p-ISSN: 2086-4280
e-ISSN: 2527-8827

Students' Errors in Solving Matrix Multiplication Problems Based on Kastolan Theory

Kartini[1]*, Tuti Alawiyah[2]

Mathematics Education Masters Study Program, Universitas Riau
Kampus Bina Widya KM 12.5, Simpang Baru, Tampan, Pekanbaru, Riau, Indonesia
[1]*kartini@lecturer.unri.ac.id; [2]tutialawiyah094@gmail.com

Artikel received: 07-07-2022, revised: 16-01-2023, published: 31-01-2023

Abstrak

Matriks merupakan salah satu materi yang diajarkan di sekolah. Namun kebanyakan siswa masih melakukan kesalahan dalam menyelesaikan soal materi matriks terutama pada perkalian matriks. Oleh karena itu perlu dilakukan analisis kesalahan yang dilakukan siswa dalam mengerjakan soal perkalian matriks untuk meningkatkan efektivitas pembelajaran. Tujuan penelitian ini adalah menganalisis kesalahan siswa dalam menyelesaikan soal perkalian matriks. Jenis penelitian ini adalah deskriptif kualitatif. Teknik pengumpulan data yang digunakan pada penelitian ini adalah teknik tes dan wawancara. Subjek penelitian terdiri dari 28 siswa kelas XI SMK Perpajakan Riau tahun ajaran 2021/2022. Analisis kesalahan siswa dilihat berdasarkan teori Kastolan yang terdiri dari kesalahan konseptual, kesalahan prosedural, dan kesalahan teknik. Hasil analisis kesalahan siswa dalam menyelesaikan soal perkalian matriks menunjukkan bahwa sebanyak 47,9% siswa melakukan kesalahan konseptual, 16,4% melakukan kesalahan prosedural, dan sebanyak 35,6% siswa melakukan kesalahan teknik.
Kata Kunci: kesalahan konseptual; kesalahan prosedural; kesalahan teknik; perkalian matriks; teori kastolan.

Abstract

Matrix is one of the materials taught in schools. However, most students still make mistakes in solving matrix material problems, especially in matrix multiplication. Therefore, it is necessary to analyze the errors made by students in working on matrix multiplication problems to increase the effectiveness of learning. The purpose of this study was to analyze the students' errors in solving matrix multiplication problems. This descriptive qualitative study collected the data by utilizing test and interview techniques. The participants consisted of 28 students of class XI of the Riau Taxation Vocational School for the 2021/2022 academic year. Analysis of student errors was based on Kastolan's theory which encompassed conceptual errors, procedural errors, and technical errors.
Keywords: conceptual errors; kastolan theory; matrix multiplication; procedural errors; technical errors.

I. INTRODUCTION

Education is one of the most important things in life. The human mind improves by education processes. The education process ideally develops the thinking processes of students starting from low level to higher level of thinking (Sari, Sukestiyarno, & Walid, 2022). Mathematics is a subject taught to students at school.

Mathematics plays an important role in life. It is a science used in almost all other branches of science (Kumar, 2017; Masfufah & Afriansyah, 2022). Various concepts and theorems of mathematics can be used in calculations and judgments in various fields. Mathematics trains critical, logical, analytical and systematic abilities in order to master other knowledge (Maryam & Zanthy, 2019; Pratama & Mardiani, 2022). In an effort to achieve the goals of learning mathematics, there are a number of obstacles that impede the achievement of the learning goals. One of the challenges in teaching mathematics is the perception of students who consider mathematics difficult (Sari et al., 2020). The difficulties are likely to create errors in the process of interpreting concepts. This will result in a decrease in students' mathematical thinking abilities.

Matrix material is one of the materials taught in mathematics. It is a mathematical lesson provided to class XI students requiring several basic competencies to be achieved. The materials studied in the matrix are types of matrices, matrix transpose, matrix similarity, matrix arithmetic operations, matrix inverse and matrix determinants. According to Khairani and Kartini (2021), there were many students who made errors on matrix problems. It occurred because the students did not understand the concept of the matrix as a result students unabled to solve the problems given.

Based on the study by Yesino et al. (2020), errors often made by students included errors in performing matrix multiplication operations, errors in solving matrix similarities, errors in solving systems of linear equations related to matrices, and errors in solving matrix determinants. This was in line with the findings of Ainin et al. (2020) that there were 9 types of errors made by students in matrix material, namely errors in changing word problems into matrix form, errors in determining the inverse matrix formula, errors in understanding the concept of matrix equations, errors in determining adjoin matrices, errors in operating numbers, errors in determining determinants matrix, errors in continuing the solving process, errors in matrix multiplication, and errors in determining the final result. Matrix multiplication is a sub-matrix of the matrix that students should master. Practically, matrix multiplication is needed to solve problems related to the equality of two matrices, the determinant of a matrix, the inverse of a matrix, and other basic algebraic operations (Sari et al., 2020).

