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From Linearity to Iteration: Navigating Polya's Problem-Solving Stages in an e-PBL Geometry Environment

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Abstrak

Penelitian ini mengkaji bagaimana mahasiswa mengalami dan memaknai tahapan pemecahan masalah Polya saat mengerjakan tugas geometri dalam lingkungan electronic Problem-Based Learning (e-PBL) yang diimplementasikan melalui sistem manajemen pembelajaran berbasis Moodle yang dikustomisasi. Penelitian ini menggunakan desain deskriptif kualitatif dengan sumber data berupa artefak pemecahan masalah tertulis, wawancara semi-terstruktur, dan jejak digital selama e-PBL. Data dianalisis menggunakan analisis tematik Braun dan Clarke dengan pengodean deduktif berdasarkan empat tahap Polya dan pengodean induktif untuk menangkap pola penalaran yang muncul. Hasil penelitian menunjukkan bahwa pemecahan masalah berlangsung secara dinamis dan rekursif. Tahap memahami dan merencanakan melibatkan penafsiran ulang representasi multimodal, sedangkan tahap pelaksanaan ditandai oleh pergeseran representasi yang memperjelas konsep. Prompt reflektif dalam e-PBL mendukung deteksi kesalahan dan penguatan konsep, serta menegaskan peran tahap evaluasi.
***Kata Kunci**: e-Problem-Based Learning (e-PBL); Pembelajaran geometri; Penalaran multimodal; Tahapan pemecahan masalah Polya.*

Abstract

This study examines how students experience and interpret Polya's problem-solving stages while working on geometry tasks in an electronic Problem-Based Learning (e-PBL) environment implemented through a customized Moodle based learning management system. Using a qualitative descriptive design, data were collected from written problem-solving artefacts, semi-structured interviews, and digital traces generated during e-PBL activities. The data were analyzed using thematic analysis following Braun and Clarke, combining deductive coding based on Polya's four stages with inductive coding to capture emerging reasoning patterns. The findings indicate that problem-solving is enacted as a dynamic and recursive process rather than a linear sequence. Understanding and planning involve repeated reinterpretation of multimodal representations, while execution is characterized by representational shifts that support conceptual clarity. Reflective prompts in e-PBL facilitate error detection and conceptual consolidation, highlighting the importance of the evaluative stage.
Keywords: e-Problem-Based Learning (e-PBL); Geometry learning; Multimodal reasoning; Polya's problem-solving stages.

I. INTRODUCTION

In digital mathematics classrooms, problem-solving increasingly unfolds within environments characterized by multimodal representations, rapid feedback, and technology-mediated interaction, posing substantive challenges to students' reasoning processes, particularly in geometry, where visual, symbolic, and conceptual information must be continuously coordinated. Recent studies indicate that effective mathematical problem-solving under such conditions requires learners not only to interpret problem contexts accurately but also to dynamically construct meaning across representations while regulating their reasoning in response to emerging information (Caviola et al., 2022; Santos-Trigo, 2024). Although Polya's four-stage problem-solving model remains influential, its original linear and sequential formulation warrants critical re-examination in light of digital learning contexts that foreground iterative reasoning, representational revision, and metacognitive adjustment (Polya, 1945; Wahab et al., 2024). In technology-rich settings, learners frequently move back and forth between understanding, planning, executing, and evaluating as they engage with digital prompts, visualizations, and feedback. This condition signals the need to explore how Polya's framework functions, adapts, or requires reconceptualization within contemporary digital inquiry environments, such as electronic Problem-Based Learning (e-PBL), which integrate multimodal scaffolds, interactive feedback, and collaborative features to support mathematical problem-solving (Jiang & Ren, 2021; Martins & Martinho, 2021; Pan & Liu, 2022; Wilang & Garcia, 2021).

Digital learning environments have reshaped how students engage with mathematical problem-solving by altering the temporal, representational, and regulatory conditions under which reasoning occurs. The increasing adoption of electronic Problem-Based Learning (e-PBL) offers inquiry settings in which learners interact with task-embedded multimodal representations, adaptive scaffold prompts that respond to emerging difficulties, and iterative feedback cycles that foreground monitoring and revision of reasoning (Casler-Failing, 2024; Kong et al., 2024; Pan & Liu, 2022). Recent empirical evidence indicates that such environments can strengthen mathematical reasoning and metacognitive monitoring by making students' intermediate thinking visible and revisable rather than treating solutions as final products (Aksu & Zengin, 2022; Bains et al., 2022; Demir & Zengin, 2023; Miller-Bains et al., 2022; Prabawanto, 2023). In geometry education, digital scaffolds and interactive visualizations are shown to support spatial reasoning by enabling learners to test, modify, and realign visual constructions with formal geometric definitions, thereby reducing representational ambiguity (Li et al., 2022; Lindgren & DeLiema, 2022; Piri & Cagiltay, 2024). Within e-PBL, these affordances are further intensified through problem scenarios that require students to repeatedly reorganize strategies in response to feedback and representational breakdowns, positioning problem-solving as an evolving process rather than a linear execution of procedures (Jelodari et al., 2025; Setyani & Susilowati, 2022; Wagino et al., 2024). Geometry constitutes a particularly salient domain for examining iterative problem-solving processes because students must continuously coordinate visual, symbolic, and spatial representations, a condition that frequently gives rise to representational mismatch and necessitates repeated reinterpretation and revision of meaning.

