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A Study of Learning Obstacles: Determining Solutions of a System of Linear Equation Using Gauss-Jordan Method

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Abstrak
Salah satu metode pencarian solusi SPL yang dipelajari di pendidikan tinggi adalah metode Gauss-Jordan. Berdasarkan data penelitian, mahasiswa melakukan kesalahan dalam melakukan operasi baris elementer (OBE), membentuk ke dalam matriks yang diperbesar, dan bahkan pemahaman mendasar mengenai reduksi baris dan ini berpengaruh dalam menentukan solusi suatu SPL. Untuk itu perlu dianalisis hambatan belajar yang terjadi dalam menentukan solusi SPL menggunakan metode Gauss-Jordan. Peneitian ini merupakan penelitian kualitatif dan merupakan bagian dari *Didactical Design Research* (DDR). Penelitian ini dilaksanakan di salah satu universitas di Nusa Tenggara Timur (NTT) dengan subjek penelitian merupakan 20 mahasiswa calon guru matematika. Teknik penelitian ini adalah tes diagnostik dan wawancara. Hasil penelitian menunjukkan beberapa hambatan yang dialami mahasiswa adalah keterbatasan pemahaman pada OBE, pengklasifikasian MEBT, dan merepresentasikan MEBT ke dalam solusi SPL. Rekomendasi dari penelitian ini adalah pengajar hendaknya membuat suatu bahan ajar yang dapat meminimalisir hambatan belajar yang dialami mahasiswa.
Kata kunci: Hambatan belajar; Metode Gauss-Jordan; Penelitian didaktis; Sistem Persamaan Linier.

Abstract
One of many methods of finding solution of a system of linear equations (SLE) studied in higher education is the Gauss-Jordan method. Based on the research, students made mistakes in performing elementary row operations (ERO), forming into augmented matrices, and even basic understanding of row subtraction and this has an effect on determining the solution of an SLE. For this reason, it is necessary to analyze the learning obstacles that occur in determining the solution of LES using the Gauss-Jordan method. This qualitative research is part of the Didactical Design Research (DDR). This research was conducted at a university in East Nusa Tenggara (NTT) involving 20 students as preservice mathematics teacher. The technique of this research is diagnostic test and interview. The results show that some of the obstacles faced by students are insufficient understanding of ERO, classifying reduced row-echelon form (RREF), and representing MEBT into SPL solutions. This study recommends that teachers create teaching materials that minimizes learning obstacles experienced by students.
Keywords: Learning obstacles; Gauss-Jordan Method; Didactical Design Research; System of Linear Equations.

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I. INTRODUCTION

In the preparation stage before learning, teachers may fail to notice some of the learner needs (Silvi & Auliya, 2022; Salamah, Susiaty, & Ardiawan, 2022). This usually occurs due to the learner diversity, but it can be minimized (Nurhanifah, 2022; Ulfa & Sundayana, 2022). Unmet learner needs can cause learners to experience learning obstacles, either in concepts or procedures (Sugiarti, 2017; Muharomi & Afriansyah, 2022).

There are three factors that cause learners experience learning obstacles (Brousseau, 1997; Doyumgaç, Tanhan, & Kiymaz, 2021), namely: Ontogeny Obstacle, Didactical Obstacle, Epistemological Obstacle. Ontogeny obstacles deal with obstacles that occur due to learners' learning readiness based on psychological aspects (Yusuf et al., 2017; Hariyani, Herman, Suryadi, & Prabawanto, 2022). The psychological aspect is closely related to the age development and the developmental level. Didactical obstacles occur due to teaching factors from the teachers. This has something to do with the teacher' style of delivering the learning materials, while epistemological obstacles are affected by the students' limited understanding of a particular context (Mahyudi & Endaryono, 2020).

