Realistic Mathematics Textbook Toward The Students’ Mathematical Generalization Skills

Fevi Rahmawati Suwanto1, Ariyadi Wijaya2, Aprisal3, Asri Fauzi4, Yunda Victorina Tobondo5

1Mathematics Education, Universitas Negeri Medan
Jalan William Iskandar, Deli Serdang, Sumatera Utara, Indonesia
fevirahmawati@unimed.ac.id

2Mathematics Education, Universitas Negeri Yogyakarta
Jalan Colombo Togyakarta No.1, Daerah Istimewa Yogyakarta, Indonesia
a.wijaya@uny.ac.id

3Mathematics Education, Universitas Sulawesi Barat
Jalan Prof. Dr. Baharuddin Lopa, Majene, Sulawesi Barat, Indonesia
aprisal@unsulbar.ac.id

4Mathematics Education, Universitas Mataram
Jalan Majapahit No. 62, Mataram, Nusa Tenggara Barat, Indonesia
asrifauzi@unram.ac.id

5Mathematics Education, Universitas Kristen Tentena
Pamona Pusalemba, Poso, Sulawesi Tengah, Indonesia
yundatobondo@gmail.com

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Abstract

Generalization is a feature that students can think mathematically. Teachers can develop teaching materials in the form of textbooks to facilitate generalization abilities. This study was the developmental stage of ADDIE model, namely the implementation and evaluation. The study produced a textbook for 8th grade even semester which was oriented to students' mathematical generalization skills. The textbook integrating a realistic mathematics education approach contained five principles including phenomenological exploration, bridging by vertical instruments, pupils' own constructions and productions, interactivity, and intertwining. The flow of activities provided more than one problem context, identifying similarities in patterns or strategies to draw conclusions related to the material as well as modeling exercises to facilitate students' mathematical generalization abilities. Based on the results of textbook implementation involving 30 8th grade students from one of a state junior high school in Yogyakarta, the percentage of students in good category met the minimum point of 75%, which reached 93.33%.

Keywords: Generalization Ability; Realistic Mathematics Education; Textbook.
I. INTRODUCTION

Generalization is the heartbeat of mathematics (Nurmawanti & Sulandra, 2020). If the teacher is not aware of generalization skills or students are not able to express their generalization thinking, the learning process does not demonstrate mathematical thinking (Mason, 1996: 65-66). As part of mathematical reasoning, generalization is not only dealing with the result but also the process because it serves the basis of mathematical concepts and ideas (Mata-Pereira & Ponteira, 1996; Suwanto & Wijaya, 2018). Thus, mathematical generalization is the ability to process and seek the resolution as the products of students' reasoning on the mathematical concepts and ideas in a general context.

Students with generalization skills will be able to combine their knowledge and experience. This combination is a creative process to construct a solution to a given problem (Hashemi et al., 2013: 212). In other words, the ability to generalize allows students' knowledge to be broader and transformed into practical knowledge (Hayuningrat & Rosnawati, 2022).

The results of TIMSS (Third International Mathematics and Sciences Study) in 2011 ranked Indonesian 8th grader at the lower category, namely 38th out of 42 participating countries (Mullis et al., 2012: 150; Mulyo, Sari, & Syarifuddin, 2019). This shows that the implementation of the education, especially in the field of mathematics, has not been fully utilized (Tomy, Maimunah, & Yenita, 2021; Nurhasanah, Syafari, & Nurfaidah, 2022; Syaputra, Hidayati, & Hasanah, 2023). Furthermore, the reasoning skill of Indonesian students is also relatively low with an average of 388, and this is far below the international average score of 500 (Mullis et al., 2012: 150). As generalization is associated with a form of inductive reasoning, it indicates that students' mathematical generalization skills are also still in the low category.

As part of the scope of mathematics learning and a critical role in problem solving skill, the students' generalization skills must be continuously improved. This improvement can be facilitated through the development of learning support tools, one of which is teaching materials such as textbooks (Yulianto, Sisworo, & Hidayanto, 2022; Auliya’ & Widjajanti, 2023). Textbooks as a reflection of the school curriculum are a good source to provide learning opportunities and at the same time a good indicator to measure students' mathematics learning opportunities (Yang et al., 2017: 2841-2842; Wardana & Damayani, 2017).