According to Widodo (2016), also Arnandi, Siregar, and Fitriawan (2022), students had different levels of difficulty in mathematics. The difficulties experienced by students could increase the chances of errors occurring when solving math problems in other subject matter (Ana & Nusantara, 2021). Factors triggering student errors in working on problems was the lack of student mastery of the concepts

p-ISSN: 2086-4280
e-ISSN: 2527-8827

and theories learned in mathematics (Mutmainah & Sari, 2019). Errors that occurred when students completed the quesion need to be analyzed. As stated by Ainin et al. (2020), students' errors in working on questions must be identified so that the teacher could be aware of various students' errors.

The analysis aimed to find out the actual events, describe the main issues and examine parts and find the relationship between the parts to obtain a conclusion (Fitriyah et al., 2020). Error analysis is a way to examine, observe, and classify errors specifically (Wardhani & Chotimah, 2021). Error analysis was carried out to obtain the types and causes of errors made by students, with the purpose that the teacher was able to find the appropriate solution as it increased the effectiveness of learning (Yesino et al., 2020).

Based on the previous analysis and studies, it was necessary to do an analysis to find out types of the students' errors in solving matrix multiplication problems. Error analysis as a rule consists of many theories that form the basis of analysis. Mathematical errors in this study were analysed based on Kastolan's theory as a benchmark for grouping the types of errors made by the students. There are three types of errors according to Kastolan, namely conceptual errors, procedural errors, and technical errors (Ulfa & Kartini, 2021; Hendriyanto et al., 2022). Conceptual errors are errors that emerge because students misuse or do not interpret concepts related to the problem (Fitriyah et al., 2020). Procedural errors are errors that emerge because students are unable to find a solution to a mathematical problem (Lutfia & Zanthy, 2019). Technical errors occur due to a lack of student accuracy in determining the results of arithmetic operations (Noviani, 2019). The findings of this study were expected to be able to provide an overview to teachers and students regarding the mistakes made by students in solving matrix multiplication problems. This study focused on describing the types of errors made by students in solving matrix multiplication problems based on Kastolan's theory.

II. METHOD

This qualitative descriptive study aimed to describe the errors made by students in solving matrix multiplication material problems. The study was conducted in the even semester of the 2021/2022 Academic Year. The participants were 28 students who had attended matrix lesson, namely class XI of the Riau Taxation Vocational School. The data collection used interview techniques and written tests consisting of five items describing the matrix multiplication material. The data analysis was carried out by testing the validity, reliability, and difficulty level of the questions.

The validity, reliability, and difficulty level of the five items were analyzed using SPSS version 26. The validation results for each item using SPSS are shown in Table 1.

Tabel 1.
Item Validation Results Using SPSS

Question Number	Correlation	Qualification
1	0.720	Very Significant
2	0.742	Very Significant
3	0.741	Very Significant
4	0.931	Very Significant

5	0.909	Very Significant

The results of the validation test of the five items using SPSS showed that the five items were categorized as valid, meaning that the five items could be used to identify errors made by students. Furthermore, the results of the item reliability test showed a correlation value of 0.812 which was included in the very high category.

In addition, an analysis was carried out to review the level of difficulty of the questions. The results of the analysis of the difficulty level of the questions using SPSS are presented in Table 2.

Tabel 2.
Results of Calculation of Item Difficulty Level Using SPSS

Question Number	Correlation	Qualification
1	0.714	Easy
2	0.571	Currently
3	0.321	Currently
4	0.392	Currently
5	0.392	Currently

The results of calculating the difficulty level using SPSS showed that question number one was in the easy category and the other four questions were in the moderate category.

III. Result and Discussion

Based on the results of the tests conducted on 28 students, there were 13 students who reached the passing grade, namely 75, while 15 other students were still under the passing grade. The highest score obtained by students was 100 and the lowest score was 4.4 with the average score of 63.7. It implied that there were a lot of errors made by the students.

Based on the data obtained, the students incorrectly answered the matrix multiplication problems. The details of the percentage of the students' results is presented in Figure 1.

