity	Constructed Individual Identity	Constructed Social Identity		Understanding of Mistakes	Teacher's Approach to Mistakes
Syauqi	A	K	Mathematics is difficult. He is motivated to pursue mathematics in the future.	Even though he does not participate with his teacher, he remains engaged in class.	He does not participate with Mr. SM unless asked to do so. Instead, he asks a friend for help.	His mistakes are "bad," which makes him feel "sad."	Mr. SM helps him, making him feel "good" after he understands.
Azmi	A	K	Mathematics is a challenge, although he enjoys learning it and is motivated to pursue it in the future.	He feels uncomfortable fully participating.	He engages with Mr. SM but feels uneasy asking for his help. Instead, he participates with other students.	For him, everyone makes mistakes, and he learns from his own.	Mr. SM "smiles" and understands. "He usually says we are all here to learn." However, Mr. SM's approach to mistakes makes him feel "stupid" by making the answers seem easier than he

Student	Offered Social Identity	Constructed Mathematical Identity	Constructed Individual Identity	Constructed Social Identity		Understanding of Mistakes	Teacher's Approach to Mistakes
							perceives them to be.
Lintang	A	A	Mathematics is his favorite subject. He is motivated to pursue mathematics in the future.	He maintains full participation.	He engages with Mr. SM when he needs help and assists other students.	He makes "a lot of mistakes" and explains that his teacher helps him.	Mr. SM doesn't seem very happy about it at first, but then he simply explains until the students understand. That makes me feel motivated to work harder.

In Mr. SM's class, Syauqi and Azmi developed a mathematical compliance identity, while Lintang developed a mathematical affiliation identity.

Syauqi was motivated to pursue mathematics in the future, reflecting his personal identity; however, he did not engage much with Mr. SM in class. Instead, he sought help from other students. He explained that he preferred to "listen" to Mr. SM teach rather than actively participate in class. Nevertheless, Mr. SM assisted Syauqi whenever he made mistakes.

Although Syauqi perceived Mr. SM's approach to mistakes positively, Azmi did not. Like Syauqi, Azmi developed a compliance identity because he felt uncomfortable fully engaging with Mr. SM. He explained that he hesitated to ask Mr. SM for help, as even though Mr. SM would "smile" and show understanding when he made mistakes, his approach—making answers seem easy when encountering challenging problems—made Azmi feel incapable.

Lintang did not perceive Mr. SM's approach to mistakes positively. Although he felt comfortable participating in class, he explained that Mr. SM did not appear "pleased" when he made mistakes. His experience with Mr. SM's approach to mistakes motivated him to work harder. Lintang developed his mathematical identity through affiliation because mathematics was his favorite subject, reflecting his personal identity.

The purpose of this study is to explore the relationship between teachers' interactions with students' mistakes and students' mathematical identities. To address our first research question regarding how teachers interact with students' mistakes, Mr. Z stated that mistakes are contagious, while Mr. SM viewed mistakes as lessons from which students can learn. Some teachers correct mistakes to ensure that students acquire accurate knowledge (Brodie, 2014), while

p-ISSN: 2086-4280
e-ISSN: 2527-8827

others may dislike the occurrence of mistakes, perceiving them as contagious (Swan, 2001).

Both teachers emphasized that mistakes are errors that need to be corrected rather than tolerated based on correct mathematical knowledge. Mr. Z corrected mistakes directly, while Mr. SM investigated the mistakes before correcting them. Simply correcting mistakes is not sufficient for students to learn from them. From a constructivist perspective, correcting mistakes does not address the underlying misconceptions or lead to the restructuring of knowledge (Brodie, 2014; Sarwadi & Shahrill, 2014). Merely correcting mistakes may not help students understand why their answers are incorrect (Brodie, 2014). Neither teacher fully embraced students' mistakes; they did not use mistakes as opportunities to generate new knowledge for all students in the classroom.