Despite substantial growth in research on problem-based learning and digital mathematics instruction, existing studies have tended to examine PBL processes, digital scaffolding, or geometric reasoning as partially overlapping but analytically separate domains. While both quantitative and qualitative investigations have generated important insights into learning outcomes, strategy use, and student engagement (Guo et al., 2023; Steen-Utheim & Foldnes, 2018), relatively little attention has been given to how Polya's problem-solving stages are experienced, negotiated, and reconfigured when embedded simultaneously within e-PBL environments and geometry tasks. In particular, there is limited qualitative evidence that traces learners' movement across Polya's stages in response to multimodal representations and digitally mediated scaffolds in geometry-specific contexts, where spatial interpretation often demands iterative shifts in understanding. Recent literature further suggests that the influence of digital scaffolds on transitions between Polya's stages remains insufficiently specified, including how such supports prompt reinterpretation, reshape planning decisions, trigger representational shifts, or foster reflective insight (Calcagni et al., 2023; Motlhaka, 2020; Pan & Liu, 2022). Moreover, although emerging studies acknowledge that students' reasoning in digital tasks is frequently non-linear, the recursive navigation of Polya's stages in e-PBL geometry settings has not yet been systematically theorized or documented through fine-grained qualitative analyses (Camelo et al., 2018; Jelodari et al., 2025; Türkoğlu & Yalçınalp, 2024).

To address these gaps, the present study investigates how students experience and interpret each stage of Polya's problem-solving process as they engage with geometry tasks in an e-PBL environment. Rather than foregrounding effectiveness measures, this study centers on students' meaning-making by integrating written problem-solving artefacts, semi-structured interviews, and digital traces to capture fine-grained cognitive and interpretive trajectories. Specifically, the study contributes by (1) documenting how students navigate Polya's problem-solving stages in a non-linear and recursive manner when engaging with geometry tasks in e-PBL contexts, thereby challenging sequential interpretations of the model; (2) explicating how digitally embedded scaffolds shape transitions between stages by prompting reinterpretation, strategic revision, and delayed reflective insight; (3) analyzing how multimodal representations mediate students' interpretation of geometric relationships and influence shifts between understanding, planning, and execution; and (4) revealing how students articulate, reorganize, and stabilize mathematical reasoning in response to conceptual uncertainty during digital inquiry. Taken together, these contributions extend the theoretical understanding of Polya's framework by situating it within technology-mediated inquiry environments and by demonstrating how e-PBL reconfigures the cognitive and metacognitive dynamics of mathematical problem-solving. As a broader implication, the findings also inform discussions on preparing learners with adaptive expertise for complex problem-solving demands in digitally enriched educational settings.

II. METHOD

This study employed a qualitative descriptive design to investigate how students experience, interpret, and regulate each stage of Polya's problem-solving process within an electronic Problem-Based Learning (e-PBL) environment. The design was selected to capture the interpretive, cognitive, and reflective dimensions of students' reasoning as they engage with authentic geometry tasks, with particular attention to the meanings constructed during digital problem-based inquiry rather than to quantified learning outcomes. Analytically, the study is guided by Polya's four-stage problem-solving model as a heuristic framework for tracing students' reasoning pathways, strategic choices, and evaluative judgments (Polya, 1945). This framework is complemented by a metacognitive regulation perspective, which illuminates how learners monitor, adapt, and revisit their thinking across problem-solving phases in response to digital scaffolds and representational challenges (Prabawanto, 2023). Informed by constructivist and digital inquiry learning perspectives, this integrated lens provides a concise yet robust basis for examining how students make sense of problems, adjust strategies, and regulate thinking in an e-PBL geometry context.

The study was conducted in an undergraduate Geometry course that implemented e-PBL using a customized Moodle-based platform designed for geometry problem-solving. The platform provided context-rich tasks, collaborative tools such as discussion forums and real-time chat, and targeted scaffolds including hints, worked examples, and reflective prompts to support interpretation, strategy revision, and reflection. Using purposive sampling, 10 to 12 second-year undergraduate students with prior geometry coursework and basic digital proficiency were selected based on consistent engagement and information-rich problem-solving artefacts. This sample size was appropriate for in-depth idiographic analysis aimed at theoretical insight rather than statistical generalization (Smith, 2018). All participants gave informed consent and were assigned pseudonyms.

Data were obtained from three complementary sources to capture students' problem-solving processes in depth. To enhance analytic transparency, each data source was systematically aligned with specific purposes in relation to Polya's problem-solving stages, as summarized in Table 1. These sources included written problem-solving artefacts, semi-structured interviews, and digital traces generated within the e-PBL platform. Together, this multi-source design enabled methodological triangulation and supported a layered interpretation of students' explicit reasoning, implicit process navigation, and metacognitive reflection during problem-solving in the e-PBL environment.

