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Singaporean and Japanese Maths Textbooks: Character, Structure, and Content

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Abstrak

Meningkatkan pengajaran matematika secara terus menerus merupakan upaya yang harus terus dilakukan. Hal ini dikarenakan matematika tidak hanya dipelajari oleh siswa sepanjang tahun tetapi juga banyak digunakan sebagai indikator kunci keberhasilan individu siswa. Salah satu faktor penting yang mempengaruhi pengajaran matematika adalah buku teks. Artikel ini membahas buku teks matematika dari dua negara, yaitu Singapura dan Jepang. Kajian terhadap kedua buku teks dilakukan terhadap karakter fisik, struktur, dan topik/isi buku teks. Kesimpulan hasil kajian ini sebagai berikut. Secara fisik, Buku teks matematika Singapura lebih sederhana dalam warna, dibandingkan buku teks matematika Jepang. Jumlah halaman Buku teks Singapura lebih banyak dari jumlah halaman Buku teks Jepang. Dari segi struktur buku, pada Buku teks matematika Singapura, terdapat beberapa bab yang dimulai dengan ringkasan materi prasyarat, kemudian diikuti dengan materi dasar, contoh dan penyelesaiannya, dan latihan. Sedangkan pada Buku teks matematika Jepang, sebagian besar diawali dengan ilustrasi mengenai topik yang dibahas, materi prasyarat, pertanyaan mendasar, ringkasan materi, contoh dan penyelesaiannya, pengayaan, dan latihan soal. Pada buku teks sekolah menengah Singapura, lebih ditekan kan pada domain Geometri dan Pengukuran, sedangkan Buku teks sekolah menengah Jepang lebih ditekankan pada domain Relasi dan Aljabar, kemudian urutan selanjutnya domain Geometri dan Pengukuran.
Kata Kunci: Buku Teks; Matematika; Sekolah Menengah.

Abstract

Continuous improvement of mathematics teaching is a continual process. The ground of the improvement is that mathematics is not only studied by students throughout the year but it is also widely used as a key indicator of students' success. One of the important factors that influence the teaching of mathematics is textbooks. This study discussed mathematics textbooks from two countries, namely Singapore and Japan. The aim was to examine the textbooks' physical character, structure, and topic/content. The study pointed out several important points. The Singaporean Math Textbook is physically simpler in colour than the Japanese Math Textbook. It also had more pages than the Japanese Textbook. Regarding the book's structure, several chapters in the Singaporean Mathematics Textbook began with a summary of the prerequisite material, followed by key topics, examples and solutions, and exercises. In Japanese Mathematics Textbooks, most chapters started with illustrations on the topics discussed, prerequisite material, essential questions, summaries, examples and solutions, enrichment, and exercises. In the Singaporean secondary school textbooks, it emphasized the Geometry and Measurement domains, while the Japanese secondary school textbooks emphasized the Relation and Algebra domains followed by the Geometry and Measurement domains.
Keywords: Mathematics; Secondary School; Textbooks.

I. INTRODUCTION

Education systems around the world have made maths a school subject that is not only studied by students throughout the year but is also widely used as a key indicator of students' success (Smith & Morgan, 2016). Therefore, the main concern of mathematics educators is to continuously improve their mathematics teaching skill. However, it is a complex activity and is influenced by a lot of factors. One of the factors is the mathematics textbooks.

In the last few decades, there have been many studies that compared mathematics curricula and mathematics textbooks from various countries, such as studies conducted by Boonlerts & Inprasitha, 2013; Fan, Mailizar, Alafaleq, & Wang, (2018); Jones & Fujita, (2013); J. Wang & Lu, 2018; Özer & Sezer, (2014); Yang et al., (2017). The textbook is one of the important curriculum materials to predict students' performance in learning mathematics (Yang et al., 2017). A study conducted by Yeap (Özer & Sezer, 2014) concluded that textbooks play an important role in the mathematics achievement of Singaporean students.