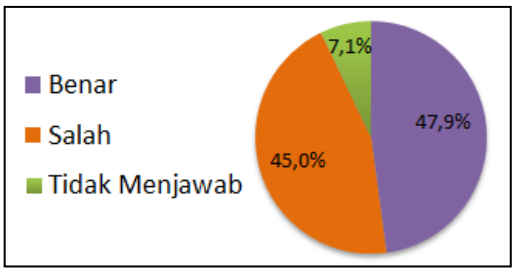

Figure 1. Percentage of Student Answers

Based on Figure 1, the percentage of students who correctly answered the matrix multiplication question was 47.9%, 45% answered incorrectly, and 7.1% did not answer. There were various types of errors that students made. The errors included conceptual errors, procedural errors, and technical errors. According to Kastolan's theory, the percentage of student errors for the three types of errors is presented in Figure 2.

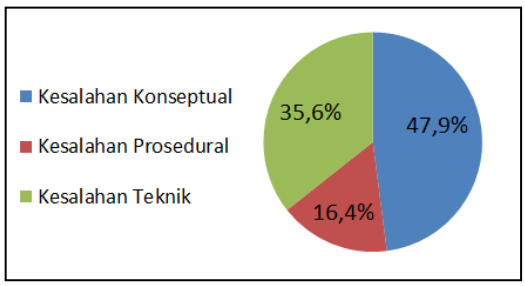

Figure 2. Percentage of Student Errors

A. Conceptual Error

Conceptual errors were errors that occur because students misunderstood or did not understand the concepts related to the problem. The indicators of conceptual errors according to Kastolan (Lestari, 2018) were 1) using the wrong formula, theorem or definition to solve a problem; 2) not

p-ISSN: 2086-4280
e-ISSN: 2527-8827

providing the formulas, theorems, or definitions to solve a problem; and 3) not providing an answer to the problem. In this study, the conceptual error indicators used were 1) misunderstanding the concept of matrix multiplication, ie, multiplying row and column elements incorrectly; and 2) not answering the questions given at all.

Based on Figure 2, it implied that the majority of students made conceptual errors with a percentage of 47.9%. The number of students who made misconceptions about the five questions were presented in Table 3.

Tabel 3.
Percentage of Concept Errors

Question Number	Many Students	Percentage
1	8	11.0%
2	6	8.2%
3	7	9.6%
4	5	6.8%
5	9	12.3%
Total	35	47.9%

Based on Table 3, it it demonstrated that the number of conceptual errors made by students on the five items wer 35 errors. From the student answer sheets, it was found that 47.9% of the students did not understand the concept of matrix multiplication. It was supported by the students' answers on the multiplication between 2×2 matrices presenting the final result in the form of a 2×1 matrix. In addition, there were some students who did not provide the completion steps at all for the questions given. Below is one of the student's answers that made a conceptual error.

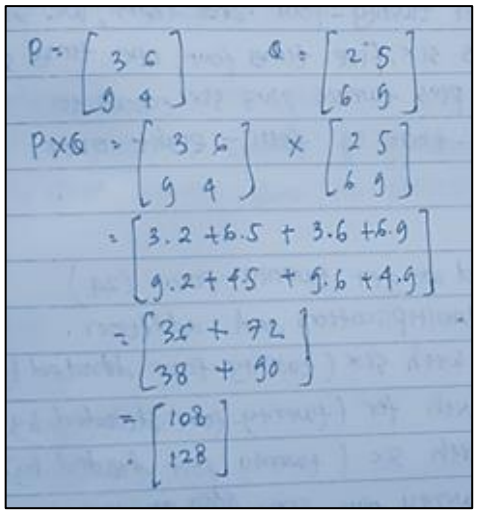

Figure 3. Percentage of Student Errors

Based on Figure 3, it was demonstrated that the SW-1 student's answered question number 2. SW-1 student made a conceptual mistake because SW-1 student did not understand the concept of matrix multiplication. Supposedly, the multiplication of two matrices is done by multiplying the number in the row of the first matrix by the number in the column of the second matrix. In addition, if there was a multiplication of two 2×2 matrices, then the result was a 2×2 matrix. From the interview results, SW-1 student stated that he did not understand the concept of matrix multiplication. SW-1 student assumed that the result of all multiplication matrices was a 2×1 matrix, namely adding up all the elements contained in each row of the matrix. The conceptual errors that occurred among students were in line with the results of the study by Natsir et al (2016) which found that the factors causing students to make conceptual errors were that they did not understand the concept and could not apply the concept.

