Letters in Algebra as The Transition from Arithmetic Thinking to Algebraic Thinking

Reni Wahyuni¹*, Tatang Herman², Siti Fatimah³

Mathematics Education Department, Universitas Islam Riau
Jalan Kaharuddin Nst No.113, Simpang Tiga, Kota Pekanbaru, Riau, Indonesia
¹*reniwahyunifkipmat@edu.uir.ac.id
Mathematics Education Department, Universitas Pendidikan Indonesia
Jalan Dr. Setiabudi No.229, Isola, Kec. Sukasari, Kota Bandung, Jawa Barat, Indonesia
²tatangherman@upi.edu, ³sitifatimah@upi.edu

Abstrak

Huruf-huruf aljabar biasanya hanya dikenal sebagai variabel oleh siswa, padahal huruf-huruf aljabar ini memiliki ragam makna pada suatu permasalahan matematis. Pada penelitian ini bertujuan untuk menyelidiki interpretasi siswa terhadap makna dari huruf-huruf aljabar dalam menyelesaikan masalah matematis dengan perspektif siswa dalam masa transisi berpikir aritmetik ke berpikir aljabar. Penelitian yang digunakan pada penelitian kualitatif dengan jenis penelitian pada penelitian fenomenologi. Partisipan penelitian ini merupakan 18 siswa SMP yang diambil dari tiga sekolah yang terdapat di Riau. Teknik pengumpul data pada penelitian ini adalah dokumen lembar kerja siswa, wawancara yang direkam secara audio dan video dan catatan lapangan penelitian. Berdasarkan temuan penelitian pada siswa kelas VII tersebut, ditemukan bahwa siswa menginterpretasikan huruf-huruf aljabar hanya sebagai pengganti dari bilangan tertentu. Siswa menjadikan huruf-huruf aljabar sebatas pengganti bilangan asli dibandingkan sebagai bilangan yang diperumum. Huruf-huruf aljabar yang dikaitkan dengan ekspresi aljabar dan operasi aljabar serta persamaan aljabar memberikan kesulitan bagi siswa dalam menyelesaikan masalah tersebut. Hal ini disebabkan siswa belum mampu melihat bahwa huruf-huruf aljabar tersebut merupakan serangkaian konsep dasar aljabar *generalized numbers* bahkan pada konsep variabel.
Kata Kunci: transisi; berpikir aritmetik; berpikir aljabar; huruf-huruf aljabar; kualitatif.

Abstract

Algebraic letters are usually only known as variables by students, even though these algebraic letters have a variety of meanings in a mathematical problem. This study aims to investigate students' interpretation of the meaning of algebraic letters in solving mathematical problems from the perspective of students in the transition period from arithmetic to algebraic thinking. The type of research used in qualitative research is phenomenological research. The participants of this study were 18 junior high school students taken from three schools in Riau. Data collection techniques in this study were student worksheet documents, audio- and video-recorded interviews, and research field notes. Based on the research findings on the seventh-grade students, it was found that students interpret algebraic letters only as a substitute for certain numbers. Students make algebraic letters as a substitute for natural numbers rather than as generalized numbers. Algebraic letters associated with algebraic expressions, algebraic operations, and algebraic equations deliver difficulties for students in solving these problems. This is because students have not been able to see that these algebraic letters are a series of basic concepts of generalized numbers in algebra, even on the concept of variables.
Keywords: transition; arithmetic thinking; algebraic thinking; algebraic letters; qualitative.

I. INTRODUCTION

Several studies have been conducted regarding students' understanding and difficulties in algebra (Asquith et al., 2007; Brizuela et al., 2015; Samo, 2009; Tiwari & Fatima, 2019). Algebra as an initial gateway to further learning (Chimoni et al., 2018; Ferrucci, 2004) has provided coherence at all school levels that algebra is an important topic in mathematics. These reasons underline the necessity to teach algebra at the elementary level (NCTM, 2000; Diva & Purwaningrum, 2023).

The need to improve the integration of algebra in mathematics learning at schools needs to be initiated from the elementary level to the intermediate level, which is a transition period from arithmetic learning to algebraic learning. The basic concept underlying this transition is when basic algebraic concepts are introduced to students (Kilhamn, 2014; Afriansyah & Turmudi, 2022). An important characteristic of the basic concept of algebra is the emergence of symbolization in the form of algebraic letters that have various meanings or roles in a problem. The existence of these algebraic letters is often considered as a substitute notation for certain numbers (Kilhamn et al., 2019). Thus, when students are encountering problems related to algebraic letters as generalized numbers, students are distracted and probably unable to solve the problem.