Linear algebra courses emphasize the fundamental knowledge of concepts and procedures (Klau et al., 2020). In general, this course is studied in the first semester of mathematics education study programs. This means that linear algebra is possible to be a prerequisite for courses in the next semester. Due to the importance of this subject, the learners' thinking skill needs to develop well to obtain deep understanding of the liner algebra (Yudi et al., 2017).

A system of linear equations (SLE) is a fundamental material. LES is studied from secondary school to higher education. In most textbooks of Linear Algebra or Elementary Linear courses. SLE is the first topic to be discussed. In addition, SLE is studied starting from secondary school to higher education. Based on the mathematics education syllabus, this material is found in the Linear Algebra for Elementary course in the early semesters.

One of the methods used to find solution of an SLE is the Gauss-Jordan elimination method. Learners are required to understand this method as it helps them to solve an SLE up to n variable (Funny, 2017). Gauss-Jordan elimination method is also an effective technique in applying elementary row operation (ERO) on upper triangular matrix and lower triangular matrix forms simultaneously (Setiadji et al., 2022).

The method works by converting an SLE into a matrix and substracting the matrix using elementary row operation (ERO) to determine the solution. Later, it forms a reduced row-echelon matrix (RREM) (Raj, 2011). The students are also required to accurately classify the SLE matrix form into

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SLE with a unique solution, many solutions, and without solution.

The errors mostly found in the operation are an operation that do not lead to row-echelon reduction and an incorrect calculation on ERO (Funny, 2017). Meanwhile, the use of these operations is considered to be equivalent with the solution process using mixed method (elimination and substitution) on the system of linear equations with two variables. A crucial error was reported by Titi Sumarni (Sumarni, 2021) that learners do not understand the conversion operation of the SLE form into an enlarged matrix form. Ironically, the learners could not proceed to the ERO and were unable to obtain the correct solution. These errors could be experienced by learners with high, moderate, and low ability (Hariati & Septiadi, 2019).

Broadly speaking, learners' weaknesses are found in term of understanding the concept and the subject matter as the support to solve a system of linear equations using Gauss-Jordan elimination method (Mahyudi & Endaryono, 2020). Therefore, this issue is necessary to be further analyzed and identified starting from analyzing and identifying the learning obstacles experienced by the learners while they are making these errors.

II. METHOD

This study used a qualitative approach as part of didactical design research (DDR). According to DDR stages, this study was at the prospective analysis stage. It was conducted before the learning process took place to find out the learning obstacles experienced by the learners (Suryadi, 2010).

The study took place in a university in Nusa Tenggara Timur (NTT). It involved 20 prospective teacher students (PTS) of mathematics education who had taken the elementary liner algebra course. The students were in semester III, IV, and VII, with diverse cognitive ability. The research instrument used was a diagnostic test to determine the solution of an SPL using Gauss-Jordan method and utilizing ERO. The study used interview techniques to confirm the results of test.

The diagnostic test presented three problems that represented the types of SLE solution; SLE with unique solutions, SLE with multiple solutions, and SPL with no solution. The results of the test indicated the learning obstacles experienced by the participants in determining SLE solution through ERO procedures they performed and the learning obstacles in identifying RREM through the application of Gauss-Jordan method.

III. RESULT AND DISCUSSION

The results dealt with the participants' answers of the diagnostic test that represented each SLE solution. The use of Gauss-Jordan elimination method certainly utilizes the application of elementary row/column operations, namely