Textbooks are essential in the learning process (Dewantara, 2019); Purnomo, Mastura, & Perbowo, 2019) as it guides teacher to plan the lesson (Alajmi, 2012). Yang, Tseng, & Wang, (2017) (2017) argued that textbooks are a good source provide students' learning opportunities as well as a good indicator to measure students' abilities. Results from the Trends in International Mathematics and Science Study (TIMSS) showed that more than half of secondary school students in countries such as Australia, Finland, Singapore, Sweden, South Africa, and Canada were taught in environments where mathematics textbooks supported the basis of the learning activities. (Jäder, Lithner, & Sidenvall, 2020). Mathematics textbooks exert a lot of influence on the learning process of mathematics (Fan, Zhu, & Miao, 2013). It occurs because textbooks are often used by teachers to adjust the learning activities and approaches. NCTM stated that students' learning opportunities were determined by their engagement with mathematical problems. Accordingly, problems in mathematics textbooks are the basis of students' opportunities to learn mathematics.

Since the 1990s there have been several international assessment programmes, including the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). The Trends in International Mathematics and Science Study (TIMSS) organised by the International Association for the Evaluation of Educational Achievement (IEA) reported the level of mathematical competence of students from several countries. Results from TIMSS 2019 revealed that Singapore was among the countries whose students achieved much higher levels of mathematical competence than other countries. The students were among the best performing students in the world, with an assessment score of 625. And the results of PISA 2018 showed that the mathematical competence of Singaporean students was ranked 2nd with an assessment score of 569. While the results of TIMSS 2019 pointed out that Japanese

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students exhibited high mathematical competence, ranked 5th with an assessment score of 593, and the 2018 PISA results also pointed out that the mathematical competence of Japanese students was ranked 6th with an assessment score of 527 (Mullis, Martin, Foy, Kelly, & Fishbein, 2020; Schleicher, 2018).

Considering the mathematics learning achievements of Singaporean and Japanese students, it is necessary to review the mathematics textbooks from both countries as a great number of problems still exist in mathematics learning achievement of Indonesian students. It was pointed out by low mathematics achievement of Indonesian students in PISA and TIMSS. The results of this study were expected to benefit the quality improvement of mathematics textbooks in Indonesia, to such an extent that they were able to support the quality of mathematics learning. The research questions were, namely: (a) What are the physical characteristics of mathematics textbooks from Singapore and Japan?; (b) What is the lesson structure of mathematics textbooks from Singapore and Japan?; and (3) What topics are found in mathematics textbooks from Singapore and Japan?

II. METHOD

This study used the content analysis method. The method aimed to explore and reveal the natural conditions of an object in the form of verbal, visual, and written data to describe and measure certain facts (Downe-Wamboldt, 1992). Using this method, the researcher examined and identified objects in a structured manner. This study involved documents as the sources, namely mathematics textbooks of Singaporean and Japanese secondary schools. The selected secondary school mathematics textbooks from Singapore were targeted grades 7 to 10, and the selected mathematics textbooks from Japan were targeted grade 7 to 11.

The Singapore mathematics textbook was the Singapore New Syllabus Mathematics (NSM) grades 7-10, written by Dr Joseph Yeo PhD, MEd, PGDE (Dist), BSc (Hons), Teh Keng Seng BSc, Dip Ed, Loh Cheng Yee BSc, Dip Ed, and Ivy Chow MEd, PGDE, BSc. The textbooks were based on the curriculum guidelines of the Singaporean Ministry of Education. Singapore's New Syllabus Mathematics (NSM) had the highest market share (about 80%) of mathematics textbooks in Singapore, and had been printed a few times until the 7th edition. The textbook contained 41 units (Yang et al., 2017). The Japanese mathematics textbook was Mathematics for Junior High School (grades 7-9), Mathematics 1: Japanese Grade 10 and Mathematics 2: Japanese Grade 11. Mathematics for Junior High School was published by the Indonesian Ministry of Education and Culture in 2018. This textbook was a translated mathematics textbook entitled "Study with Your Friends Mathematics for Junior High School". The Japanese textbook had been translated into English. Based on the review of SEAMEO Qitep in Mathematics in 2015, this mathematics textbook was considered to be a high-quality book. The textbook contained 21 units. Mathematics 1:

Japanese Grade 10 and Mathematics 2: Japanese Grade 11 were published by the American Mathematica Society. The mathematics textbooks were originally in Japanese and had been translated into English by The University of Chicago School Mathematics Project (UCSMP). The textbooks contained 13 units.