Table 1.
Data Sources, Formats, and Analytic Purposes Aligned with Polya's Problem-Solving Stages

Data Source	Format or example	Purpose Aligned with Polya's Stage
Written Artefacts	Solution sheets, planning notes, reflective journals	To capture explicit reasoning across understanding, planning, execution, and evaluation stages
Digital Traces	GeoGebra construction history, time-stamped clicks on hints, discussion forum logs	To observe implicit, process-oriented navigation, representational shifts, and tool use across stages
Interviews	Semi-structured, screen recorded retrospective walkthroughs	To elicit students' metacognitive interpretation of their own problem-solving processes

Data were analyzed using Thematic Analysis following Braun and Clarke's (2006, 2019) six-phase framework, supported by Atlas.ti to facilitate systematic coding and theme development. All artefacts, interview transcripts, and digital traces were first read iteratively to build familiarity with students' reasoning. Initial codes were generated deductively from Polya's four stages and inductively from emerging patterns in students' accounts, with code co-occurrences informing theme construction. Codes were then clustered into preliminary themes, reviewed for coherence, and refined to ensure analytic clarity. Final themes were defined in relation to Polya's model within the e-PBL context and reported through a synthesized narrative supported by representative excerpts, ensuring transparent links between coding decisions and interpretive claims.

Trustworthiness was ensured through triangulation across artefacts, interviews, and digital traces; member checking in which participants reviewed interpretive summaries of their reasoning; and maintaining an audit trail documenting coding decisions and analytic reflections. Ethical procedures included informed consent, anonymization, secure data handling, and adherence to institutional research ethics guidelines.

III. RESULT AND DISCUSSION

A. Result

The findings illustrate how students experienced and interpreted each stage of Polya's problem-solving process while engaging with geometry tasks in an e-PBL environment. Using reflexive thematic analysis, students' written artefacts, interview transcripts, and digital traces were coded deductively based on Polya's four stages and inductively to capture cross-case patterns. Triangulated evidence traced how students understood problems, developed strategies, carried out procedures, and evaluated solutions in interaction with e-PBL scaffolds. The analysis revealed that engagement with Polya's stages was largely iterative rather than linear, shaped by digital prompts, multimodal representations, and collaborative features. The table below synthesizes these findings by aligning Polya indicators with representative evidence and the resulting coded themes.

Clarifying Givens and Resolving Diagram–Text Mismatch

Students' engagement with the geometry task revealed that understanding the problem did not operate as a fixed preliminary stage, but unfolded as an iterative interpretive process. In the hexagonal prism task, students were required to coordinate verbal givens describing spatial dimensions with diagrammatic representations generated in GeoGebra. Initial attempts to make sense of the problem frequently resulted in interpretive tension when textual descriptions and visual cues did not immediately align, prompting students to reread the problem statement and reassess their interpretations. One student reflected this process by stating, *"I re-read the question several times because the picture and text didn't match my first interpretation"* (S3-WA1). Figure 1 provides concrete visual evidence of this difficulty, showing a collapsed three dimensional representation in which the student projected skew line segments onto a single plane. This projection indicates a fundamental misunderstanding of the geometric definition of spatial angles and directly explains why the student's planning stage stalled, as the constructed diagram no longer preserved the required spatial relations. Such responses indicate that confusion functioned as a productive trigger for sense making rather than a terminal obstacle.

When students encountered mismatches between diagrams and text, they began to question the epistemic status of each representation, treating diagrams as provisional models that required validation against textual givens. They moved repeatedly between rereading the problem, revising diagrams, and reconsidering spatial relations, indicating a form of cognitive conflict triggered by incompatible representations. This conflict was mediated and gradually resolved through multimodal interaction within the e-PBL environment, including dynamic visuals, textual cues, and scaffold prompts. In this process, rereading functioned as a deliberate interpretive strategy for resolving ambiguity rather than a mechanical repetition of instructions.

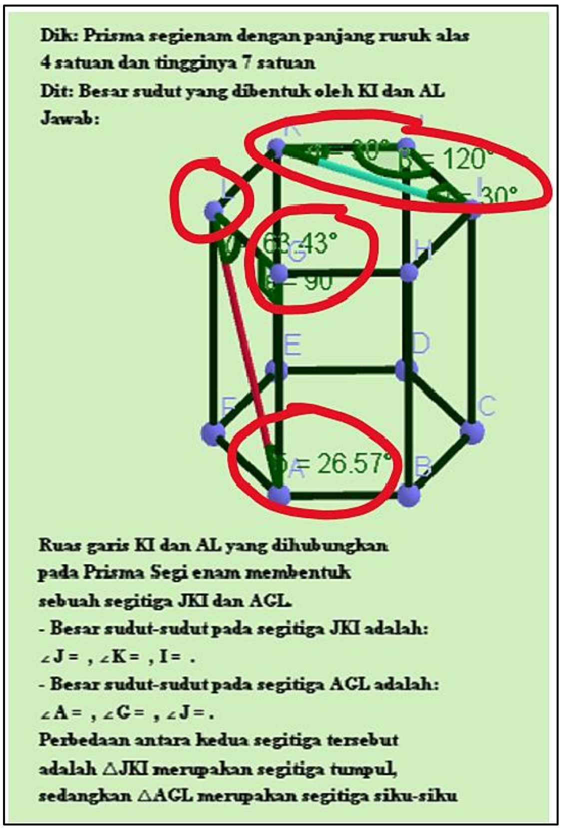

Figure 1. Student generated three dimensional representation exhibiting a diagram text mismatch, in which spatial relations between line segments are collapsed into a planar projection.