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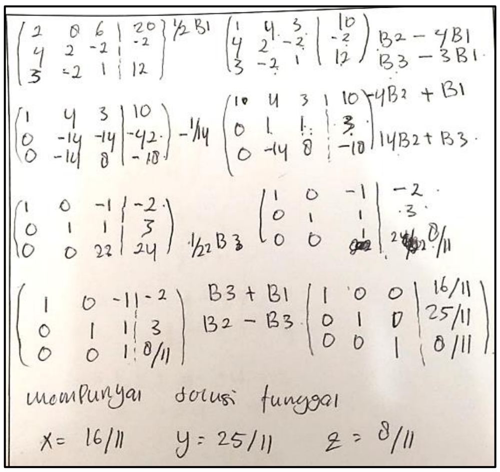
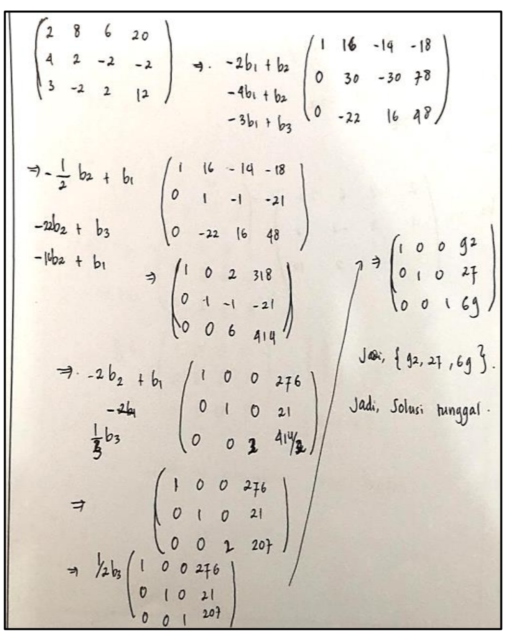
multiplying an equation by a non-zero constant, adding a multiple of an equation to another equation, and interchanging two equations (Anton et al., 2005). Hereafter, they are called operation 1, operation 2, and operation 3 respectively.

1) SLE with a unique solution



Picture 1. Answer type 1.



Picture 2. Answer type 2.

This section provided an SLE problem with a unique solution. According to picture 1 and 2, the same problem generated varied answers, although both of them showed a unique solution. In the picture 1, the participants had form a main 1 in the entry of row 1 column 1 by applying operation 1. It was followed by the application of operation 2 to form zero entries in row 2 and 3 column 1. In general, the participants had a clear direction, namely forming a main 1 in each row and then making zero entries in other entries beside the main 1 (Raj, 2011). Then, they formed an RREM, in this case, the identity matrix. However, the error occurred in the calculation process of filling in the entries after the basic operations. For instance, during the operation *b3*-*3b*1 in row 3 column 3, it should be -8, but the participant wrote 8. This caused incorrect data entries at the later stages.

In Picture 2, the participant wrote entry 1 in row 1 column 1 in the first stage of the enlarged matrix calculation. This means the participant had already understood the strategy to solve the enlarged matrix. The first step was to determine or form the main 1. The determined objective was not relevant to the basic operation performed. To form the main 1 in row 1 column 1, operation 1 may be applied by multiplying row 1 with constant $\frac{1}{2}$ by $\frac{1}{2}b1$ symbol. However, the

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participant applied operation 2 and it was not relevant to the desired result.

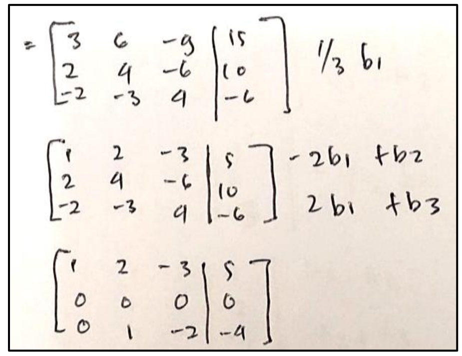
Based on the interview results, the learning obstacle was found during the elementary row operation in each row and it became more complicated if the entry encountered required fractional values. This obstacle was mostly encountered at almost every stage of the thinking skill developments, whether at the primary, secondary, or hinger education level ((Smith & Powell, 2011).

This is due to the lack of understanding of the concept of fractions and lack of experience in encountering problems related to fractions. This finding was in line with a study conducted by Brown and Quinn (2006) that revealed 50% of 143 participants who took basic algebra courses were unable to find the sum and the multiplication results of a fractional operation.

