Cognitive Conflict Strategy: Mathematical Representation Based on Cognitive Style

Mohamad Salam
Mathematics Education, Universitas Halu Oleo
Kendari, Sulawesi Tenggara, Indonesia
mohamad.salam@uho.ac.id

Article received: 2022-09-20, revised: 2023-01-17, published: 2023-04-30

Abstract
This study aimed to describe types of students’ mathematical representations based on the cognitive style in solving integral problems after learning cognitive conflict strategies. This exploratory research used a descriptive-qualitative approach. The participants were two students of Mathematics Education Department at Halu Oleo Universit, class of 202, consisting of one student with a reflective cognitive style and one student with an impulsive cognitive style. The research data were obtained through tests and interviews, and analyzed qualitatively. The findings pointed out that the student with a reflective cognitive style performed images or geometric representations, verbal representations, and symbolic representations, as well as translated various types of representations, starting from geometric representations to verbal representations, and then to symbolic representation and vice versa. Meanwhile, the student with an impulsive cognitive style performed symbolic representations, inaccurate geometric representations, but lacked of verbal representations, mathematical representations, and translation of various types of representation.

Keywords: cognitive conflict strategies; cognitive style; mathematical representation.
I. **INTRODUCTION**

Mathematics is one of the lessons that is objective, logical, and practical (Jawad et al., 2021; Lai et al., 2015) as a symbolic language that allows humans to think about problem solving and utilized to connect one problem to another (Sukardjo & Salam, 2020; Yarmohammadian, 2014), for that reason, teachers must be able to thoroughly observe the problems in the classroom to find appropriate learning patterns in solving problems according to the student’s characteristics (Salam et al., 2019). Complex problems could be resolved by visual, symbolic, and verbal representation (Yang & Sianturi, 2022).

Representation is a description of an object in a mathematical equation, computer simulation, picture, graphic, diagram, or in other forms to explain the cognitive process in understanding mathematical ideas (Geyer & Kuske-Janßen, 2019; Sahendra et al., 2018; Salam, 2022), encouraging the students to develop cognitive styles by describing abstract mathematical ideas into sensible forms which assist their comprehension (Sari & Darhim, 2020).

Representation is a factual form of a person’s thinking about the problem, and used as a measuring tool to obtain a solution (Inayah & Nurhasanah, 2019; Silviani, Mardiani, & Sofyan, 2021). Mathematical representation is a process to design mathematical models of real problems into mathematical concepts or abstract symbols (Yuanita et al., 2018; Linda & Afriansyah, 2022), with the purpose that it is easier to understand the concept (Kokkonen & Schalk, 2021; Yakubova et al., 2020). The indicators of mathematical representation ability consist of: (1) visual, to present information or data in the form of diagrams, graphs, tables, and drawings or geometric patterns in solving and explaining problems; (2) symbolic, to compile equations or mathematical models and make conjectures on number patterns and solve problems involving mathematical expressions; and (3) verbal, to solve problems in written forms or in the form of stories according to the problems presented (Taofik & Juandi, 2022).

The phenomenon that occurs among students of Mathematics Education Department of Halu Oleo University Kendari showed that the students are having difficulties to represent Integral problems into various mathematical representations (geometric representation, verbal representation, and symbolic representation). The students perform incorrect mathematical representations while solving Integral problems. The students still experience a variety of barriers and errors in solving integrals due to lack of understanding of the concept. Errors in performing representations emerge misconceptions that influence the representations of the problems. Investigating students' misconceptions might be conducted by applying cognitive conflict strategies (Mufit et al., 2020).

A cognitive conflict strategy is a learning process designed to arise mental conflicts on the discrepancy between given information and students’ prior knowledge with the purpose that the information affects the students’ cognitive structures (Gusnidar et al., 2018; Sundayana & Parani, 2023). Cognitive conflict is an analogy of...
discrepancy between the learning condition and the student’s cognitive development (Supena et al., 2021; Yustinaningrum & Lubis, 2019), that can be identified by verbal cues or the student’s statements towards the conflict (Maharani & Subanji, 2018). One of the cognitive conflict indicators is when the student realizes the discrepancy in the interpretation during their information construction process (Guarana & Hernandez, 2016; Fauzan, Kusnadi, & Sofyany, 2023).