Digital scaffolds embedded in the e-PBL environment played a mediating role in this negotiation process. Log data indicate that hints were typically accessed at moments

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when students' interpretations stalled, particularly when critical spatial relations such as ignored angles or implicit projections were overlooked. Rather than supplying procedural solutions, scaffolds redirected students' attention toward these neglected elements. As one learner noted, *"The hint helped me notice the angle I ignored"* (S4-LOG1). Through this mechanism, scaffolding functioned as a catalyst for resolving cognitive conflict by reorienting students' attention toward geometrically salient features rather than correcting errors directly.

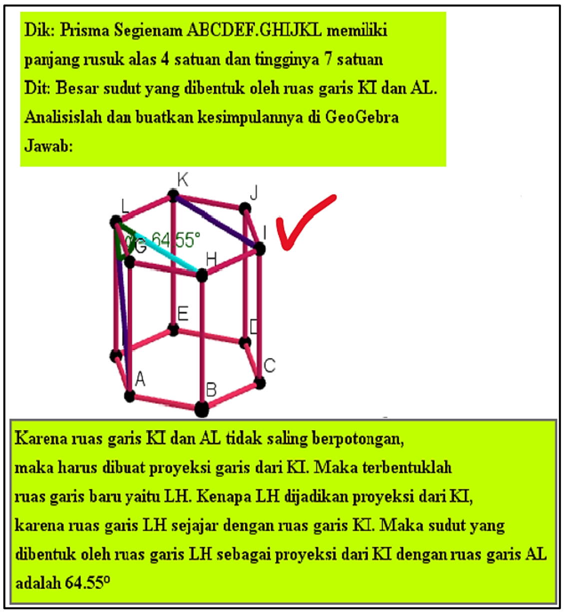

Figure 2. Ideal spatial representation for angle determination, preserving three dimensional relations required for accurate geometric interpretation.

Analysis of students' visualizations revealed clear differences in representational consistency. Students who reached correct solutions maintained the spatial relations of line segments KI and AL and constructed appropriate auxiliary lines, resulting in GeoGebra models and angle measures aligned with formal definitions of spatial angles. In contrast, incorrect solutions stemmed from a collapse of three-dimensional relations into a planar projection, producing invalid geometric models that undermined both computation and strategic planning. During verification, students with accurate representations were able to confirm consistency between constructed models and measured results, while those with flawed representations failed to detect such discrepancies. Viewed through Polya's framework, these patterns show that understanding the problem was repeatedly revisited through representation construction and verification, with digital scaffolding supporting iterative movement across stages by making representational breakdowns visible and revisable within the e-PBL environment.

Strategy Revision through Scaffold-Driven Restructuring

Planning within the e-PBL environment emerged as a provisional and revisable process rather than a fixed strategic commitment. In the square pyramid task requiring the determination of the distance between line segments BT and DT, students initially proposed tentative plans that were frequently reassessed as they interacted with embedded examples and strategy hints. One student noted, *"My plan changed after checking the example in e-PBL… I realized my first idea wouldn't work"* (S8-WA2), indicating metacognitive monitoring of plan adequacy rather than unsystematic trial and error. Moments of uncertainty often led students to deliberately consult digital scaffolds, as reflected in the comment, *"I didn't know the next step, so I opened the strategy hint and rewrote my plan"* (S6-INT1). Such actions signal recognition of an impasse and an intentional epistemic shift prompted by system feedback. Planning thus functioned as a dialogic, externally mediated activity in which plans operated as working hypotheses, repeatedly tested and revised in response to representational demands and scaffolded guidance.

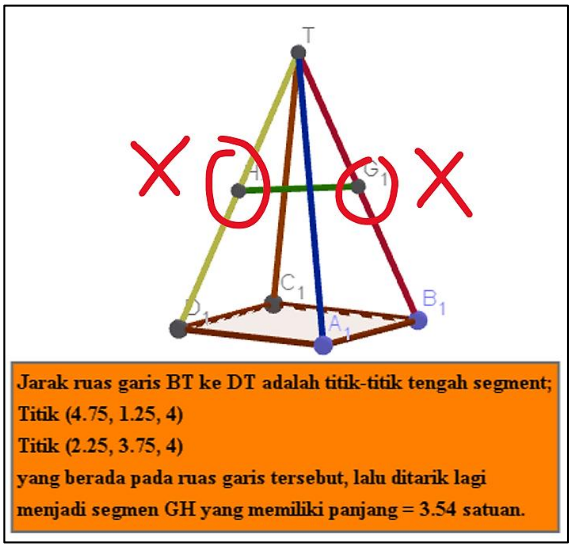

Figure 3. Student generated representation showing a strategy related spatial mismatch in determining the distance between line segments due to an inappropriate auxiliary construction.