The national curriculum explicitly states that algebraic letters are studied in grade VII with the topic of algebraic forms (As'ari et al., 2017). In this topic, students are introduced to symbolization by algebraic letters known as variables. In the national curriculum sequence, before students learn algebra, students first learn arithmetic (Sugiman et al., 2016). Students will study arithmetic for 6 years, followed by algebra (Hidayanto et al., 2014). The sequence of this mathematics learning encourages continuous learning of arithmetic or known as generalized arithmetic (Çelik & Güneş, 2013; Pitta-pantazi et al., 2019).

In line with the studies, according to Chimoni et al. (2018) arithmetic learning would involve numbers and operations, while algebraic learning involved algebraic letters and other operations. Furthermore, algebraic learning also includes relationships between numbers in arithmetic, as well as the representation of a problem that surrounds the algebra (Kaput, 2008). Algebra learning is also known as abstract learning. The abstract nature of algebra is often considered challenging and unpleasant (Warren et al., 2016; Lisan, Risnawati, & Setyawan, 2023).

Some studies stated that difficulties in learning algebra were found among the student mathematization process, the process of student understanding of algebraic letters, the process of student understanding of algebraic expressions, the process of using algebraic operations and the process of understanding the meaning of the equals sign (Fulop, 2015; Jupri et al., 2014; Astutik, & Purwasih, 2023). Of the several difficulties in learning algebra stated in the study, one of the results of the study, namely the difficulty in the process of understanding algebraic letters, is an important problem in learning algebra and gaining attention among educators (Christou & Vosniadou, 2012; Khalid et al., 2020; Warren et al., 2016). The rationale is

related to algebraic letters as a foundation for students to learn algebra. This is in line with Malisani and Spagnolo (2009) who stated that the use of algebraic letters was a requirement to learn algebra and basic knowledge in the transition phase of arithmetic thinking to algebraic thinking. Furthermore, Malisani and Spagnolo also revealed that students were struggling to learn algebraic letters because they had trouble determining algebraic letters as a substitute for certain numbers. Students are puzzled when several algebraic letters appear in a mathematical problem. They find it difficult to interpret the algebraic letters whether the algebraic letters are a substitute for the value of a particular number or a substitute for the range of values of numbers.

With the recent studies, Mashazi, (2013) stated that there were some learning barriers that occur in learning algebra. The barriers were mainly dealing with students' understanding of algebraic letters. The difficulty was rooted from the diverse roles of algebraic letters. According to Christou et al. (2007), the tendency of students to use their experiences at the basic level in the form of numbers and arithmetic operations caused students to be distracted in interpreting algebraic letters. In addition, students had a tendency to simplify algebraic answers into a single answer. According to Stacey & Macgregor, (2001), the occurrence of erroneous interpretation in students was related to the development of students' cognitive level from concrete to abstract. In a study conducted by Küchemann (1981), it was found that various interpretations were given by students to the letters of algebra from the study. There are six students' interpretations of algebraic letters, which are *the letter evaluated means interpreted as numerical values through trial and error, the letter not used means interpretes as irrelevant, the letter as an object or labels, the letter as a specific unknown, the letter as generalized number and the letter as variable means interpreted with awareness of functional relationships*. Of the six student interpretations of algebraic letters, it was found that students were able to interpret algebraic problems on letters as evaluated, with a percentage of 92%, meaning that students interpreted the algebraic letters as a number value, while other algebraic problems related to students' interpretation of algebraic letters provide the lowest percentage of interpretation of algebraic letters as variables, namely only 6%, which means students had difficulty in understanding algebraic letters as variables (Kilhamn, Röj-Lindberg, & Björkqvist, 2019). Küchemann's research showed that students' difficulties in interpreting algebraic letters appeared when the algebraic letters acted as variables. In fact, the role of algebraic letters as variables is a concept for students to learn functions (Molina et al., 2018).

Students' interpretation of algebraic letters as a substitute for a certain number, a substitute for a range of number values and a substitute for a general number is a transition process of students' thinking which is first dealing with numbers and number ranges then later introduced to a generalized number. The transition process

is identified as the transition of students' thinking. The transition thinking on numbers, range of values in numbers, and the symbolization of these numbers into algebraic letters guides students towards algebraic thinking. The process requires conceptualizing of thinking processes both in the form of numbers and in generalizing numbers (Molina et al., 2018).