The demand for textbook development to facilitate student mathematical generalization ability is also supported by several reasons. First, the available textbooks do not facilitate mathematical generalization ability because they are too procedural and do not provide opportunities for students to do modeling activities and low reflective tasks Panhuizen, (2003: 51-60). Second, there are only few teachers who develop their own learning textbooks. The evidence is represented by the lower percentage of the teachers who use their own modules. 86% of grade VIII junior high school mathematics teachers in Yogyakarta are using math textbooks provided by the government as a reference and guide in the learning process (Suwanto
Third, based on the results of a questionnaire given to junior high school mathematics teachers in Yogyakarta, it revealed that no teachers have developed a textbook oriented to students' mathematical generalization ability (Nurhikmayati & Jatisunda, 2019).

One of the constructive approaches that can be applied to textbooks to develop students' mathematical generalization skills is Realistic Mathematics Education (RME). This approach uses real world contexts of everyday life and provide problems that students can envisage as the basis of mathematics learning (Afriansyah, 2016). Thus, the students are involved in a meaningful learning process (Hadi, 2017: 37; Fitri, Fitri, & Jufri, 2022).

Gravemeijer (2010: 43) defines RME as an interactive teaching theory between teachers and students or among students, centered on certain problems to support students to build or rediscover mathematical concepts. Through this approach, the teacher emphasized its role as a facilitator while students actively proceed to the social context of the classroom to discover or rebuild mathematical concepts (Zaneta, 2022). This rediscovery of mathematical concepts is related to the term mathematization.

Mathematization can be defined as the process of representing realistic models into mathematical language (Suwanto & Wijaya, 2018: 304; Afriansyah, 2022). It is divided into two types, namely horizontal mathematization and vertical mathematization. Horizontal mathematization is known as the process of using models to solve specific problems, whereas using these models to make generalizations, formalizations, is known as vertical mathematization (Dickinson et al., 2011: 48). Progressive mathematization in RME, both vertically and horizontally, is based on five teaching principles stated by Treffers (1987: 248-250). They are phenomenological exploration, bridging by vertical instruments, pupils' own constructions and productions, interactivity, and intertwining (Mutiarahman, Edriati, & Suryani, 2023).

Through phenomenological exploration, the learning process begins with providing real problems in the textbook to foster students' mathematical generalization skills. This is in line with NCTM (2000: 335) who revealed that mathematical generalization could be introduced effectively by giving problems that helped students see important aspects of the idea to be generalized.

Furthermore, through bridging vertical instruments, students' construction and production, interactivity, and intertwining by emphasizing analogy, classification, and structure in mathematics learning can also facilitate students' mathematical generalization skill. This demonstrates that generalization is a mathematical process in RME itself which is related to patterns and relationships (Wijaya, 2012: 42). Thus, the textbook developed from the principles of the RME approach is expected to indirectly facilitate students' mathematical generalization skill.

In addition, Jupri, Usdiyana, & Sispiyati (2020: 105) stated that RME principles in the form of reality, levels, and linkages can be used to develop teaching materials on
numerical pattern material. One of the learning activities contains generalization of numerical patterns that facilitate students' mathematical generalization skills. Therefore, the development of teaching materials in the textbooks to improve students' mathematical generalization skills can be conducted by applying RME principles.

II. METHODS

This research is two of the five results at the developmental stages of the ADDIE model (Analysis, Design, Development, Implementation, Evaluation) adapted from (Branch, 2009), namely the implementation and evaluation stages. The study produced a mathematics textbook for junior high school students in grade VIII for even semester with RME approach. The effectiveness of the textbook in facilitating students' mathematical generalization skills was investigated through a test instrument given to 30 students of one of the VIII classes of a Junior High School in Yogyakarta in the even semester of the 2017/2018 academic year. The descriptive test was presented after the trial of the textbook had been developed at the implementation stage. The quantitative data of the test results obtained were then analyzed at the evaluation stage using the conversion presented in Table 1 (Widoyoko, 2017). The developed book was said to be effective if the test results of students' mathematical generalization ability were at least 75% in the good category.

III. RESULT AND DISCUSSION

The even semester junior high school mathematics textbook developed from the RME approach consisted of five chapters (main materials), namely the Pythagorean Theorem, circles, flat-sided spaces, statistics, and probability. The quality of this textbook not only met the validity and practicality criteria (Suwanto & Wijaya, 2021) but also the effectiveness criteria to foster students' mathematical generalization skills. This effectiveness were demonstrated by the results at the evaluation stage after the implementation.