Students who demonstrate cognitive conflict should be resolved immediately because it will interfere with the construction process of further knowledge. The learning process infused by cognitive conflict strategy follows these stages: (1) pre-conflict stage, which is the stage before the conflict, to create a state of cognitive conflict; (2) cognitive conflict stage; and (3) conflict resolution stage, which is the stage of resolving cognitive conflict (Sujana et al., 2019). One of the important stages in the learning process with cognitive conflict strategy is the conflict resolution (Parwati & Suharta, 2020). The key to the successful learning process with cognitive conflict strategy is to assure that students reconstruct the correct concept in developing their knowledge (Calleja & Formosa, 2020), which help them to perform a correct mathematical representation (Parwati & Suharta, 2020).

One of the variables that differ the student’s characteristics in mathematics learning is the cognitive style. The cognitive style relies on the veracity of the individual’s characteristics in collecting, analysing, storing, thinking, and utilizing the information to solve problems (Kamid et al., 2020). The cognitive style is classified into: reflective and impulsive (Ismaeel & Mulhim, 2021; Shoimah et al., 2018). A student who spends longer time in answering the question, but usually answers carefully and correctly is called reflective cognitive style, whereas a student who spends shorter time to solve the problems, but lacks of careful thought and usually gives incorrect answers is called impulsive cognitive style (Muhtarom et al., 2018; Astutik & Purwasih, 2023).

Various studies on describing the ability to represent mathematical problems has been conducted, to mention: Setyawati (2020), Yenni & Sukmawati (2020), Kusumawarandi & Mega (2021) investigated the students' mathematical representation ability based on self-efficacy, learning motivation, and reflective-impulsive cognitive style; Nurfitriyanti et al. (2020) investigated the students' mathematical representation ability based on mathematical cognition in the problem-based learning process; Azizah et al. (2019) explored the students’ mathematical representation ability based on the students’ cognitive styles through problem-based learning; and Amalia et al. (2020) investigated the mathematical representation ability based on the cognitive styles after giving problem based learning assisted by Geogebra and realistic mathematics education model.

Based on the above explanation, this study examined the students’ ability of mathematical representation in terms of cognitive style after being given cognitive
conflict strategy learning and aimed to: (1) describe the types of mathematical representations performed by students with reflective cognitive styles in solving integral problems after completing the cognitive conflict strategy learning process; (2) describe the types of mathematical representations made by students with cognitive-impulsive style in solving integral problems after completing the cognitive conflict strategy learning process.

II. Method

This research was an exploratory study using a qualitative approach involving the students of the Department of Mathematics Education, Halu Oleo University, class of 2021 who were taking integral calculus courses.

The participants were selected through some steps: (1) the cognitive style test given to determine the students’ impulsive-reflective cognitive styles based on Jerome Kagan (1965), using Matching Familiar Figures Test (MFFT) developed by Warli (2010), consisting of 13 items (Herianto, 2020), (2) the selection of two participants that represented reflective and impulsive cognitive style. The two participants possessed a similar ability in differential calculus.

The data were collected by using tests and interviews. The test consisting of 2 items of integral problems was provided to the participants, as follows:

1. Calculate $\int_{-2}^{6} f(x) \, dx$, if:
   
   $f(x) = \begin{cases} 
   x + 2; & -2 \leq x < 0 \\
   2; & 0 \leq x < 4 \\
   x - 2; & 4 \leq x \leq 6 
   \end{cases}$

2. Calculate $\int_{0}^{6} f(x) \, dx$,

if $f(x) = \begin{cases} 
\lfloor x - 2 \rfloor; & 0 \leq x < 4 \\
2; & 4 \leq x \leq 6 
\end{cases}$

The interview was conducted to verify the students’ answers of the written test. The data, then, were analysed through data reduction, data presentation and conclusion drawing (Sarosa, 2021).