This study was developed to describe the characteristics of mathematics textbooks. The analysis focused on three aspects, namely: Physical characteristics of mathematics textbooks, Lesson structure, Topics found in mathematics textbooks.

The analysis of the physical characteristics included the number of all pages in each book series, the number of units or chapters in each book series. The lesson structure included the presentation of the lesson in each chapter. The table of contents in each book series was examined to determine the topic list of the book. To obtain a more detailed description, the content of each unit was examined to ensure the accuracy. Prior to that, the researcher divided the mathematics content domains presented in the textbooks into four domains, namely: the domains of Numbers and Operations, Geometry and Measurement, Relations and Algebra, Statistics and Probability.

III. RESULT AND DISCUSSION

Mathematics textbooks are likely to contribute in improving the quality of the learning process. In recent years the number of studies related to mathematics textbooks has increased. The results of a study conducted by Chang & Silalahi, (2017) showed that of the 44 studies reviewed, some of the subject topics taught in mathematics textbooks showed standard analysis and distributive properties were the largest topics, especially those related to standard analysis, and problem solving was the highlighted topic of the book. In addition, the five mathematical contents in the mathematics textbooks showed that Numbers and Operations, as well as Algebra and arithmetic, predominated the subject topics. Furthermore, addition and subtraction were the most studied topics.

In this section, the results of the analysis on the textbook physical characteristics, the structure of the lessons, and the topics were described.

A. Physical Characteristics of the Maths Textbooks

Singaporean mathematics textbooks displayed a simple cover with a picture of a white notebook and a drawing compass on it, and numbers indicating the grade level. Each grade level in Singaporean maths textbooks was covered in one volume. Japanese mathematics textbooks was designed with coloured covers, and numbers indicating the grade level. Similar to the Singapore textbooks, each grade level in the Japanese maths textbooks was covered in one volume. In the Japanese textbooks, before the table of contents, there was a page containing instructions on how to use the textbook. Singapore textbooks had more pages than Japanese mathematics textbooks (about 1,836), while Japanese textbooks consisted of 1,409 pages. The physical characteristics of the textbooks are presented in Table 1.

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Table 1.
Physical Character in Secondary School Mathematics Textbooks

	Class	Number of Pages in the Textbook	Percentage of Pages in Each Maths Content Design				Number of chapters	Number of chapters in each Maths Content Domain			
			Numbers and Operations	Geometry and Measurement	Relations and Algebra	Statistics and Probability		Numbers and Operations	Geometry and Measurement	Relations and Algebra	Statistics and Probability
Singapore	7	458	0,18	0,30	0,28	0,09	15	3	5	5	2
	8	558	-	0,32	0,43	0,18	17	-	5	9	3
	9	440	0,09	0,68	0,13	-	11	1	8	2	-
	10	380	0,08	0,28	0,21	0,32	8	1	2	2	3
	Total	1.836	0,08	0,39	0,27	0,15	51	5	20	18	8
Japan	7	304	0,16	0,23	0,33	0,08	7	1	2	3	1
	8	252	-	0,27	0,36	0,09	6	-	2	3	1
	9	308	-	0,28	0,38	0,05	8	-	3	4	1
	10	261	0,10	0,28	0,50	-	5	0,5	2	2,5	-
	11	284	-	0,33	0,23	0,66	8	-	3	2	3
	Total	1.409	0,05	0,28	0,36	0,18	34	1,5	12	14,5	6

The Geometry and Measurement domain in Singapore maths textbooks covered a higher percentage of pages compared to the other domains (about 39%), followed by the Relations and Algebra domain (about 27%). According to Table 1, Singaporean maths textbooks highly concerned on the Geometry and Measurement domain in Grade 9, which was almost twice as many pages as the other grades (about 68%). In the domain of Relations and Algebra, it was emphasized in Grade 8, which was almost three times the number of pages of Grade 9 textbooks (about 43%). the Numbers and Operations domain were emphasized at grade 7 (about 18%). Whereas in Singapore maths textbooks in the Statistics and Probability domain, the greatest emphasis was on grade 10 (around 32%). Grade 7, 9, and 10 mathematics textbooks concerned with the Geometry and Measurement domain (about 30%, 68%, and 28%), in contrast to the other grade mathematics textbooks, Grade 8 mathematics textbooks highlighted the Relations and Algebra domain (about 43%), followed by the Geometry and Measurement domain.