Analysis of students' visualizations revealed that unsuccessful strategy revision was closely associated with misrepresentation of spatial relations during execution. In the student-generated representation, the distance between line segments BT and DT was treated as a positive value because the student constructed an auxiliary segment connecting the two lines, implicitly representing them as non intersecting. This strategic assumption overlooked the fact that both segments intersect at point T, a relation that mathematically implies a distance of zero. This error illustrates a failure of metacognitive evaluation, as the student did not reassess the conceptual validity of the constructed representation before proceeding with execution. The mismatch illustrates how an inadequately revised plan can propagate into execution without immediate detection.

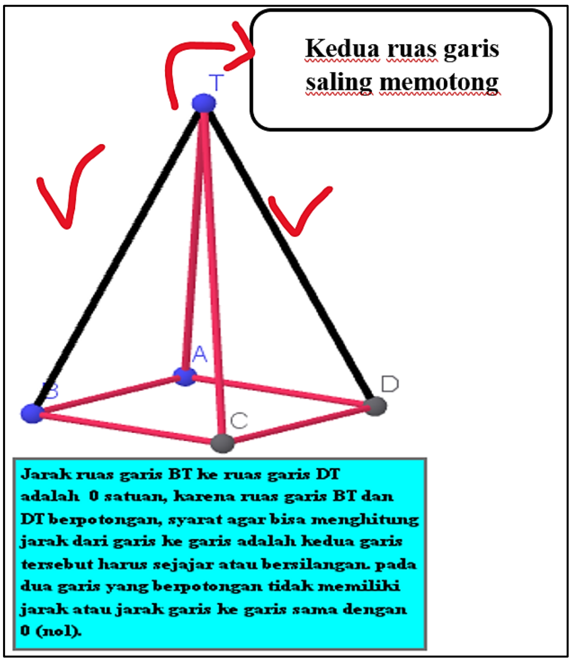

Figure 4. Ideal spatial representation for distance determination, illustrating alignment between geometric meaning, visual construction, and numerical result.

In contrast, the ideal representation preserved the intersection of BT and DT at the apex T, aligning the diagram with the formal definition of distance between intersecting line segments and producing coherence between geometric meaning, visual construction, and numerical results. This shift reflects metacognitive regulation, as students evaluated representational adequacy and realigned their strategy through conceptual restructuring rather than mere procedural adjustment. Viewed through Polya's framework, devising a plan did not operate as a discrete pre-execution stage but was revisited in response to representational breakdowns during execution. Iterative movement between planning and carrying out the plan was thus enabled by scaffold-driven epistemic reorientation within the e-PBL environment.

Representational Shifts and Verification during Execution

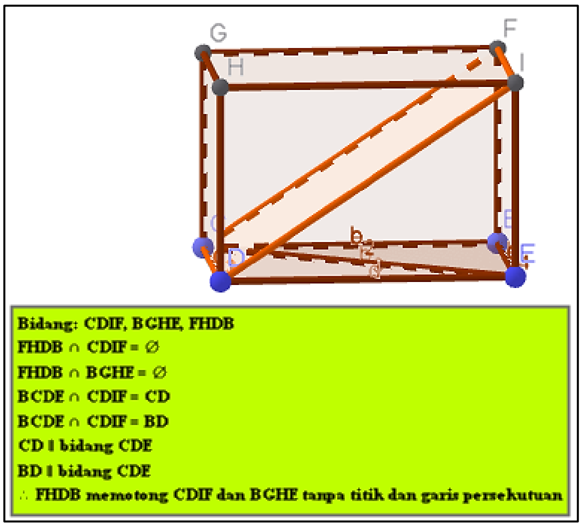

Figure 5. Student generated representation exhibiting an execution stage mismatch, where plane intersections are inaccurately aggregated without preserving invariant spatial relations.

Execution in the e-PBL environment unfolded as an active process of verification and representational movement, in which procedural work remained closely tied to conceptual monitoring. In the rectangular prism task involving the planes OPQR, KNOR, and LMPQ, students continued to cross-check numerical steps against constructed diagrams even after a plan had been established. As one student noted, *"Once I fixed my plan, the steps became clearer, but I still checked the figure to avoid mistakes"* (S1-WA2), indicating that execution involved ongoing validation rather than mechanical application. When discrepancies arose, students often shifted representations, returning to diagram construction to resolve numerical confusion, as reflected in the log entry, *"I switched to drawing the shape again because the numbers didn't make sense"* (S2-LOG1). This transmediational shift from symbolic to visual representation shows that execution functioned as a meaning-making activity, enabling students to realign calculations with spatial structure. Consequently, execution remained dynamically linked to understanding and planning, rather than operating as an isolated procedural stage.