B. Procedural Error

Procedural errors were caused by the the students' inability to solve math problems. The indicators of procedural errors according to Kastolan(Puji Lestari, 2018) were 1) the disorganization of the steps in solving the problem; 2) inability to manipulate steps to answer a problem; 3) did not solve the problem to its simplest form. The procedural error indicator presented in this study was not solving the problem to its simplest form.

According to Figure 2, the procedural error was the smallest number of errors made by students with a percentage of 16.4%. The percentage of procedural errors made by students on the five question items is presented in Table 4.

Tabel 4.
Percentage of Procedural Errors

Question Number	Many Students	Percentage
1	0	0.0%
2	1	1.4%
3	0	0.0%
4	7	9.6%
5	4	5.5%
Total	12	16.4%

Based on Table 4, it implied that the number of procedural errors made by the students on the five items was 12. From the student answer sheets, it was found that the reason of the errors was that the students had not been able to solve the matrix multiplication problem to its simplest form. Students only provided multiplication between rows and columns without calculating the results of the multiplication. The students argued that multiplication between 3×3 matrices require a lot of calculations, so the students were reluctant to solve them.

The following is a snippet of procedural errors made by the students on question number 4.

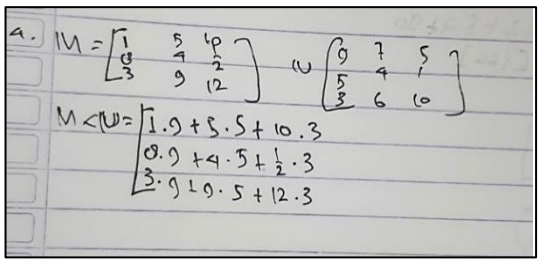

Figure 4. Percentage of Student Errors

Based on Figure 4, SW-2 students' answered to question number 4. SW-2 students made a procedural error because they did not complete the last step. From the results of the interviews, SW-2 student was having difficulty in calculating on large matrices because of lots of numbers that had to be calculated. The difficulties experienced by SW-2 student were due to lack of practices. Procedural errors made by students were also present in the study by Sari & Najwa(2021) which found that procedural errors occurred because students were not able to do the manipulation steps properly and perform integer operations causing the errors in determining the final result.

C. Technical Error

Technical error is an error that occurs due to lack of accuracy in determining the result of an arithmetic operation or in short negligence. The indicators of technical errors according to Kastolan (Puji Lestari, 2018) were an error in using calculation rules and in determining the result of a mathematical operation. Indicators of technical errors presented in this study were errors in the operation proces and

p-ISSN: 2086-4280
e-ISSN: 2527-8827

determining the results of an arithmetic operation.

Based on Figure 2, the percentage of students who made technical errors was 35.6%. The number of the students who made technical errors in the five questions is presented in Table 5.

Tabel 5.
Percentage of Technical Errors

Question Number	Many Students	Percentage
1	0	0.0%
2	5	6.8%
3	12	16.4%
4	5	6.8%
5	4	5.5%
Total	26	35.6%

Based on Table 5, there were 26 technical errors made by the students on the five items. From the student answer sheets, it was found that the students frequently made mistakes on the multiplication operations, especially the multiplication of negative numbers. In addition, the students also made mistakes in the sum of negative numbers.

The following is an excerpt of one of the technical errors made by students in question number 3.

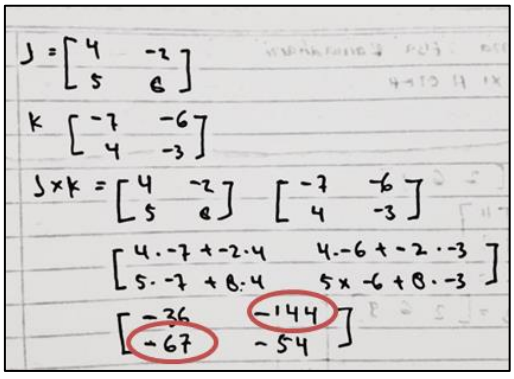

Figure 5. SW-3's Answer to Number 3

Based on Figure 5, SW-3 student answered question number 3. SW-3 student made a technical error because there was a mistake in calculating the result of an arithmetic operation. Based on the interview results, SW-3 student admitted that double-checking was supposedly done to the final result. The final result of question number 3 should be $\begin{bmatrix} -36 & -18 \\ -3 & -54 \end{bmatrix}$. SW-3 student made a calculation error on the elements of the first row, the second column and the elements of the second row, the first column. In the elements of the first row, the second column, the error was multiplying all the numbers, research$4 \times (-6) \times (-2) \times (-3)$, which resulted in -144. In the elements of the second row, the first column, student SW-3 incorrectly worked on the operation of negative numbers, $5 \times (-7) + 8 \times 4 = -35 + 32$. SW-3 student assumed that the results of $-35 + 32 = -67$ was correct. Technical errors made by students wew in line with Widyaningrum (2016) which stated that technical errors occurred because students were calculating hastily and were not working carefully on the questions.