Basically, the participants had understood that the enlarged matrix was substracted to obtain the solution, but they did not understand how to apply the operation, choose possible operations to reduce the rows, and reason why each operation is unique.

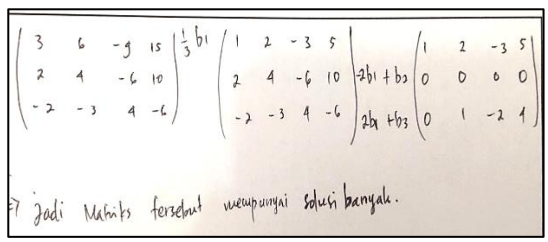
2) SLE with multiple solutions

In the instrument item 2, an SLE with multiple solutions was presented. Firstly, the picture 3 showed that the reduced matrix forms were not RREM. Secondly, the participant were unable to represent the matrices into an SLE solution.



Picture 3. Answer type 3.

The finding in Picture 4 was the result of reduced matrices that did not form an RREM. The participant stopped the operation at classifying the type of the reduced matrices into an SLE with multiple solutions. The participant could not write the parameterized solution.



Picture 4. Answer type 4.

Based on the interview results, the participant did not understand the RREM form. The respondent only understood that the enlarged matrices were required to form the identity matrix, whereas an RREM does not have to be an identity matrix. In term of SLE solutions, the participant admitted that he did interpret the result of the reduced matrices into an SLE solution or a parameterized solution

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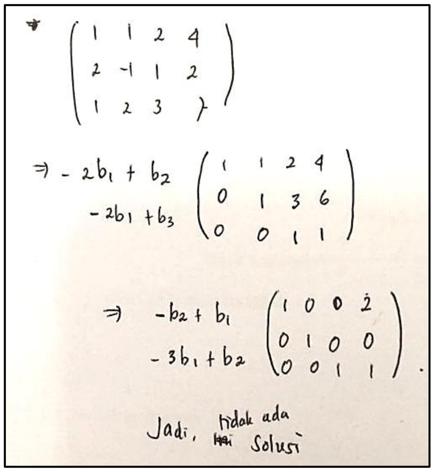
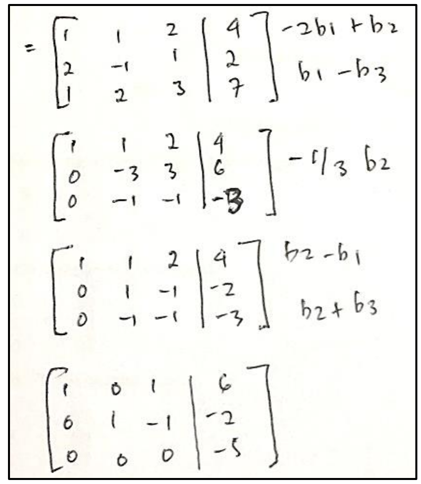
because he did not understand SLE with multiple solutions.

3) SLE with no solution



Picture 5. Answer type 5.

In Picture 5, the participant concluded that the reduced matrix form was equivalent to an SLE with no solution. In fact, the matrix entries, except the 4th column, formed an identity matrix that represented *x, y,* and *z* solution. When confirmed through an interview, the participant argued when a zero entry was generated in the 4th column, the SLE had no solution. On the other hand, the entry of the 4th column was, in fact, the solution of the desired variable value.



Picture 6. Answer type 6.

Picture 6 showed that the participant failed to classify the result of the reduced matrices into an SLE solution. It was indicated by the participant's inability to draw a conclusion on this instrument item 3. When confirmed, the participant stated that he was unable to form the SLE solution because the last row from column 1 to column 3 had zero entries which made him unable to write the variables. From the statement, it was concluded that the participant did not understand the SLE with no solution or reduced matrix type with no solution.