In Japanese mathematics textbooks, the highest percentage of pages contained Relations and Algebra, followed by the domain of Geometry and Measurement (about 36% and 28%). Grade 7, 8, 9 and 10 textbooks emphasised the domain of Relations and Algebra (about 33%, 36%, 38%, 50%). In the 11th grade Japanese mathematics textbooks, the Statistics and Probability domain was highlighted (about 66%). In the domain of Relations and Algebra, it was highlighted on Grade 10, which had more pages than Grade 9 textbooks (about 50%). The Geometry and Measurement domain was the main topics on grade 11 (about 33%). The Statistics and Probability domain were highlighted on grade 11, which had almost five times as

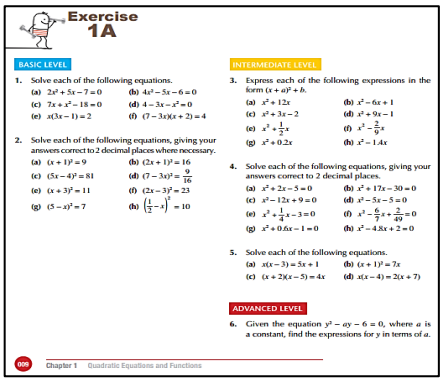
many pages as the grade 9 textbook (about 66%). And in the Numbers and Operations domain, it was presented in grade 7 (about 16%).

In both Singaporean and Japanese mathematics textbooks, the Numbers and Operations domain covered the least percentage of the books (around 8% and 5% respectively). There were more topics in Singaporean mathematics textbooks than in the Japanese textbooks, pointed out by the number of chapters in Singapore mathematics textbooks. As showed by table 2, the number of chapters in Singapore maths textbooks was 51, while Japanese maths textbooks were 34 chapters.

B. Lesson Structure

Singaporean maths textbooks consisted of chapters starting with a recap of the prerequisite material, followed by the basic material, examples and solutions, and a series of exercises. These exercises were similar to the examples. In between the material, students were given the opportunity to explore their knowledge.

The textbooks provided an opportunity to investigate, discuss and reflect. Each sub-chapter in the Singaporean textbooks always ended with a practice problem before entering the next sub-chapter. Each problem was categorised into three levels of difficulty: easy, medium and difficult (Figure 1).


Exercise 1A

BASIC LEVEL

1. Solve each of the following equations.
(a) $2x^2 + 5x - 7 = 0$ (b) $4x^2 - 5x - 6 = 0$
(c) $7x + x^2 - 18 = 0$ (d) $4 - 3x - x^2 = 0$
(e) $x(3x - 1) = 2$ (f) $(7 - 3x)(x + 2) = 4$

2. Solve each of the following equations, giving your answers correct to 2 decimal places where necessary.
(a) $(x + 1)^2 = 9$ (b) $(2x + 1)^2 = 16$
(c) $(5x - 4)^2 = 81$ (d) $(7 - 3x)^2 = \frac{9}{16}$
(e) $(x + 3)^2 = 11$ (f) $(2x - 3)^2 = 23$
(g) $(5 - x)^2 = 7$ (h) $\left(\frac{1}{2} - x\right)^2 = 10$

INTERMEDIATE LEVEL

3. Express each of the following expressions in the form $(x + a)^2 + b$.
(a) $x^2 + 12x$ (b) $x^2 - 6x + 1$
(c) $x^2 + 3x - 2$ (d) $x^2 + 9x - 1$
(e) $x^2 + \frac{1}{2}x$ (f) $x^2 - \frac{2}{9}x$
(g) $x^2 + 0.2x$ (h) $x^2 - 1.4x$