The procedures of this study were: (1) integrating the cognitive conflict learning strategy in the integral lesson regarding “Fundamental Theorems of Calculus”, that consists of (a) pre-conflict stage, (b) cognitive conflict stage, and (c) conflict resolution stage, resolving the cognitive conflict, (2) providing the cognitive style test, (3) selecting the participants of the study, (4) providing integral test to the participants, (5) interviewing the participants to verify the test results, (6) validating the data with the triangulation technique by examining the validity of the written test results against the interview results, (7) analysing the data qualitatively, and (8) drawing conclusions.

III. Result and Discussion

A. The participant with reflective cognitive style

Before answering question number 1, the participants with reflective cognitive style were first asked to perform image representation or geometric representation, and the results are presented in Figure 1.
According to the image representation in Figure 1, the participant with reflective cognitive style obtained the result of 16 units of area. After representing the problem geometrically, the participant performed a verbal representation, namely explaining the relationship between the problem and the geometric representation that had been made. The participant argued that the integral result of the given function could be found by calculating the area of the bounded area with: (1) line \( y = x + 2 \), line \( x = -2 \), axis \( x \), and line \( x = 0 \); (2) line \( y = 2 \), line \( x = 4 \), \( x = 0 \), and axis \( x \); (3) line \( y = x - 2 \), axis \( x \), line \( x = 4 \) and line \( x = 6 \). According to the geometric representation made, the participant stated that the result of the integral function equalled to the area of triangle L1, the area of rectangle L2, and the area of trapezoid L3.

Furthermore, by using the theorem:
\[
\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx,
\]

the participant performed symbolic representation as showed in Figure 2.

Based on the symbolic representation in Figure 2, the participant with reflective cognitive style found similar results with the geometric representation and the verbal representation to solve the problem was 16. Based on the question number 1, the participant translated the geometric representation to verbal representation, the geometric to symbolic representation, and verbal to symbolic representation, or vice versa. It implied that the reflective participant performed one type of representation to another, namely from geometric representation to verbal representation and then to symbolic representation or vice versa.

To answer question number 2, the participant performed visual or geometric representation, illustrated by Figure 3.
After drawing geometric representation, the participant performed verbal presentation by explaining the correlation question number 2 with the geometric representation that had been made. The participant argued that the integral results of the formula in question number 2 could be obtained by calculating the area bordered by (1) line \( y = x - 2 \), axis \( x \), line \( x = 0 \) and line \( x = 2 \); (2) line \( y = -x + 2 \), axis \( x \), line \( x = 2 \) and line \( x = 4 \); and (3) line \( y = 2 \), axis \( x \), line \( x = 4 \) and line \( x = 6 \). In other words, the integral result of the formula equaled to the sum of the areas of two triangles \((L_1+L_2)\) and a rectangular \((L_3)\). The result obtained by the participant was 8. Using the theorem \( \int_{a}^{c} f(x) \, dx = \int_{a}^{b} f(x) \, dx + \int_{b}^{c} f(x) \, dx \), the participant performed symbolic representation, illustrated by Figure 4.

\[
\begin{align*}
\int_{0}^{2} f(x) \, dx &= \int_{0}^{2} (2x - x^2) \, dx + \int_{2}^{4} (x - 2) \, dx + \int_{4}^{6} (2x - 2) \, dx \\
&= \left[ x^2 - \frac{x^3}{3} \right]_{0}^{2} + \left[ \frac{x^2}{2} - 2x \right]_{2}^{4} + \left[ x^2 - 2x \right]_{4}^{6} \\
&= 2 + 2 + 4 \\
&= 8.
\end{align*}
\]

Figure 4. The symbolic representation made by the student with reflective cognitive style on question number 2

B. The participant with impulsive cognitive style

When the participant with impulsive cognitive style was answering question number 1, the participant represented the problem geometrically first, and the result is presented by Figure 5.