According to the findings and analysis of the participants' diagnostic test results, there were several learning obstacles experienced by the learners. Firstly, it dealt with the understanding of ERO in Gauss-Jordan method. This obstacle might be derived from the participants and the teachers. Based on the interview results, there were obstacles related to basic knowledge of number operations,

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especially fractions, as well as unfamiliarity with problems related to fractions. It might epistemologically be interpreted that the participants not only lacked the understanding of ERO but also didactically lacked problems or items dealing with the issue. In addition, the teaching materials used did not explain the function of each ERO procedure applied to the enlarged matrices. It was indicated by the participants who were unable write the type of ERO used to form a particular entry in the matrices.

The second obstacle dealt with classifying RREM. In the previous discussion, the participants mistakenly understood the identity matrix as the only type of RREM. Based on the interview results, the participants had studied RREM and its prerequisite. However, on the test they identified RREM as an identity matrix. This issue was categorized as epistemological obstacles.

The third obstacle dealt with representing RREM into an SLE solution. The participants classified the RREM types limited to multiple solutions and no solution. They were having difficulties to form parameters in multiple solutions and determine the type of matrices with no solution. This obstacle was categorized as epistemological obstacles.

Teachers, teaching materials, and the learners are required to work together to minimize the obstacles that might occur in the teaching and learning process. It is suggested that teachers prepare and adjust the materials to the learners' characteristics (Yudi et al., 2017).

Although the textbooks are provided in the digital forms, the learners have not utilized continuously. The available textbooks do not fit the learning obstacles experienced by the learners so that they do not develop the learners' understanding. With respect to this issue, the learners are highly dependent on the lectures in the classroom, whereas, the material will not be able to be delivered thoroughly due to limited time (Mahyudi et al., 2017). There might be possibilities of misconception and misprocessing in the classroom. If the learners do not consult with the reference books, they might develop a false understanding of the materials.

IV. Conclusion

The conclusion of this study is that there were several obstacles experienced by the participants, namely: limited understanding of ERO, RREM classification, and representing RREM into an SLE solution.

This study recommended that it is necessary to provide learning materials that explain ERO in detail and relevant to mixed methods, conduct a special strategy to apply ERO efficiently, and create a mapping of RREM types with SLE solutions.

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BIBLIOGRAPHY

Anton, H., Rorres, Chris., & Black, Christine. (2005). *Elementary linear algebra, with applications. Student solutions manual*. J. Wiley.

Brouseau, G. (1997). *Theory of Didactical Situation in Mathematics.* Dordrecht: Kluwer Academic Publisher.

Brown, G., & Quinn, R. (2006). *Algebra students' difficulty with fractions*. *Australian Mathematics Teacher, 62*(4), 28–40.

Doyumgaç, I., Tanhan, A., & Kiymaz, M. S. (2021). Understanding the most important facilitators and obstacles for online education during COVID-19 through online photovoice methodology. *International Journal of Higher Education*, *10*(1), 166-190.

Funny, R. A. (2017). Using Matlab To Help Students Understanding Gauss-Jordan Methods. *Angkasa: Jurnal Ilmiah Bidang Teknologi*, *9*(2), 75. https://doi.org/10.28989/angkasa.v9i2.176

Hariati, A., & Septiadi, D. D. (2019). Analysis of students' mistakes in solving system of linear equation in three variables: A case on HOTS problems. *International Journal on Teaching and Learning Mathematics*, *2*(1), 29. https://doi.org/10.18860/ijtlm.v2i1.7616

Hariyani, M., Herman, T., Suryadi, D., & Prabawanto, S. (2022). Exploration of Student Learning Obstacles in Solving Fraction Problems in Elementary School. *International Journal of Educational Methodology*, *8*(3), 505-515.

Klau, K. Y., Siahaan, M. M. L., & Simarmata, J. E. (2020). *An Identification of Conceptual and Procedural Understanding : Study on Preservice Secondary Mathematics Teacher*. *11*(2), 339–350.

Mahyudi, Ariani, N. M., & Ramadianti, W. (2017). Desain bahan ajar aljabar linier. *Kalamatika*, *2*(1), 1–14.