4. Solve each of the following equations, giving your answers correct to 2 decimal places.
(a) $x^2 + 2x - 5 = 0$ (b) $x^2 + 17x - 30 = 0$
(c) $x^2 - 12x + 9 = 0$ (d) $x^2 - 5x - 5 = 0$
(e) $x^2 + \frac{1}{4}x - 3 = 0$ (f) $x^2 - \frac{6}{5}x + \frac{2}{49} = 0$
(g) $x^2 + 0.6x - 1 = 0$ (h) $x^2 - 4.8x + 2 = 0$

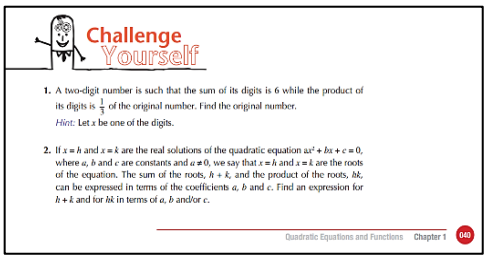
5. Solve each of the following equations.
(a) $x(x - 3) = 5x + 1$ (b) $(x + 1)^2 = 7x$
(c) $(x + 2)(x - 5) = 4x$ (d) $x(x - 4) = 2(x + 7)$

ADVANCED LEVEL

6. Given the equation $y^2 - ay - 6 = 0$, where a is a constant, find the expressions for y in terms of a.

Figure 1. Examples of sub-chapter exercises in Singaporean maths textbooks

After a series of material and exercises, the textbooks provided the summary of the material covered. It was followed by a series of end-of-chapter exercises presented in the form of descriptions and ended with a *challenge yourself,* which helped students to reinforce students' comprehension (Figure 2).


Challenge Yourself

1. A two-digit number is such that the sum of its digits is 6 while the product of its digits is $\frac{1}{3}$ of the original number. Find the original number.
Hint: Let x be one of the digits.

2. If $x = h$ and $x = k$ are the real solutions of the quadratic equation $ax^2 + bx + c = 0$, where a, b and c are constants and $a \neq 0$, we say that $x = h$ and $x = k$ are the roots of the equation. The sum of the roots, $h + k$, and the product of the roots, hk, can be expressed in terms of the coefficients a, b and c. Find an expression for $h + k$ and for hk in terms of a, b and/or c.

Figure 2. Example of *Callenge Yourself* in a Singapore maths textbook

There were four chapters in Japanese mathematics textbooks at each grade level started with a review of the prerequisite material or material that had already been learned and would be used in the chapter being discussed (Figure 3 and 4). then it was followed by a series of basic questions. The textbook briefly presented the material related to what was discussed in the chapter. There were also examples of problems and how to solve them, enrichment and practice problems to understand the material. There was also a section to explore students' knowledge through the investigation and observation

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columns. Each sub-chapter in the Japanese textbook mostly ended with enrichment presented in the form of a description before entering the next sub-chapter. After a series of materials and exercises, Japanese maths textbooks always concluded the chapter with a series of end-of-chapter exercises presented in the form of descriptions.

Figure 3. Examples of illustrations in Japanese maths textbooks

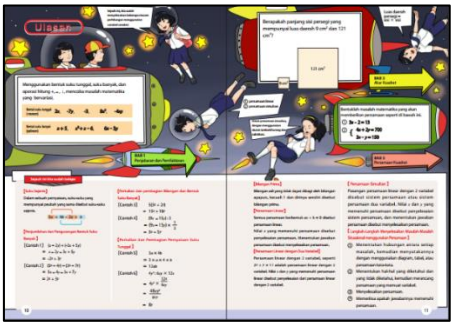

Figure 4. Example of a review in a Singapore maths textbook

C. Maths Content Found in Maths Textbooks

Table 2 and Table 3 presents the distribution of topics from the four domains covered in Singaporean and Japanese secondary school mathematics textbooks based on grades. In Singaporean textbooks for grades 7 and 10, the topics covered all the content domains of mathematics, while in grades 8 and 9 only some domains were covered. In the mathematics textbooks, the domains of Geometry and Measurement and Relations and Algebra were found in grades 7, 8, 9, and 10, while the domains of Numbers and Operations were only found in grades 7, 9, and 10. The domains of Statistics and Probability were only found in grades 7, 8, and 10 mathematics textbooks.