IV. CONCLUSION

There were three types of errors from Kastolan's theory as the guide to classify the errors made by students, namely conceptual errors, procedural errors, and technical errors. From the results of the analysis, the study concluded that 1) students made conceptual errors of 47.9% including misunderstanding the concept of matrix multiplication, ie, multiplying row and column elements incorrectly; and did not answer the questions at all, 2) students

made procedural errors of 16.4% because they could not provide the final answer or the simplest form, and 3) technical errors made by students of 35.6% were due to errors in determining the result of an arithmetic operation.

Based on the conclusions, the study suggested that in the learning process, the teacher needed to pay attention to the level of students' understanding of the concept of the material being taught, especially matrix multiplication material by encouraging students that if the $A_{m\times n}$ matrix was multiplied by a $B_{n\times m}$ matrix it would produce a $C_{m\times m}$ matrix. In addition, students should be encouraged to practice independently regarding the lesson topic. Through exercises, it was expected to minimize the errors made by students in working on mathematical problems.

BIBLIOGRAPHY

Ainin, N., Hartono, W., & Aripin, J. (2020). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Matriks dan Kaitannya dengan Motivasi Belajar Matematika Pada Kelas XI. *Euclid*, *7*(2), 137–147. https://doi.org/10.33603/e.v7i2.3122

Ana, E. N., & Nusantara, T. (2021). Analisis Kesulitan Siswa dalam Menyelesaikan Soal Aritmatika Sosial Analysis of Student Difficulties in Solving Problems of Social Arithmetic. *Jurnal Pendidikan Matematika Dan Sains*, *7*(1), 1–4.

Arnandi, F., Siregar, N., & Fitriawan, D. (2022). Media Pembelajaran Matematika Menggunakan Smart Apps Creator pada Materi Bilangan Bulat di Sekolah Dasar. *Plusminus: Jurnal Pendidikan Matematika*, *2*(3), 345-356. https://doi.org/10.31980/plusminus.v2i3.2194

Fitriyah, I. M., Pristiwati, L. E., Sa'adah, R. Q., Nikmarocha, N., & Yanti, A. W. (2020). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Cerita Koordinat Cartesius Menurut Teori Kastolan. *Al-Khwarizmi: Jurnal Pendidikan Matematika Dan Ilmu Pengetahuan Alam*, *8*(2), 109–122. https://doi.org/10.24256/jpmipa.v8i2.1002

Hendriyanto, A., Fitriana, L., Ishartono, N., Mustofa, B., & Sahara, S. (2022, November). Identification of student errors based on the kastolan theory: A case study of "Kawung Batik pattern". In *AIP Conference Proceedings* (Vol. 2566, No. 1, p. 020006). AIP Publishing LLC.

Khairani, B. P., & Kartini, K. (2021). Analisis Kesalahan Siswa Kelas XI SMA Pada Materi Matriks. *Mosharafa: Jurnal Pendidikan Matematika*, *10*(3), 505–514. https://doi.org/10.31980/mosharafa.v10i3.981

Kumar, S. (2017). Use of mathematics by adults in daily life. *International Journal Peer Reviewed Journal Refereed Journal Indexed Journal*, *3*(9), 345–347.

Lutfia, L., & Zanthy, L. S. (2019). Analisis Kesalahan Menurut Tahapan Kastolan dan Pemberian Scaffolding dalam Menyelesaikan Soal Sistem Persamaan Linear Dua Variabel. *Journal on Education*, *1*(3), 396–404. https://doi.org/10.24252/auladuna.v5i1a9.2018

Maryam, M. S., & Zanthy, L. S. (2019). Smk Pada Materi Matriks Dengan

Penerapan. *Journal on Education*, *01*(03), 155–165.

Masfufah, R., & Afriansyah, E. A. (2022). Analisis Kemampuan Literasi Matematis Ditinjau dari Gaya Belajar Siswa SMP pada Pembelajaran Daring. *Jurnal PERISAI: Jurnal Pendidikan dan Riset Ilmu Sains*, *1*(1), 1-13.