Analysis of students' visualizations indicates that execution stage errors often stemmed from incomplete verification of spatial relations. In the student-generated representation, planes OPQR, KNOR, and LMPQ were treated as sharing a common line of intersection, despite the absence of consistent geometric justification. This interpretation resulted from aggregating partial intersections without systematically examining whether each plane shared common points or lines with the others. The visual artifact captures a breakdown in transmediation, where representational shifts occurred without preserving invariant geometric relations, leading to conclusions that diverged from the solid's actual spatial structure.

In contrast, the ideal representation preserved the distinct intersection relations among the planes, with OPQR intersecting KNOR and LMPQ separately, while plane LMN shared no common line or point of intersection with OPQR. This visualization shows how careful verification during execution, through consistent tracing of intersection lines and spatial relations, aligned constructed diagrams with formal geometric definitions. Transmediation was successful because meaning was maintained across visual and symbolic forms via deliberate representational checking.

Figure 6. Ideal spatial representation of plane intersections, illustrating correct geometric relations among planes in the rectangular prism.

Viewed through Polya's framework, carrying out the plan did not operate as an isolated operational stage but involved iterative movement between action and verification. Execution thus emerged as a site of conceptual testing and revision, reinforcing non-linear navigation across Polya's stages within the e-PBL geometry environment.

Reflective Detection of Errors and Delayed Insight

Evaluation functioned as a cognitively productive stage in which misconceptions became visible and open to revision. Several students identified conceptual errors only after completing their solutions and engaging with reflective prompts, as illustrated by the comment, *"While writing the reflection, I realized the angle I used earlier was wrong"* (S5-WA3). This delayed detection suggests that the evaluation created a critical cognitive pause in metacognitive monitoring, which was largely absent during execution. Diagram-based validation emerged as a key evaluative strategy, with students comparing final answers against constructed representations, as one participant explained, *"I checked my answer by comparing it with the diagram; the result didn't match, so I corrected it"* (S7-INT2). Through this reflective scrutiny, evaluation enabled students to reconcile symbolic results with spatial meaning. Viewed in relation to earlier stages, evaluation did not function as a terminal checkpoint but as a mechanism for reopening understanding and planning when inconsistencies surfaced, reinforcing iterative movement across Polya's stages within the e-PBL geometry environment.

Synthesizing the Process: A Model of Recursive, Scaffold-Mediated Transitions

Across cases, students' problem-solving trajectories in the e-PBL environment challenge a linear reading of Polya's stages. Rather than progressing sequentially from understanding to evaluation, students engaged in recursive movement across stages that formed flexible clusters of activity, such as repeated transitions between understanding and planning or between execution and evaluation. When intermediate outcomes conflicted with expectations, students frequently suspended ongoing actions and returned to earlier stages to reassess their interpretations or strategies. Log data capture this adaptive regulation, as one student reported, *"I went back to the first step after seeing the output didn't match what I expected"* (S7-LOG2). Such movement did not indicate breakdown or confusion, but functioned as a coherence-restoring mechanism triggered by representational mismatch or conceptual misalignment. This synthesized pattern of recursive movement is illustrated in Figure 7.