According to Powell et al., (2016) students need to be facilitated in conceptualizing their way of thinking, such as paying attention to the transition of students' arithmetic thinking to students' algebraic thinking. In the arithmetic stage, students tend to think about numbers and calculations while in the algebraic stage, students are trying to make connections between algebraic letters and numbers, perform algebraic operations and accept algebraic expressions as the final answers (Kieran, 2004).

However, studies on making connections between algebraic letters and numbers, performing algebraic operations and accepting algebraic expressions as the final result of student answers has not been developed much at the national setting. Previous studies mainly concerned with the students' difficulties in learning algebra. Therefore, studies on making relationships between algebraic letters and numbers, performing algebraic operations and accepting algebraic expressions as the final result of student answers in understanding algebra still need to be explored and developed. In particular, this study aimed to investigate students' understanding of the role of algebraic letters in solving problems in the perspective of students' thinking transition

from arithmetic thinking to algebraic thinking starting from how students make the relationship between algebraic letters and numbers, perform algebraic operations and accept algebraic expressions as the final result of students' answers.

II. METHOD

The research approach used in this study was qualitative research utilizing phenomenological approach (Merriam & Tisdell, 2016). This type of research was appropriate with the research objectives which provided an overview of the phenomenon of students' understanding in interpreting algebraic letters in a mathematical problem.

The study involved 7th grade junior high school students who were in the odd semester as the participants. The number of students was 18 students within the age of 13 years old. Students were also residents nearby, approximately 3 kilometers from school.

This study was conducted in three ways starting from the study of literature related to algebraic letters, the design of test questions and the design of estimated students' interpretations of algebraic letters in mathematical problems, and finally the test of test questions to participants.

The test questions consisted of eight questions involving algebraic letters. Before the test questions were tested on students, the test questions had first been validated regarding the feasibility and readability of the test questions. For this reason, the researchers conducted a validity test with three junior high school teachers and three lecturers from the mathematics education

department. These 3 math teachers had teaching experience in grade 7th and working experience for approximately 5 years, while the three lecturers were teaching at universities in Riau. After the test questions were declared feasible, the researchers conducted tests on the questions to students.

Students took the paper-based test. In solving the problem, students were given the opportunity to write all the arguments. Since the test was conducted during covid-2019 pandemic and students did math learning for only 60 minutes, this test was carried out over a period of two days. On the first day, students worked on multiple choice questions and on the second day students worked on description questions. Each test was set to 60 minutes to complete. After the test was carried out, interviews were conducted regarding the test result, conducted openly in the school hall and adhered to the covid-2019 health protocol.

The data of this study were collected from student worksheet documents, audio and video recordings of interview results. The audio was a recording from the recording gadgets, while the video recording was generated from a handycam.

The data was later analyzed by testing, coding, and reducing the result of students' works and interviews. At the final stage, the final data were presented and interpreted.

III.   RESULT AND DISCUSSION

Based on the results of data analysis, it was discovered that students completing the arithmetic thinking to algebraic

thinking transition representation test related to algebraic letters provided different interpretations. It was illustrated by the test of the transitional representation from arithmetic thinking to algebraic thinking in the following mathematical problem.

"If $a + b = 15$, then the value of $2a + 2b = \cdots$"

On the test, there were two different algebraic letters in the equation. Furthermore, the two equations involved algebraic expressions that required students' observation regarding the role of these algebraic letters in the mathematical problem and the value of other algebraic expressions.

Table 1 presents the data from 18 participants of the study.

Table 1.
Name and participants' responses towards algebraic letters

No.	Name	Responses
1.	AT1	replacing letters with numbers a = 7 and b = 8
2.	AT2	replacing letters with numerical expression 2 × 15
3.	BT1	replacing letters with numbers a = 10 and b = 5
4.	BT2	replacing letters with numerical expression 2 × 15
5.	CT1	replacing letters with numbers a = 7,5 and b = 7,5
6.	CT2	replacing letters with numerical expression 2 × 15
7.	AS1	replacing letters with numerical expression 2 × 15
8.	AS2	replacing letters with numbers a = 10 dan b = 5
9.	BS1	no answer
10.	BS2	replacing letters with numbers a = 10 and b = 5
11.	CS1	replacing letters with numbers a = 8 dan b = 7
12.	CS2	replacing letters with numbers a = 10 dan b = 5
13.	AR1	replacing letters with numbers a = 7

		dan b = 8
14.	AR2	replacing letters with numerical expression 2 × 15
15.	BR1	no answer
16.	BR2	no answer
17.	CR1	replacing letters with numbers a = 10 dan b = 5
18.	CR2	replacing letters with numerical expression 2 × 15