Other research also indicates that how teachers respond to students' mistakes can influence their conceptual understanding. For example, a study by Booth et al. (2014) found that when students' mistakes were used as discussion material in class, they tended to develop a deeper understanding of mathematical concepts compared to simply receiving direct corrections from the teacher. Additionally, a study by Santagata (2005) showed that teachers who encouraged the exploration of mistakes through reflective discussions helped students develop better metacognitive skills in solving mathematical problems. Thus, a more dialogic approach to mistakes, such as the one applied by Mr. SM, aligns more closely with constructivist learning approaches than merely correcting mistakes without further analysis, as seen in Mr. Z's approach.

The second research question concerns how teachers' approaches to mistakes shape students' mathematical identities. In Mr. Z's class, being a successful mathematics learner meant avoiding mistakes, whereas in Mr. SM's class, success in mathematics was defined by learning from mistakes. The classroom environment plays a crucial role in providing students with opportunities to engage in learning (Cornelius & Herrenkohl, 2004; Richardson & Mishra, 2018). The way teachers approach mistakes, or how their approach is perceived, influences how students participate in class and construct their identities. However, students also play an active role in shaping their mathematical identities, which are supported by both personal and social identities.

In Mr. Z's class, Iqbal felt guilty about making mistakes, whereas Dimas avoided making mistakes altogether. Iqbal hesitated to participate, while Dimas had minimal interaction with Mr. Z. Feelings of shame when making mistakes are common in classrooms (Heyd-Metzuyanim et al., 2015). In Mr. SM's class, Lintang experienced positive emotions when interacting with Mr. SM. His personal and social identity supported his identification with the offered social affiliation identity, allowing him to participate fully in class. Similarly, Azmi also experienced positive emotions when corrected by Mr. SM. However, he still felt guilty about making

mistakes and remained uncomfortable participating in class.

Other studies have also shown that teachers' approaches to errors influence students' mathematical identities and engagement in learning. For example, research by Black and Wiliam (2009) found that a classroom environment that encourages the use of mistakes as learning opportunities can boost students' confidence and participation in mathematical discussions. Additionally, a study by Langer-Osuna (2018) highlighted that students who feel safe making mistakes and discussing them openly are more likely to develop a positive mathematical identity and confidence in problem-solving. This aligns with the findings in Mr. SM's class, where a reflective approach to mistakes provided a more supportive learning experience, although its impact varied depending on students' social identities and personal emotions. Conversely, as seen in Mr. Z's class, an approach that emphasizes correction without further exploration tended to make students hesitant to participate, thereby limiting their development of a strong mathematical identity.

IV. CONCLUSION

The exploration of the relationship between teacher-student interactions regarding mistakes and students' mathematical identities reveals that teachers' approaches to errors play a crucial role in shaping students' learning experiences and mathematical identities. A teacher's approach is influenced by their perspective on the importance of student reasoning and the types of social identities they facilitate in the classroom. In this study, all students accepted the social identities offered by their teachers, highlighting the significant role teachers play in shaping students' mathematical identities. A sense of belonging within the learning environment emerges as a key factor in fostering student engagement. Therefore, considering the connection between personal identity and social identity is essential in developing a positive mathematical identity that empowers students in learning mathematics.

REFERENCES

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability, 21*(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2014). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 34*, 24–34. https://doi.org/10.1016/j.learninstruc.2014.03.004

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, *85*, 221–239.

Cornelius, L. L., & Herrenkohl, L. R. (2004). Power in the classroom: How the classroom environment shapes students' relationships with each other and with concepts. *Cognition and Instruction*, *22*(4), 467–498.

Dellantonio, S., & Pastore, L. (2021). Ignorance, misconceptions and critical thinking. *Synthese*, *198*(8), 7473–7501.

Dorko, A. (2019). Generalization, assimilation, and accommodation. *The Mathematics Educator*, *28*(2).

Gardee, A. (2021). Relationships between teachers' practices and learners' mathematical identities. *International Journal of Mathematical Education in Science and Technology*, *52*(3), 377–403.