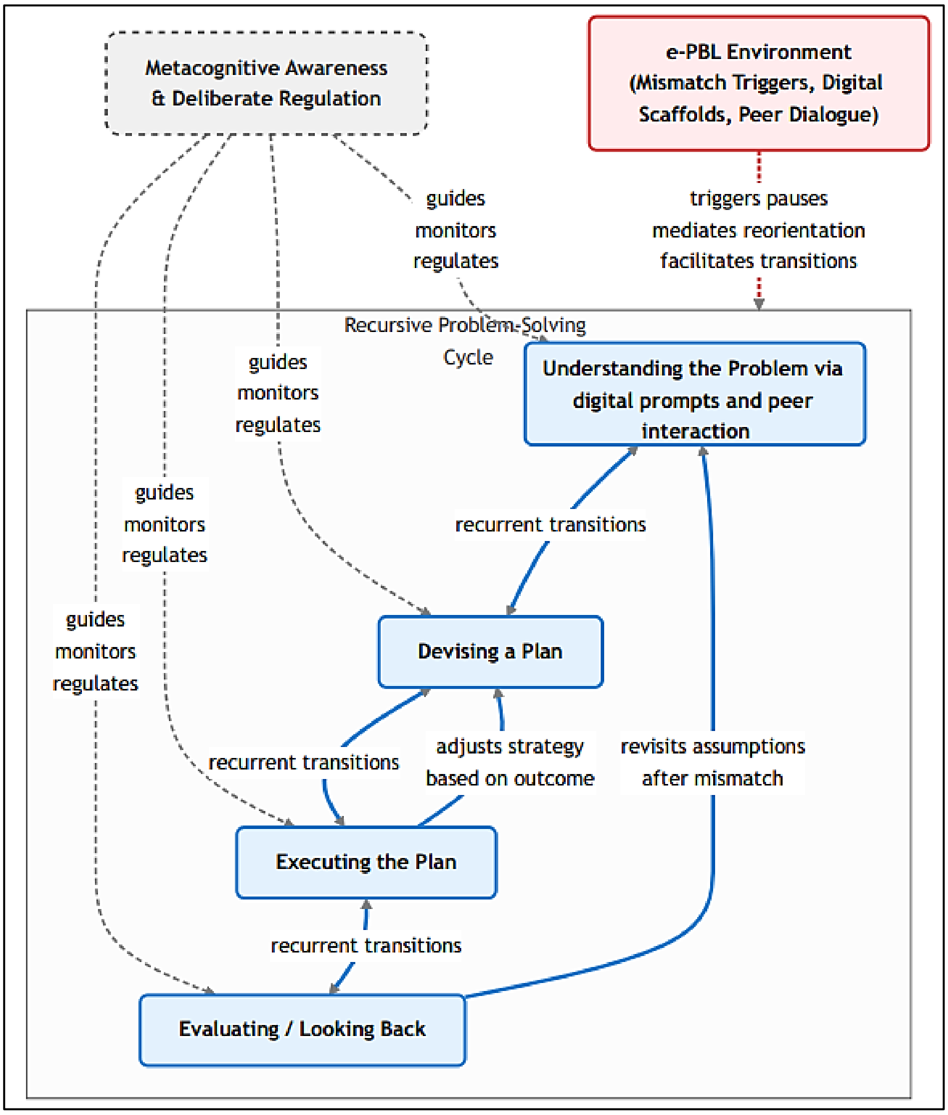

Figure 7. A data-grounded conceptual model of recursive, scaffold-mediated transitions across Polya's problem-solving stages in an e-PBL geometry environment.

As illustrated in Figure 7, the findings support a model of problem-solving in which Polya's stages operate as a recursive cycle rather than a linear sequence. Understanding, planning, execution, and evaluation are dynamically interconnected, with students repeatedly revisiting earlier stages when representational mismatch disrupts coherence between symbolic, visual, and conceptual forms. This circular configuration highlights how conceptual difficulty functions as a trigger for stage revisiting rather than as a signal of failure.

Transitions across stages were frequently scaffold-mediated rather than internally driven alone. Digital prompts, strategy hints, and collaborative exchanges facilitated epistemic reorientation, enabling students to suspend unproductive actions and reorganize their reasoning. Metacognitive regulation operated across the entire cycle, guiding and monitoring movement between stages instead of appearing as a separate phase.

Interview and log data indicate that these recursive shifts were guided by metacognitive awareness rather than by trial-and-error. As one participant explained, *“I repeated planning because I misunderstood the earlier step”* (S9-INT1). Stage revisiting often spanned multiple stages, with students returning to understanding after execution or reengaging planning during evaluation, reflecting a dynamically reorganized problem-solving process.

Collaborative features within the e-PBL environment further amplified recursive sense-making. Peer dialogue provided external reference points for evaluating reasoning, as noted by one student: *"The chat with friends made it easier to see if my thinking was correct"* (S6-LOG1). Such interaction frequently prompted renewed movement across stages when alternative perspectives exposed unnoticed inconsistencies.

Finally, these findings reconceptualize problem-solving in e-PBL geometry contexts as a recursive, scaffold-mediated process. Representational mismatch initiates stage revisiting, digital scaffolds support epistemic reorientation, and iterative refinement leads to conceptual stabilization, reframing Polya's framework as a dynamic system of interrelated reasoning activities rather than a fixed sequence of steps.

B. Discussion

The findings of this study challenge a linear reading of Polya's problem-solving model and position students' engagement in e-PBL as a recursive and meaning-making process rather than a sequential progression of stages. While classical interpretations of Polya's framework often imply an orderly progression from understanding to evaluation, recent literature has begun to question this assumption by documenting nonlinear reasoning in digital learning environments (Santos-Trigo, 2024; Wahab et al., 2024). Our findings strongly align with this non-linear perspective; however, they extend it by demonstrating that students' recursive movement across stages is systematically patterned rather than chaotic or random. Recursive returns to earlier stages were consistently triggered by representational mismatches or conceptual misalignment, suggesting that iteration functioned as a regulatory mechanism rather than a breakdown of problem-solving competence.

This patterned recursivity is particularly evident in the understanding phase, where students repeatedly revisit problem statements and diagrams when encountering inconsistencies between textual and visual information. Such behavior reflects what Türkoğlu & Yalçınalp (2024) describe as iterative sense-making in geometry problem-solving, in which understanding is progressively refined through reinterpretation. In the e-PBL context, multimodal prompts intensified this process by exposing tensions between representations that needed to be resolved before progress could continue. This observation aligns with prior findings that students often struggle to integrate textual and visual elements in complex mathematical tasks (Adarsh et al., 2025;

Haque, 2024), yet it also shows that such struggle can be epistemically productive when supported by appropriate digital affordances.

Specifically, features such as dynamic construction, revision functions, and immediate visual feedback lowered the epistemic cost of error, enabling students to experiment with alternative interpretations and to revisit earlier stages when constructed diagrams or numerical results conflicted with conceptual expectations. This flexibility positioned iteration as a core mechanism of meaning making rather than a corrective afterthought.