![Figure 1. The Distribution of The Textbooks to The Students](image)

The implementation of the textbook in the pilot class involving 30 junior high school students in grade VIII was successfully conducted. After the distribution of textbooks to each students' group (see Figure 1), students were asked to do and discuss the activities contained in the book.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Skor</th>
<th>Kategori</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X &gt; \bar{X}<em>i + 1.8 \times s</em>{bi}$</td>
<td>Very good</td>
<td></td>
</tr>
<tr>
<td>$\bar{X}<em>i + 0.6 \times s</em>{bi} &lt; X \leq \bar{X}<em>i + 1.8 \times s</em>{bi}$</td>
<td>Good</td>
<td></td>
</tr>
<tr>
<td>$\bar{X}<em>i - 0.6 \times s</em>{bi} &lt; X \leq \bar{X}<em>i + 0.6 \times s</em>{bi}$</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>$\bar{X}<em>i - 1.8 \times s</em>{bi} &lt; X \leq \bar{X}<em>i - 0.6 \times s</em>{bi}$</td>
<td>Below average</td>
<td></td>
</tr>
<tr>
<td>$X \leq \bar{X}<em>i - 1.8 \times s</em>{bi}$</td>
<td>Poor</td>
<td></td>
</tr>
</tbody>
</table>

Note:
- $\bar{X}_i$ = perfect mean
- $s_{bi}$ = perfect standard deviation
- $X$ = empirical score
Students' engagement in the activities is presented in Figure 2.

Figure 2. Students Discussing the Activities in The Textbook.

After the pilot class, the generalization test was provided to test the textbook effectiveness on the students. The result of the test is provided in Table 2.

Table 2.
The Students’ Generalization Skills

<table>
<thead>
<tr>
<th>Category</th>
<th>Number of students</th>
<th>Percentage</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very high</td>
<td>16</td>
<td>53.33 %</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>12</td>
<td>40 %</td>
<td></td>
</tr>
</tbody>
</table>
| Average    | 2                  | 6.67 %     | 79.93 (
| Below average | 0              | 0 %        |         |

Table 2 shows that the average value of students' mathematical generalization skill is relatively high, namely 79.93. There were no students in the below and poor categories, but there were few in the average and high categories. In the average category there are only 2 students with a percentage of 6.67%. The rest, 12 students were in the high category and 16 students were in the very high category with a percentage of 40% and 53.33% respectively. The number of students who were at least in the high category is 28 students with a percentage of 93.33% > 75%. Thus, the developed product was considered effective in terms of students' mathematical generalization ability.

Table 3.
The Indicators of The Students’ Generalization Skills

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Average</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the nature of an object or phenomenon</td>
<td>3.43</td>
<td>Very good</td>
</tr>
<tr>
<td>Identify the similarity of an object, trait, or situation</td>
<td>3.62</td>
<td>Very good</td>
</tr>
<tr>
<td>State the rules, patterns, strategies, or ideas of an object or phenomenon</td>
<td>3.48</td>
<td>Very good</td>
</tr>
<tr>
<td>Define an object that meets a given relationship, pattern, or phenomenon</td>
<td>2.40</td>
<td>Average</td>
</tr>
<tr>
<td>Use general ideas or strategies to solve problems</td>
<td>3.50</td>
<td>Very good</td>
</tr>
<tr>
<td>Modify general ideas or strategies to solve problems</td>
<td>2.75</td>
<td>Good</td>
</tr>
</tbody>
</table>

Then, the analysis of the effectiveness in terms of mathematical generalization skill was derived from the percentage of indicator achievements. Based on Table 3, the highest average score obtained by students was 3.62. This showed that the students considered identifying the similarity of an object, trait, or situation as the simplest activity. While the lowest overall average score obtained by students was 2.40. This meant that students considered defining an object that fulfills a given relationship, pattern, or phenomenon as the most complex one.

Table 4.
The Relationship of The Indicators of Mathematical Generalization Skills and The Principles of RME

<table>
<thead>
<tr>
<th>Indicators</th>
<th>The principles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the nature of an object or phenomenon</td>
<td>Phenomenological exploration, intertwining</td>
</tr>
<tr>
<td>Identify the similarity of an object, trait, or situation</td>
<td>Through interactivity:</td>
</tr>
</tbody>
</table>
State the rules, patterns, strategies, or ideas of an object or phenomenon

| bridging with vertical instruments, students’ construction and production, intertwining |

Define an object that meets a given relationship, pattern, or phenomenon

Use general ideas or strategies to solve problems

Modify general ideas or strategies to solve problems

The effectiveness of the RME-oriented textbook to the mathematical generalization skills was certainly inseparable from the implementation of the principles of the approach itself (Note Table 4). Through the application of phenomenological exploration of using context in the book, the students were facilitated to connect two or more problems, situations, ideas, or mathematical objects related to the material being studied (see Figure 3). In other words, this encouraged students to be able to identify the natures of an object or phenomenon.