Table 2.
Topics Covered in Singapore Textbooks at Each Grade Level

Domain	Singapore			
	Grade 7	Grade 8	Grade 9	Grade 10
Numbers and Operations	▪ Prime Numbers, Greatest Common Factor and Least Common Multiple ▪ Integers, Rational Numbers and Real Numbers ▪ Number pattern		▪ Power numbers and standardised forms	▪ The set ▪ Numbers, and percentages
Geometry and Measurement	▪ Basic Geometry (Point, Line, Plane, Angle, Angle formed by two parallel lines cut by a transversal line)	▪ Conformity and Kesebang ▪ Geometric Transformation ▪ Trigonometric Ratios ▪ Volume and	▪ Coordinate System ▪ Function Graphs and Solutions ▪ Advanced trigonometry	▪ Vector ▪ Geometry and Measurement (Angles of triangles and polygons, congruence

	▪ Triangles, Rectangles and Polygons ▪ Geometric Construction ▪ Perimeter and Area of a Flat Plane ▪ Volume and Surface Area of Prisms and Cylinders	Surface Area of Rectangular Limas, Cones and Spheres ▪ Symmetry ▪ The set ▪ Pythagorean Theorem	▪ Trigonometry Applications ▪ Arc Length, Sector Area and Radian Measure ▪ Test for similarity and congruence ▪ Area of flat shapes and Volume of congruent spaces ▪ Circle	and congruence, pythagorean theorem and trigonometry, measurement, coordinate system, vectors in two dimensions, properties of circles) ▪
Relations and Algebra	▪ Basic Algebra and Algebraic Manipulation ▪ Linear Equations and Simple Inequalities ▪ Linear Functions and Graphs ▪ Percentage ▪ Ratio, Scale, and Speed	▪ Equivalent Comparison and Inverse Comparison ▪ Linear Graphs and Simultaneous Linear Equations ▪ Translation and Factoring of Quadratic Equations ▪ Further Translation and Factoring of Quadratic Equations ▪ Quadratic Equations and Graphs ▪ Fractions and Algebraic Formulas ▪ Relationships and Functions	▪ Quadratic Equations and Functions ▪ Linear Inequality	▪ Matrix ▪ Comparison, Ratio, Scale and Speed ▪ Algebraic Manipulations and Formulas ▪ Equations and Inequalities ▪ Functions and Graphs ▪ Graphs in Practical Situations
Statistics and Probability	▪ Forecasts and Estimates (data and uncertainty) ▪ Statistical Data Processing	▪ Chance of a Single Event ▪ Statistical Diagram ▪ Average Statistical Data		▪ Chance of Occurrence ▪ Statistical Data Analysis ▪ Probability and Statistics

Table 3.
Topics Covered in Japanese Textbooks at Each Grade Level

Domain	Japan				
	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11
Numbers and Operations	▪ Positive and Negative Numbers			▪ Real Numbers ▪ Numbers and Sets	
Geometry and Measurement	▪ Flat Buildings	▪ Investigating the	▪ Equivalence	▪ Point Coordinates	▪ Vector ▪ Differentials

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	▪ Build Space	Properties of Geometric Buildings ▪ Triangle and Quadrilateral	▪ Circle ▪ Pythagorean Theorem	▪ Straight Line ▪ Circle ▪ Inequality Regions ▪ Trigonometric Ratios	and Their Applications ▪ Integrals and their Applications (Sure and Indefinite Integrals)
Relations and Algebra	▪ Algebraic Forms, Maths Sentences ▪ Linear Equation ▪ Equivalent Comparison and Inverse Comparison	▪ Simplifying Algebraic Forms ▪ System of Equations ▪ Linear Function	▪ Translation and Factoring Square Root ▪ Quadratic Equation ▪ Function	▪ Integral Expression ▪ Integral and fraction types ▪ Equations and Inequalities ▪ Algebraic Expressions and Proofs ▪ Function	▪ Function Types (Exponential Function, Logarithm Function, Trigonometric Function) ▪ Sequences (Arithmetic and Geometric Sequences)
Statistics and Probability	▪ Using Data	▪ Opportunities	▪ Sample Survey		▪ Probability ▪ Statistics ▪ Computers and Flowcharts