Students' planning behavior further illustrates how digital scaffolds mediated recursive transitions between stages. Consistent with digital PBL research, learners frequently revised their strategies after consulting hints or example-based supports, reflecting scaffold-triggered metacognitive shifts (D. Chang et al., 2021; H.-Y. Chang et al., 2022). However, beyond merely prompting metacognition, scaffolds in this study functioned as cognitive pivots that enabled students to exit unproductive loops, such as persistent misunderstandings, and to transition into a different stage of problem-solving. Planning thus emerged not as a forward-looking procedural step but as a site of epistemic reorientation in which learners reorganized conceptual relations in response to feedback and representational failure.

Importantly, students did not engage with digital scaffolds indiscriminately. The selective use of hints and supports indicates that scaffold use operated as a conditional mediation process, reflecting learners' agency in determining when external assistance meaningfully enhanced strategy formation. This selectivity echoes prior findings that scaffolding is most effective when learners are ready to interpret and integrate support, rather than when it functions as a directive mechanism that automatically produces conceptual change (Abrahamson et al., 2020; Bjork & Bjork, 2020). Thus, scaffold effectiveness emerges from its contingent uptake within learners' meaning making processes rather than from its procedural presence alone.

During execution, students engaged in frequent representational shifts among diagrams, symbolic manipulation, and verbal reasoning, reinforcing the view that geometry problem-solving requires continuous coordination of multiple representations (Engelbrecht & Borba, 2024). These shifts were particularly pronounced when numerical results conflicted with conceptual expectations, prompting students to redraw figures or reinterpret spatial relations as a form of transmediation. This behavior mirrors findings that dynamic visualization enhances conceptual integration and reduces procedural error in geometry learning (Borji et al., 2021; Milinković, 2025), while also supporting the interpretation of execution as a meaning-making process rather than mere procedural enactment.

Finally, these findings reconceptualize Polya's model in digital inquiry contexts by introducing the notion of digital stage-gating, in which progression across problem-solving stages depends on interactions with e-PBL affordances such as representational feedback, strategy hints, and peer dialogue. Within this view, stage transitions are scaffold-mediated events situated in a distributed cognitive system involving learners, digital tools, and the learning community, rather than solely internally timed cognitive shifts.

From a learning analytics perspective, diagnostically meaningful events such as repeated diagram revision, frequent returns to earlier stages, or clustered access to specific strategy hints can be identified through log data as indicators of representational breakdown or conceptual uncertainty. Detecting these patterns creates opportunities for adaptive system design in which targeted scaffolds are triggered at moments of epistemic need, aligning analytic feedback with the recursive problem-solving processes observed in this study.

Despite these contributions, the study is limited by its small, purposive sample and single e-PBL platform, which restricts generalizability. Future research should examine diverse technological designs and mathematical domains, and combine qualitative analysis with learning analytics to trace the frequency, sequencing, and effectiveness of recursive problem-solving pathways.

IV. Conclusion

This study demonstrates that in e-PBL environments, Polya's problem-solving model is best understood not as a linear sequence but as a recursive and scaffold mediated process. The core contribution of this study lies in theorizing the mechanisms that underpin this reconceptualization, namely digital stage gating, epistemic reorientation, and transmediation. Rather than progressing through fixed stages, students navigated clusters of problem-solving activity in which transitions were triggered by representational mismatch, mediated by digital scaffolds and peer dialogue, and regulated through metacognitive monitoring. By foregrounding these mechanisms, the study reframes Polya's model from an internal cognitive script into a distributed system of reasoning involving the learner, digital tools, and the learning community.

From a pedagogical perspective, the findings further suggest that assessment in geometry learning should attend to students' problem-solving processes rather than focusing exclusively on final answers. Recursive movement across Polya's stages, representational shifts, and moments of revision provide critical evidence of students' conceptual understanding and metacognitive regulation, particularly in e-PBL contexts where such process-oriented indicators are observable through interactions with digital representations and scaffolded supports.

From a practical perspective, the findings offer concrete guidance for the design of e-PBL environments in geometry learning. Rather than deploying scaffolds uniformly or based on task completion time, designers should engineer explicit breakpoints within e-PBL tasks that trigger scaffolds in response to diagnostically meaningful events, such as inconsistent alignment between diagrams and textual givens, implausible geometric constructions, or conflicting numerical and visual results.

For example, a diagram text inconsistency or an implausible

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construction can function as a breakpoint that activates prompts asking students to justify spatial relations or reconstruct diagrams using alternative representations. Similarly, repeated revision of a construction or frequent returns to earlier problem-solving stages may signal conceptual uncertainty and warrant reflective prompts that encourage students to re-examine underlying assumptions.

Such design moves operationalize digital stage gating by ensuring that progress through problem-solving stages is contingent on conceptual coherence rather than procedural advancement. Future research should extend this work by examining recursive problem-solving pathways across different mathematical domains and platform designs, and by integrating learning analytics with qualitative analysis of digital traces to investigate the frequency, sequencing, and effectiveness of scaffold mediated transitions.

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