Tabel 1 Showed that most students interpreted the algebraic letters as a substitute for the number which were replaced with several numbers. There were 3 students who replaced the algebraic letters with the numbers a=7 or a=8 and b=8 or b=7, while there were 5 students who replaced the algebraic letters with the numbers a=10 and b=5. In fact, there was also one student who replaced algebraic letters with the numbers a=7.5 and b=7.5. In addition, there were 6 students replacing algebraic letters based on the form of algebraic expression, namely 2a, which meant that students replaced 2a = 15 then students did the operation, namely 2×15. On the other hand, there were also 3 students who did not answer the question and did not provide the reason for their action. The five different responses indicated that students were able to solve the given problem by interpreting algebraic letters as a substitute for certain numbers in an unknown quantity.

Further observation on the students' worksheets discovered that students provided answers by determining certain numbers that were suitable for the equation containing the algebraic letters. Here is student A's worksheet.
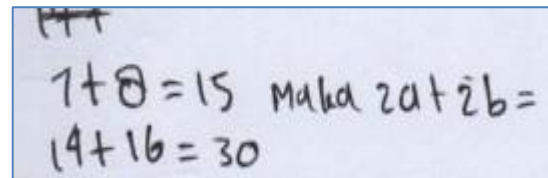

Figure 1. Student A's answer sheet

Figure 1 illustrated that the students could solve the problem correctly. The student solved the problem by replacing the algebraic letters in the equation a + b = 15 with a certain number value so that students replaced the letter 'a' with 7 and the letter 'b' with 8 then the student formed the algebraic equation into a number equation, namely 7 + 8 = 15.

Based on the observation, after the student wrote the equation, they remarked that the numerical equation had matched the algebraic equation in the problem, namely a + b = 15 and 7 + 8 = 15. Therefore, students continued to perform calculation by replacing the values of a and b in the algebraic expression $2a + 2b$ to obtain $14 + 16 = 30$.

To find out the students' thinking process when solving the problem, the following interview was conducted.

P: How did you solve the equation?

S: (the student is reading the question) If $a + b$ equal to 15 then $2a + 2b$ equal to? I think the answer is (pointing at $a + b = 15$), 15. Because this one is 15, so I tried to change the letters like this $a = 7$ dan $b = 8$, so I got 15 as the result and I counted again $7 + 8 = 15$. Because this is similar (the student is pointing to the algebraic and numerical equations), so 2 times 7 plus 2 times 8 so $14 + 16 = 30$.

P: Why did you replace a and b?

S: Because we need to find the value. I was supposed to find the value of a and b.

P: Were there any alternatives to solve this equation?

S: No, there weren't. I think this is the easiest way.

When students were asked how to solve the problem, students explained that they replaced the algebraic letters with a certain number and each algebraic letter had one certain number. Students explained that the algebraic letters could be replaced with certain numbers. To ensure that the algebraic letters that were replaced with certain numbers were correct, students checked the answer by performing the calculation of the numerical equation 7 + 8 = 15. Afterwards, the students changed the algebraic expression 2a + 2b into a numerical expression 14 + 16. Thus, the students found that the value of the expression was 20. From the interview excerpts, it was also discovered that the students found it easy to solve these mathematical problems by replacing algebraic letters in a variety of certain quantities.

In addition, there was also a different answer from student A. The following is student B's worksheet in solving the problem.

Figure 2. Student B' answer

Figure 2 illustrated that the student solved the equation incorrectly. The way thes student solved the problem was that students separated the algebraic expression 2a + 2b into algebraic expressions 2a and 2b then students wrote

15×2 = 30a and 15×2 = 30b. The student continued the operation by forming 30a+30b=60.

To find out the student' thinking process when solving the problem, the following interview was conducted.

P: How did you solve the equation?

S: (the student is reading) if $a + b$ equal to 15 then $2a + 2b$ equal to?. Hmmmm (the student is thinking and observing the answer he had written). It's hard. I actually do not understand this.