After analysing the content of the secondary school mathematics textbooks from the two countries, the study pointed out that in Singapore's mathematics textbooks the Geometry domain took a larger percentage compare to the other three domains and the next one was the Algebra domain. Meanwhile, in Japanese mathematics textbooks the Algebra domain predominated. There were some topics in Singaporean textbooks that were taught earlier than in Japan. Trigonometric ratios, Pythagorean theorem, linear functions, quadratic equations, fractions, probability and statistics were also taught earlier in Singaporean mathematics textbooks than in Japan.

From the descriptions in Tables 2 and 3, there were some topics in Singaporean secondary school mathematics textbooks that were not presented in Japanese mathematics textbooks. In the Numbers and Operations domain, Japanese mathematics textbooks only presented types of numbers and sets, while the topics of greatest common factor, least common multiple, number patterns, percentages, signed numbers and standard forms were not presented. In the Geometry and Measurement domain, the topics of symmetry, coordinate systems, graphs of functions and their solutions, advanced trigonometry, applications of trigonometry, arc length, sector area and radian measure were not presented in the Japanese textbooks, while the topics of inequality regions, differentials and their applications, integrals and their applications (definite and indefinite integrals) were not presented in the Singaporean secondary

school mathematics textbooks. In the domain of Relationships and Algebra, the topics of percentages, ratios, scales and rates, linear graphs and simultaneous linear equations, translation and factoring of quadratic equations, Relationships and functions, linear inequalities, matrices, Algebraic manipulations and formulae, functions and graphs were not presented in Japanese secondary school mathematics textbooks, whereas the topics of quadratic inequalities, simultaneous equations, polynomials, square roots, functions (types of functions and sets and functions), Algebraic proofs (proving equations, proving inequalities, direct and indirect proofs), series (arithmetic and Geometric series), systems of equations were not presented in Singapore secondary school mathematics textbooks. In Statistics and Probability, the topics of estimation and forecasting (data and uncertainty), statistical diagrams, statistical data analysis were not presented in Japanese secondary school mathematics textbooks, while the topics of computers and flowcharts, sample surveys were not presented in Singapore secondary school mathematics textbooks.

This study analysed mathematics textbooks used in two different countries: Singapore and Japan. The analysis focused on describing the physical features of the books, the structure of the lessons, and the mathematical content of the textbooks.

D. Singaporean Maths Textbook

Singapore's ministry of education has the authority to approve textbooks to be printed and distributed to students. it involves publishers to develop textbooks based on the syllabus. To maintain the quality of Singapore's textbooks, the the textbook was reviewed by curriculum specialists, teachers, and academics from universities in Singapore. The use of textbooks in Singapore schools is not a mandatory, but if schools prefer to use ones, they are instructed to use textbooks that have been approved by the Singapore Ministry of Education. (Oates, 2014).

Singapore maths textbooks offered more opportunities for effective teaching of Geometry and Measurement, as seen from Singapore maths textbooks, the Geometry and Measurement domain in Singapore maths textbooks covered a higher percentage of page counts compared to other domains. The second highest topic coverage dealt with the domain of Relationships and Algebra. The Singaporean maths textbooks provided space for investigation, discussion, and thinking time, with the purpose that students were encouraged to explore their knowledge. Some secondary school topics were taught earlier than in Japan.

In the mathematics textbooks, each question at the end of a sub-topic was grouped into three levels of difficulty: easy, medium, and difficult. The results of a study conducted by Vicente, Sánchez, & Verschaffel (2020) pointed out some example problems in Singapore textbooks illustrating that there were several problem-solving steps, including: Information (collecting the given information); Reasoning, Operation; Questioning; Rechecking (Rechecking the results). The study conducted by Özer & Sehazer, 2014 explained that Singaporean students were introduced to mathematical

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concepts one or two years earlier than Turkish students.