Geometric representation was incorrect because it was not supported by the graph of the function given in the question. The graph was not equal to the graph of the function defined by: (1) line \( y = x + 2 \), axis \( x \), line \( x = -2 \) and line \( x = 0 \); (2) line \( y = 2 \), axis \( x \), line \( x = 0 \) and line \( x = 4 \); and (3) line \( y = x - 2 \), axis \( x \), line \( x = 4 \) and line \( x = 6 \). The participant did not perform verbal representation. Using the theorem: \( \int_{a}^{c} f(x) \, dx = \int_{a}^{b} f(x) \, dx + \int_{b}^{c} f(x) \, dx \) the participant obtained the result as showed by Figure 6:

\[
\begin{align*}
\int_{0}^{6} f(x) \, dx &= \int_{0}^{2} f(x) \, dx + \int_{2}^{4} f(x) \, dx + \int_{4}^{6} f(x) \, dx \\
&= \left[ (x^2 + 2x) \right]_{0}^{2} + \left[ 2x^2 \right]_{2}^{4} + \left[ (x^2 - 2x) \right]_{4}^{6} \\
&= (4 + 4) + 16 + (16 - 8) \\
&= 16.
\end{align*}
\]

Figure 5. The image representation made by the student with impulsive cognitive style on question number 1

The integral result using symbolic representation was not identical to the integral result using image or geometric representation. The participant was not
able to translate one representation to another.

To solve question number 2, the participant graphed the function as the image or geometric representation, illustrated by Figure 7.

![Figure 7](image)

Figure 7. The image representation made by the student with impulsive cognitive style on question number 2

The image made by the participant was not illustrated the graph of the function in question number 2 because it was incorrect. The participant did not perform verbal representation. Using the theorem:

\[ \int_{a}^{c} f(x)dx = \int_{a}^{b} f(x)dx + \int_{b}^{c} f(x)dx \]

the participant performed symbolic representation, as illustrated by Figure 8.

![Figure 8](image)

Figure 8. The symbolic representation made by the participant with impulsive cognitive style on question number 2

The integral result from the symbolic representation was not identical with the result from the geometric representation. The participant was not able to translate one type of representation to another one.

C. Discussion

After completing cognitive conflict strategy learning, the participant with reflective cognitive style was able to solve the integral problem accordingly. The participant could demonstrate conceptual changes for their misconceptions. Based on the correct conceptual understanding, the participant easily performed the geometric, verbal and symbolic representation and translated one representation to another well. The participant obtained the same results in solving the first and second problems by performing geometric, verbal and symbolic representation. The participant made translations from one type of representation to another, namely from geometry representation to verbal representation, then to symbolic representation or vice versa. The participant explained that the basic concept of integral was area and integral could be used to calculate an area.

The implementation of cognitive conflict strategy learning at the stage of conflict resolution could help students with reflective cognitive style construct conceptual changes from misconceptions to the correct concept. This was in line with (Calleja & Formosa, 2020) stating that the result of the cognitive conflict strategy learning aimed to assist student to make conceptual changes. Cognitive conflict strategy learning might impact on improving students’ mathematical representation skills because students were encouraged to make conceptual
changes to the misconceptions that influenced the mathematical representations. Presenting mathematical ideas through various representations would highly influence the students' mathematical understanding, therefore students were encouraged to practice constructing their own representations to strengthen their skills and conceptual understanding used in solving problems. (Jonassen & Carr, 2020; Rau & Matthews, 2017). The participant with reflective cognitive style represented question number 1 and 2 into various representation conducted slowly and deliberately to generate appropriate representation and solution. It was supported by the individual’s characteristics that solved the problem slowly, but deliberately and gradually in order to get correct answers (Cintamulya, 2019).