Mahyudi, & Endaryono. (2020). Learning Obstacles Konsep Operasi Baris Elementer Ditinjau Dari Hambatan Epistemologi. *Inovasi Pendidikan Matematika Desember Jurnal*, *3*(1), 48–59.

Muharomi, L. T., & Afriansyah, E. A. (2022). Kemampuan Koneksi Matematis dan Kemandirian Belajar Siswa pada Materi Sistem Persamaan Linear Dua Variabel. *Leibniz: Jurnal Matematika*, *2*(2), 45-64.

Nurhanifah, N. (2022). Kemampuan berpikir kreatif matematis siswa kelas viii smp pada materi geometri. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *1*(2), 161-172. https://doi.org/10.31980/powermathedu.v1i2.2228

Copyright © 2023 Mosharafa: Jurnal Pendidikan Matematika

Raj, L. L. (2011). Focusing Learning on Concepts of Introductory Linear Algebra using MAPLE Interactive Tutors. *Integration of Technology Into Mathematics Education: Past, Present and Future*, 439–448.

Salamah, S., Susiaty, U. D., & Ardiawan, Y. (2022). Instrumen Three-Tier Test Berbasis Kemampuan Representasi Matematis untuk Mengetahui Miskonsepsi Siswa. *Plusminus: Jurnal Pendidikan Matematika*, *2*(3), 391-404. https://doi.org/10.31980/plusminus.v2i3.2205

Setiadji, B. H., Supriyono, & Indriastuti, A. K. (2022). The Use of Gauss-Jordan Elimination Method in Determining the Proportion of Aggregate Gradation. *Lecture Notes in Civil Engineering*, *216*, 431–440. https://doi.org/10.1007/978-981-16-7949-0_38

Silvi, K. L., & Auliya, N. N. F. (2022). Bahan Ajar E-LKS Berbasis Realistic Mathematics Education Materi Sistem Persamaan Linear Dua Variabel. *Plusminus: Jurnal Pendidikan Matematika*, *2*(3), 381-390. https://doi.org/10.31980/plusminus.v2i3.2223

Smith, L., & Powell, J. (2011). An Alternative Method to Gauss-Jordan Elimination : Minimizing Fraction Arithmetic. *The Mathematics Educator*, *20*(2), 44–50.

Sugiarti, L. (2018). Kesulitan Siswa Dalam Menyelesaikan Soal Operasi Bentuk Aljabar. *Prosiding Seminar Nasional Etnomatnesia Yogyakarta*. Hal. 325-329.

Sumarni, T. (2021). Analisis Kesalahan Mahasiswa Menyelesaikan Sistem Persamaan Linier dengan Menggunakan Metode Gauss Jordan pada Mata Kuliah Aljabar Linier Elementer. *MENARA Ilmu*, *XV*(02), 102–109.

Suryadi, D. (2010). *Metapedadidaktik dan Didactical Design Research (DDR): Sintesis Hasil Pemikiran Berdasarkan Lesson Study*. Bandung: FPMIPA Universitas Pendidikan Indonesia.

Ulfa, N. C. A., & Sundayana, R. (2022). Kemampuan representasi matematis siswa pada materi bilangan berdasarkan self-confidence. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *1*(2), 193-200. https://doi.org/10.31980/powermathedu.v1i2.2231

Yudi, M., Ariani, N. M., & Ramadianti, W. (2017). Desain Bahan Ajar Mata Kuliah Alajabar Linear Untuk Mengembangkan Kemampuan Berfikir Kreatif Matematis. *KALAMATIKA Jurnal Pendidikan Matematika*, *2*(1), 1. https://doi.org/10.22236/kalamatika.vol2no1.2017pp1-14

Yusuf, Y., Titat, N., & Yuliawati, T. (2017). Analisis Hambatan Belajar (Learning Obstacle) Siswa SMP Pada Materi Statistika. *AKSIOMA*, *8*(1), 76.

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