Singaporean mathematics textbooks emphasized compound problems that required the use of two or three forms of representation (i.e., pure mathematical, oral, visual, and compound forms) (Wang & Yang, 2016). It appeared that Singaporean textbooks provided relatively balanced opportunities to solve problems with multiple representations (pure mathematics, oral, visual, and combined forms) for students (Yang et al., 2017). Singaporean students tended to have more experience with problems in visual and combined forms, as shown by the results of a study conducted by Yang et al. (2017) that Singaporean mathematics textbooks had a higher percentage of problems in visual and compound forms, (about 33.7%). In addition, the Singapore primary school mathematics textbooks (MPHM) concerned on visual forms combined with other forms of representation (Wang & Yang, 2016). A study conducted by Yang et al. (2017) showed that non-application problems in Singapore textbooks were more dominant than application problems (about 86.2%). This suggested that the problems in Singapore textbooks were not contextualised in real-world situations. In addition, Singapore had a higher percentage of closed questions (z = 11.8) (Wang & Yang, 2016).

E. Japanese Maths Textbook

Japanese mathematics educators claimed that Japanese mathematics teaching had changed significantly over the past 50 years. It was influenced by various factors, one of which was the textbooks. Textbooks plays an important role in the learning process, what is taught and how mathematics is taught, thus, textbooks significantly affect students' opportunities to learn (Watanabe, 2014). The topics on the textbooks for grades 7, 8, 9 and 10 emphasised the domain of Relations and Algebra. This was supported by the percentage of the number of pages in the domain of Relations and Algebra was higher than other domains, followed by the domain of Geometry and Measurement.

Japanese mathematics textbooks covered a higher percentage of pages than others in the domain of Relations and Algebra, followed by the domain of Geometry and Measurement. Japan considered reasoning in Geometry to be important (Jones & Fujita, 2013). This was confirmed by a study conducted by Jones & Fujita, (2013) stating that the content in Japanese mathematics textbooks focused more on Geometric proofs using congruence. Congruence of Geometric figures was the content to introduce mathematical proofs in the Japanese curriculum. This was in line with the dominance of Geometric proofs in the Geometry curriculum in Japan (Miyakawa, 2017).

Like other countries, Japanese maths textbooks had also changed over time. The latest edition of the Main Mathematics Textbook series included more resources to assist teachers in teaching through problem solving and to help students learn through problem solving. The book series contained more alternative approaches to problem solving, providing diagrams

intended to help students solve problems independently. In addition, Japanese mathematics textbooks included a separate page teaching students how to take notes effectively, and each textbook included several examples of how to take notes to develop students' mathematical thinking and problem-solving skills (Takahashi, 2016).

In Japanese mathematics textbooks, it concerned more on the direct proof of Geometry statements, accompanied by activities that encouraged students to form conjectures. Besides, the Japanese textbooks provided direct proof arguments, especially in exercises and sample questions (Fujita & Jones, 2014). The questions in Japanese mathematics textbooks were more complex and difficult. (Özer & Sehazer, 2014). The way mathematical proofs constructed in the textbooks was through proving various geometric statements (Fujita & Jones, 2014).

IV. CONCLUSION

The conclusions of this review were as follows. In Singapore's mathematics textbooks, some chapters began with a recap of prerequisite material, followed by basic material, examples and solutions, and a series of exercises. Meanwhile, in Japanese mathematics textbooks, the chapter was started by some illustrations of images on the topic and ended with a review of the prerequisite material then followed by a series of basic questions to get to know the material being discussed, a summary of the material, examples and solutions, enrichment and exercises to understand the material. In terms of content, mathematics textbooks from both countries contained similar content/topics. Singaporean secondary school textbooks emphasized the Geometry and Measurement domain, whereas Japanese secondary school textbooks emphasized the Relation and Algebra domain, followed by the Geometry and Measurement domain.

Related to the results of this study, it was suggested to to design mathematics textbooks as Singapore and Japanese textbooks, especially dealing with the structure of the book. The basis was that both countries were eminent in mathematics worldwide.

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