The participant with impulsive cognitive style drew geometric representations for problem numbers 1 and 2 incorrectly because there was mathematical misconception that affected the representation of the given problem, for example the concept of drawing linear function graphs had not been mastered by by the participant, according to the results of the interview. The participant could represent problem number 1 and 2 verbally. The participant could represent question number 1 and two into symbolic representation correctly, by applying the theorem: \[ \int_{a}^{c} f(x)dx = \int_{a}^{b} f(x)dx + \int_{b}^{c} f(x)dx, \] the participant was able to determine the solution of question number 1 and 2 correctly. After conceptual changes, the participant comprehended the concept of the absolute value to solve question number 2 symbolically.

After completing cognitive conflict strategy learning, the participant was not able to represent the problem into various representations because his understanding regarding the concepts of mathematical representations was not well developed (Styoningtyas & Hariastuti, 2020), and he was not able to make a connection among mathematical topics (Febrianti Habel & Susilowaty, 2021). The students were encouraging to acquire sufficient understanding and knowledge to represent a mathematical problem. It was in line with Niss & Højgaard (2019) stating that solving mathematical problem required sufficient knowledge and understanding. The participant with impulsive cognitive style did not demonstrate adequate understanding and knowledge of the mathematical concepts underlying the concept of integral material to solve question number 1 and 2. The fact was that when the participant was drawing the geometric representation, he was not able to make the function graph of problem number 1 and 2 correctly, even though the mathematical concepts related to function drawing had been learned in the Differential Calculus course and at the previous education level. Therefore, cognitive conflict strategy learning was applied to the Differential Calculus course, to make conceptual changes on the topic of Function Graphs, by training students to draw various function graphs. The participant solved the given problems into geometric representations quickly, but not thoroughly resulting in incorrect representations. This was supported by the
characteristics of the individuals with impulsive cognitive style who usually solve problems quickly, but less accurately which cause a lot of errors (Cintamulya, 2019; Purwaningsih et al., 2019). Impulsive cognitive style was affected by an emotional mind, which demonstrated the nature of being fast but careless (Aini et al., 2020; Purnama et al., 2021).

IV. CONCLUSION

Based on the findings and discussion, the study concluded that the type of mathematical representations made by the student with reflective cognitive style in solving integral problems after being taught with cognitive conflict learning strategy was able to represent images or geometric representations, verbal representations and symbolic representations and translate a various types of representations to another, namely from geometric representations to verbal representations, then symbolic representations and vice versa. Meanwhile, the type of mathematical representation performed by students with impulsive cognitive style in solving integral problems after being taught with cognitive conflict strategy learning was symbolic representation, incorrect geometric representation, but did not perform verbal representation, mathematical representation, and translation of various types of representation.

Based on the conclusions, the researcher recommended teaching integral calculus with the representation of images or geometry, verbal representation, and symbolic representation in explaining the concept and consideration of the cognitive style of students.

REFERENCES


Cintamulya, I. (2019). Analysis of students’ critical thinking skills with reflective and impulsive cognitive styles on
conservation and environmental knowledge learning. *Asia-Pacific Forum on Science Learning and Teaching*, 20(1).


https://doi.org/10.29333/iji.2021.14351a


https://doi.org/10.15575/ja.v8i1.17915


https://doi.org/10.1080/1045988X.2019.1627999


https://doi.org/10.1080/03055698.2021.1740977


https://doi.org/10.11648/j.pbs.20140301.16


https://doi.org/10.1371/journal.pone.0204847


**AUTHOR’S BIOGRAPHY**

Dr. Mohamad Salam, S.Pd, M.Si.

![Dr. Mohamad Salam, S.Pd, M.Si.](image) Born in Raha, 17 April 1969. Teaching staff at Halu Oleo University. Bachelor of Mathematics Education at Halu Oleo University, Kendari, graduated in 1994; Master of Mathematics Bandung Institute of Technology, Bandung, graduated in 2000; and S3 Education Technology, Jakarta State University, Jakarta, graduated in 2019.