Mutmainah, D. S., & Sari, P. C. (2019). Analisis Kesulitan Belajar Siswa Kelas Xi Pada Materi Matriks Ditinjau Dari Kemampuan Komunikasi Matematis Smk Bina Insan Bangsa. *Journal of Education*, *1*(2), 430–439.

Natsir, N., Tandiayuk, M. B., & Karniman, T. S. (2016). Profil kesalahan konseptual dan Prosedural siswa dalam menyelesaikan soal cerita himpunan di kelas VII SMP 1 Siniu. *Jurnal Elektronik Pendidikan Matematika Tadulako*, *3*(4), 440-453.

Noviani, J. (2019). Analisis Kesalahan Tahapan Kastolan Dan Pemecahan Masalah Model Polya Pada Mata Kuliah Matematika Finansial. *Jurnal Ilmiah Pendidikan Matematika Al Qalasadi*, *3*(1), 27–39. https://doi.org/10.32505/qalasadi.v3i1.891

Pratama, B. A., & Mardiani, D. (2022). Kemampuan berpikir kritis matematis antara siswa yang mendapat model problem-based learning dan discovery learning. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *1*(1), 83-92. https://doi.org/10.31980/powermathedu.v1i1.1918

Pujilestari. (2018). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Matematika Sma Materi Operasi Aljabar Bentuk Pangkat Dan Akar. *Jisip*, *2*(1), 226–232.

Sari, F. Y., Sukestiyarno, S., & Walid, W. (2022). Kemampuan Berpikir Kreatif Matematis Siswa SMP Ditinjau dari Adversity Quotient. *Plusminus: Jurnal Pendidikan Matematika*, *2*(3), 357-368. https://doi.org/10.31980/plusminus.v2i3.2172

Sari, P. D., Syafi'i, M., & Nurwiatin, N. (2020). Analisis Kesulitan Siswa Dalam Penyelesaian Soal Matematika Pada Materi Matriks. *Prosiding Seminar Nasional Pendidikan STKIP Kusuma Negara II*, 388–396.

Sari, R. A., & Najwa, W. A. (2021). Analisis Kesalahan Siswa dalam Menyelesaikan Penjumlahan Bilangan Bulat Berdasarkan Teori Kastolan. *Jurnal Sekolah Dasar*, *6*(1), 77–83. https://doi.org/10.36805/jurnalsekolahdasar.v6i1.1288

Ulfa, D., & Kartini, K. (2021). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Logaritma Menggunakan Tahapan Kesalahan Kastolan. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *5*(1), 542–550. https://doi.org/10.31004/cendekia.v5i1.507

Wardhani, I. K., & Chotimah, S. (2021). Analisis Kesalahan Siswa Smk Dalam Menyelesaikan Soal Materi Matriks. *JPMI Jurnal Pembelajaran Matematika Inovatif*, *4*(5), 1299–1306. https://doi.org/10.22460/jpmi.v4i5.1299-1306

Widodo, S. A. (2016). Kesalahan Dalam Pemecahan Masalah Divergensi Pada Mahasiswa Matematika. *AdMathEdu : Jurnal Ilmiah Pendidikan Matematika, Ilmu Matematika Dan Matematika Terapan*, *4*(1). https://doi.org/10.12928/admathedu.v4i1.4810

Widyaningrum, A. Z. (2016). Analisis Kesulitan Siswa dalam Mengerjakan Soal Cerita Matematika Materi Aritmatika Sosial Ditinjau dari gaya Belajar Siswa Kelas VII SMP Negeri 5 Metro Tahun Pelajaran 2015/2016. *Iqra'*, *I*(2), 166–190.

Yesino, L., Ikromi, S. L., & Baharsyah, R. F. (2020). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Materi Matriks Sesuai dengan Minat Belajar Matematika Siswa. *Seminar Nasional Pendidikan Matematika*, *2*, 418–424.

AUTHOR'S BIOGRAPHY

Dr. Kartini, M.Si.

Born in Bagan Punak, July 3 1972. Lecturer in Mathematics Education, FKIP, University of Riau. Study S1 Mathematics Education, University of Riau, graduated in 1997; Master of Mathematics Bandung Institute of Technology, graduated in 2001; Doctor of Mathematics Education, Indonesian University of Education, graduated in 2012.

Tuti Alawiyah, S.Pd.

Born on Pulau Kijang, June 11 1999. Studying S1 Mathematics Education, University of Riau, graduating in 2021; Student of Master of Mathematics Education FKIP, University of Riau, Pekanbaru.