In the book's 'activity' (interactivity), which was the application of bridging with vertical instruments, students’ construction and production, and intertwining, the students were provided more than one context and then asked to look for relationships or patterns between contexts to move into more general conclusions (see Figure 4). The use or modification of these general ideas or strategies in problem solving was also facilitated by the book through sub-chapter exercises or chapter evaluations. Thus, the principles of the RME approach developed in the textbook had facilitated students’ mathematical generalization skills through its indicators.

The textbook with RME approach proved effective to improve mathematical generalization skills, supported by Veloo et al., (2015) and Dani et al., (2017) which concluded that RME was better than the conventional approach in terms of students' mathematical generalization skills. Similar research results also confirmed that one of

<table>
<thead>
<tr>
<th>Period</th>
<th>Export (Kg)</th>
<th>Import (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>515.569</td>
<td>841.143.541</td>
</tr>
<tr>
<td>2015</td>
<td>519.197</td>
<td>851.911.801</td>
</tr>
<tr>
<td>2016</td>
<td>399.167</td>
<td>1,703.178.517</td>
</tr>
<tr>
<td>2017</td>
<td>3,531.751</td>
<td>736.590.685</td>
</tr>
</tbody>
</table>

Tabel 4.1 Ekspor-Impor Beras Indonesia

<table>
<thead>
<tr>
<th>Periode</th>
<th>Ekspor</th>
<th>Impor</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
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</table>


Figure 3. Utilizing Contexts in The Textbook.

Figure 4. Textbooks Activities.
the RME principles, namely bridging with vertical instruments that was closely related to mathematical modeling, had an effect on students’ mathematical generalization reasoning skill (Suryani, 2016).

IV. CONCLUSION

The mathematics textbook for junior high school students in grade VIII even semester developed from the principles of the RME approach was effective in terms of mathematical generalization ability with a percentage of 93.33%, greater than the lower average score, 75% of students in the good category. The application of the five principles of the realistic mathematics approach in the textbook through the flow of activities providing more than one problem context, identifying similarities in patterns or strategies to draw conclusions related to the material and providing modeling exercises in this textbook, facilitated students’ mathematical generalization skills. Thus, the developed textbook could be used in the learning process as an effort to foster students' mathematical generalization skills. In further development and research, it is expected that teachers or researchers could develop teaching materials with the RME approach oriented to the communicative skills and mathematical generalization skills at different educational levels or learning topics.

REFERENCES


**RIWAYAT HIDUP PENULIS**

**Fevi Rahmawati Suwanto, M.Pd.**

Lecturer in the Mathematics Education Study Program at Medan State University. Graduated from Bachelor of Mathematics Education at Riau Islamic University in 2015 and graduated from Masters in Mathematics Education at Yogyakarta State University in 2018.
Ariyadi Wijaya, M.Sc., Ph.D.

Lecturer in the Mathematics Education Study Program at Yogyakarta State University. Graduated from S1 in Mathematics Education at Yogyakarta State University in 2004, graduated from S2 and S3 Mathematics Education at Utrecht University in 2008 and 2015.

Aprisal, M.Pd.

Lecturer in the Mathematics Education Study Program at the University of West Sulawesi. Graduated from S1 in Mathematics Education at Makassar State University in 2014 and graduated from S2 in Mathematics Education at Yogyakarta State University in 2018.

Asri Fauzi, M.Pd.

Lecturer in the Elementary School Teacher Education Study Program at the University of Mataram. Graduated from Bachelor of Mathematics Education at STKIP Hamzanwadi in 2015 and graduated from Master of Mathematics Education at Yogyakarta State University in 2018.

Yunda Victorina Tobondo, M.Pd.

Lecturer in the Mathematics Education Study Program at Tentena Christian University. Graduated from S1 Mathematics Education at Sanata Dharma University in 2015 and graduated from S2 Mathematics Education at Yogyakarta State University in 2018.