P: What do you understand then?

S: I think it's $2a$ and $2b$. Hmmmm (quiet). And this is $2a + 2b$, because $a + b = 15$ so it means 15 multiplied by 2.

P: why did you multiply 15 by 2?

S: because it's $2a$ and 15, and 15 multiplied by 2 was 30. And this one is 30 for a and this 30 is for b. So, I summed the numbers, $30a$ and $30b$ equal to 60.

P: where did you get 15 from?

S: I got it from this equation $a + b = 15$. So, the values of the letters are known as 15 (pointing out that a equal to 15).

When student B was asked the same question as student A, namely how to solve the problem, student B gave a different explanation from student A. Student B also took quite a long time to remember and explain the answer. After a while waiting for the student's answer, student B gave an explanation of equations and algebraic expressions. Student B explained that a + b = 2a, which was 15 and a + b = 2b, which was 15. Thus, student B interpreted 2a = 15, because in the problem there was also 2a, so the student assumed that there were 2 times the value of 15, namely 15 multiplied by 2 = 30. Likewise done with 2b, to explain how he got the value of 30, the

student wrote 30a and 30b. Finally, he wrote 30a+30b=60.

From the student's answer, it was concluded that the student had not fully comprehend the meaning of algebraic expressions, algebraic expression operations and algebraic equations. From the observations made during the interview, student B also focused on the numbers contained in the algebraic letters only, thus the student could not pay attention to the letters and numbers simultaneously that were necessary in the problem.

The results showed that seventh grade students had diverse interpretations in interpreting algebraic letters in mathematical problems. Most students interpreted algebraic letters as a fixed unknown quantity. This was similar to the study by Christou & Vosniadou (2012) which stated that students tended to replace algebraic letters with natural numbers both when the algebraic letters appear in algebraic expressions and in equations.

Students were thinking that algebraic letters in a mathematical problem must be able to be replaced by a certain number. Thus, they interpreted that algebraic letters were only a substitute for certain numbers not as a generalized number. This interpretation implied that students found it difficult to describe that the algebraic letters were generalized numbers and even students would also find it difficult to form the meaning of the algebraic letters as variables, namely varying known quantities (Brizuela et al., 2015).

Whereas in the perspective of school algebra, algebraic letters were not just a substitute for certain numbers but had a variety of roles, especially towards generalized and variable numbers. On the other hand, it should be noted that algebraic letters that become generalized numbers would require students' analysis on the meaning of algebraic letters and numbers simultaneously instead of just focusing on one thing, either algebraic letters or numbers. This was in accordance with the question given by Kieran (2004) that in developing algebraic thinking as a transition of thinking from arithmetic to algebra, students were required to focus on both aspects of algebraic letters and numbers. If they observe the algebraic letters or numbers only, students would have difficulty in solving problems (Brizuela et al., 2015; Steinweg et al., 2018).

IV. CONCLUSION

Based on the research objectives, namely investigating students' interpretations in interpreting algebraic letters in mathematical problems, it was concluded that students would probably interpret algebraic letters as a substitute for a certain number rather than as a generalized number. These algebraic letters could be something that was not used when there was an algebraic expression, algebraic operation or algebraic equation. However, this had not yet been reflected in the students' interpretation in solving problems. The inclination of students to regard algebraic letters as a substitute for specific numbers provided the impression that students were thinking in an arithmetic manner. The study came to the fascinating conclusion that students' interpretations in solving algebraic letter

problems were still focused on the difficulty of transforming a notation to the form of a natural number. In fact, seventh-grade students were expected to have algebra learning experiences including algebraic letters in the sense of generalized numbers. This required the attention of both educators and mathematics education experts while delivering algebra learning materials. What may be investigated further is how to construct a learning sequence based on algebraic letters, beginning with the meaning as a substitution for specific numbers and progressing until students discovered the variable values of the algebraic letters.

ACKNOWLEDGEMENT

REFERENCES

Afriansyah, E. A., & Turmudi, T. (2022). Prospective teachers' thinking through realistic mathematics education based emergent modeling in fractions. *Jurnal Elemen*, *8*(2), 605-618.

As'ari, A. R., Tohir, M., Valentino, E., Imron, Z., & Taufiq, I. (2017). *Matematika SMP/MTs Kelas VII Semester 1* (A. Lukito, A. Mahmudi, Turmudi, M. N. Priatna, Y. Satria, & Widowati (eds.); Edisi Revi). Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbud.