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: the state of the art and future directions: Review and introduction to ZDM Special Issue on Identity in Mathematics Education. *ZDM*, *51*(3), 361–377.

Heyd-Metzuyanim, E., Tabach, M., & Nachlieli, T. (2015). Opportunities for learning given to prospective mathematics teachers: Between ritual and explorative instruction. *Journal of Mathematics Teacher Education*, *16*(9), 547–574.

Hogg, M. A., & Reid, S. A. (2006). Social identity, self-categorization, and the communication of group norms. *Communication Theory*, *16*(1), 7–30.

Langer-Osuna, J. M. (2018). The social life of mathematical learning: Exploring identity, participation, and meaning in school mathematics practices. *Mathematical Thinking and Learning, 20*(2), 77–90. https://doi.org/10.1080/10986065.2018.1403543

Löwstedt, M., & Räisänen, C. (2014). Social identity in construction: enactments and outcomes. *Construction Management and Economics*, *32*(11), 1093–1105.

Marks, A., & O'mahoney, J. (2014). Researching identity: A critical realist approach. In *P. K. Edwards, J. O'mahoney, & S. Vincent (Eds.), Studying organizations using critical realism: A practical guide* (pp. 66–85). Oxford: Oxford University Press.

Matusov, E., von Duyke, K., & Kayumova, S. (2016). Mapping concepts of agency in educational contexts. *Integrative Psychological and Behavioral Science*, *50*, 420–446.

Paul, E. (2022). Communities of practice: Learning, meaning, and identity. *British Journal of Educational Psychology*, *72*, 460.

Radatz, H. (1980). Students' errors in the mathematical learning process: a survey. *For the Learning of Mathematics*, *1*(1), 16–20.

Radovic, D., Black, L., Williams, J., & Salas, C. E. (2019). Towards conceptual coherence in the research on mathematics learner identity: A systematic review of the literature. *Educational Studies in Mathematics*, *99*, 21–42.

Richardson, C., & Mishra, P. (2018). Learning environments that support student creativity: Developing the SCALE. *Thinking Skills and Creativity*, *27*, 45–54.

Rushton, S. J. (2018). Teaching and learning mathematics through error analysis. *Fields Mathematics Education Journal*, *3*(1), 1–12.

Santagata, R. (2005). Practices and beliefs in mistake-handling activities: A video study of Italian and US mathematics lessons. *Teaching and Teacher Education, 21*(5), 491–508. https://doi.org/10.1016/j.tate.2005.03.004

Sarwadi, H. R. H., & Shahrill, M. (2014). Understanding students' mathematical errors and misconceptions: The case of year 11 repeating students. *Mathematics Education Trends and Research*, *2*(1), 1–10.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22.

Swan, M. (2001). Dealing with misconceptions in mathematics. In P. Gates (Ed.), *Issues in Mathematics Teaching* (pp. 147–165). Routledge.

AUTHOR'S BIOGRAPHY

Dr. Drs. Irwani, M.Kes.

Born in Petaling, May 2, 1965. Teaching staff at Muhammadiyah University Gresik. Study S1 Mathematics Education, Sriwijaya University, Palembang, graduated in 1991; S2 Biostatistics Airlangga University, Surabaya, graduated in 1999; and S3 Mathematics Education, Surabaya State University, Surabaya, graduated in 2016.

Dr. Sri Suryanti, M.Si.

Born in Ponorogo, September 13, 1985. Teaching staff at Surabaya State University. He studied S1 Mathematics Education, Muhammadiyah Gresik University, graduated in 2010; S2 Mathematics ITS, Surabaya, graduated in 2013; and S3 Mathematics Education, State University of Malang, Malang, graduated in 2023.

Dr. Lestariningsih, M.Pd.

A lecturer in Mathematics Education Department at Universitas Negeri Surabaya. Her primary research focus is on mathematical literacy,where she explores innovative ways to enhance students' understanding and application of mathematical concepts in both academic and real-world contexts.