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle School Mathematics Teachers' Knowledge of Students' Understanding of Core Algebraic Concepts : Equal Sign and Variable. *Mathematical Thinking and Learning*, *9*(2), 249–272. https://doi.org/10.1080/10986060701360910

Astutik, E. P., & Purwasih, S. M. (2023). Field Dependent Student Errors in Solving Linear Algebra Problems Based on Newman's Procedure. *Mosharafa: Jurnal Pendidikan Matematika*, *12*(1), 169-180.

Brizuela, B. M., Blanton, M. L., Gardiner, A. M., Newman-Owens, A., & Sawrey, K. (2015). Children's Use of Variables and Variable Notation to Represent Their Algebraic Ideas. *Mathematical Thinking and Learning*, *17*(1), 34–63. https://doi.org/10.1080/10986065.2015.981939

Çelik, D., & Güneş, G. (2013). Different grade students' use and interpretation of literal symbols. *Kuram ve Uygulamada Egitim Bilimleri*, *13*(2), 1168–1175.

Chimoni, M., Pitta-Pantazi, D., & Christou, C. (2018). Examining early algebraic thinking: insights from empirical data. *Educational Studies in Mathematics*, *98*(1), 57–76. https://doi.org/10.1007/s10649-018-9803-x

Christou, K. P., & Vosniadou, S. (2012). What Kinds of Numbers Do Students Assign to Literal Symbols ? Aspects of the Transition from Arithmetic to Algebra. *Mathematical Thinking and*

Learning, *14*, 1–27. https://doi.org/10.1080/10986065.2012.625074

Christou, K. P., Vosniadou, S., & Vamvakoussi, X. (2007). Students' Interpretations of Literal Symbols in Algebra. *Re-Framing the Conceptual Change Approach in Learning and Instruction*, January, 283–297.

Diva, S. A., & Purwaningrum, J. P. (2023). Strategi Mathematical Habits of Mind Berbantuan Wolfram Alpha untuk Meningkatkan Kemampuan Berpikir Kritis Siswa dalam Menyelesaikan Bangun Datar. *Plusminus: jurnal pendidikan matematika*, *3*(1), 15-28.

Ferrucci, B. J. (2004). Gateways to Algebra at the Primary Level. *The Mathematics Educator*, *8*(1), 131–138.

Fulop, Z. (2015). Transition from arithmetic to algebra in primary school education. *Teaching Mathematics and Computer Science*, *2*, 225–248. https://doi.org/10.5485/TMCS.2015.0398

Hidayanto, E., Purwanto, Subanji, & Rahardjo, S. (2014). *Transisi dari berpikir aritmetis ke berpikir aljabaris*. *July*.

Jupri, A., Drijvers, P., & Heuvel-Panhuizen, M. van den. (2014). Difficulties in Initial Algebra Learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710. https://doi.org/10.1007/s13394-013-0097-0

Kaput, J. J. (2008). What Is Algebra? What is Algebraic Reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 8–17). Taylor&Francis Group, LLC.

Khalid, M., Yakop, F., & Ibrahim, H. (2020). Year 7 students' interpretation of letters and symbols in solving routine algebraic problems. *Qualitative Report*, *25*(11), 4167–4181. https://doi.org/10.46743/2160-3715/2020.4062

Kieran, C. (2004). Algebraic Thinking in The Early Grades : What is it? *The Mathematics Educator*, *8*(1), 139–151.

Kilhamn, C. (2014). When Does a Variable Vary ? Identifying Mathematical Content Knowledge for Teaching Variables. *Nordic Studies in Mathematics Education*, *19*(3–4), 83–100.

Kilhamn, C., Röj-Lindberg, A.-S., & Björkqvist, O. (2019). School Algebra. In C. Kilhamn & R. Saljo (Eds.), *Encountering Algebra* (pp. 1–12). Springer International Publishing Switzerland. https://doi.org/10.1007/978-3-030-17577-1

Küchemann, D. (1981). Algebra. In K. M. Hart (Ed.), *Children's Understanding Mathematics* (pp. 102–119). John Murray.

Lisan, L. F., Risnawati, L., & Setyawan, B. W. (2023). Implementasi prinsip aritmatika sosial dalam transaksi jual beli pedagang di sekitar kampus uin sayyid ali rahmatullah tulungagung. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *2*(2), 129-138.

Malisani, E., & Spagnolo, F. (2009). From Arithmetical Thought to Algebraic Thought : The Role of The"Variable". *Educational Studies in Mathematics*, *71*, 19–41.

https://doi.org/10.1007/s10649-008-9157-x

Mashazi, S. (2013). Learners' Explanations of The Errors They Make in Introductory Algebra. *Pythagoras*, *9*, 14–33.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative Research: A Guide to Design and Implementation* (Fourth). Jossey-Bass.

Molina, M., Ambrose, R., & Rio, A. (2018). First Encounter with Variables by First and Third Grade Spanish Students. *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds: The Global Evolution of an Emerging Field of Research and Practice*, *2017*, 261–280.

NCTM. (2000). *Principles and Standards for School Mathematics*. The National Council of Teachers Mathematics.

Pitta-pantazi, D., Chimoni, M., & Christou, C. (2019). Different Types of Algebraic Thinking : an Empirical Study Focusing on Middle School Students. *International Journal of Science and Mathematics Education*. https://doi.org/ 10.1007/s10763-019-10003-6 Different

Powell, S. R., Kearns, D. M., & Driver, M. K. (2016). Exploring the connection between arithmetic and prealgebraic reasoning at first and second grade. *Journal of Educational Psychology*, *108*(7), 943–959. https://doi.org/10.1037/edu0000112

Samo, M. A. (2009). Students' Perceptions about the Symbols, Letters and Signs in Algebra and How Do These Affect Their Learning of Algebra: A Case Study in a Government Girls Secondary School Karachi. *International Journal for Mathematics Teaching and Learning*, 35.

Stacey, K., & Macgregor, M. (2001). Curriculum Reform and Approaches to Algebra. In R. Sutherland, T. Rojano, A. Bell, & R. Lins (Eds.), *Perspectives on School Algebra* (Vol. 22, pp. 141–153). Springer. https://doi.org/10.1007/0-306-47223-6_8

Steinweg, A. S., Akinwunmi, K., & Lenz, D. (2018). Making Implicit Algebraic Thinking Explicit : Exploiting National Characteristics of German Approaches. In C. Kieran (Ed.), *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds, ICME-13 Monographs* (pp. 283–307). Springer International Publishing AG. https://doi.org/ 10.1007/978-3-319-68351-5_12

Sugiman, Sumadyono, & Marfuah. (2016). *Karakteristik Siswa SMP dan Bilangan* (N. Priatna, Y. Anggraena, & S. Wardhani (eds.)). Direktorat Jendral Guru dan Tenaga Kependidikan Kementerian Pendidikan dan Kebudayaan.

Tiwari, C., & Fatima, R. (2019). Secondary School Students' Misconceptions in Algebra Concepts. *Mahatma Gandhi Central University Journal of Social Sciences*, *1*(1).

Warren, E., Trigueros, M., & Ursini, S. (2016). Research on the learning and teaching of Algebra. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (pp. 73–108).

Sense   Publishers.
https://doi.org/10.1007/978-94-6300-561-6_3

AUTHOR'S BIOGRAPHY

Dr. Reni Wahyuni, S.Pd. M.Pd.

Born in Pekanbaru, January 10, 1986. Teaching staff at Universitas Islam Riau. Study S1 Mathematics Education, Universitas Riau, Pekanbaru, graduated in 2008; Master of Mathematics Education, Universitas Sriwijaya, Palembang, graduated in 2011; and Doctoral Degree in Mathematics Education, Universitas Pendidikan Indonesia, Bandung, graduating in 2023.

Prof. Dr. Tatang Herman, M.Ed.

Born in Garut, on October 11, 1962. Professor at Universitas Pendidikan Indonesia. Studying S1 Mathematics Education at IKIP Bandung 1989. Continuing Masters at Deakin University Melbourne Australia graduated in 1996, and Doctoral Degree at Universitas Pendidikan Indonesia graduated in 2006.

Siti Fatimah, M.Si, Ph.D.

Born on August 23, 1968. Is an Associate Professor at the Universitas Pendidikan Indonesia. Graduated with a Bachelor's degree at Universitas Pendidikan Indonesia in 1992. Continuing with a Master's degree at Universitas Gajah Mada, graduating in 1996, and a Doctoral Degree at Utrecht University, the Netherlands, graduating in 2002.