Mathematical Representation Ability of Junior High School Students: A Case Study of Students’ Cognitive Ability through Realistic Mathematics Education

Intan Kemala Sari1*, Ahmad Nasriadi2, Yuli Amalia3, Musfiya4, Farida Agus Setiawati5

1,2,3,4Mathematics Education Department, Universitas Bina Bangsa Getsempena Jalan Tanggul Krueng Lamnyong no 34, Kota Banda Aceh, Aceh, Indonesia 1*intan@bbg.ac.id

5Mathematics Education Department, Universitas Negeri Yogyakarta Jalan Colombo Yogyakarta No 1, Sleman, Daerah Istimewa Yogyakarta, Indonesia

Article submitted: 04-01-2023, revised: 27-06-2023, published: 31-07-2023

Abstract
Mathematical representation ability is very important to develop. This study aimed to analyze students' mathematical representation abilities, as well as the forms of mathematical representations that appear. This study used case studies of four Grade VII junior high school students with moderate abilities, which were carried out in Recognizing Algebraic Shapes. The research was carried out in the odd semester of the 2022/2023 academic year in three meetings. The data collection techniques utilized questionnaires, interviews, and problem-solving analysis sheets. The results showed that class VII students were students at the early abstraction stage, so their mathematical representation abilities had not emerged and developed well. The form of representation that appears was symbolic representation with a low type where there was still a gap between concrete thinking and initial abstraction abilities. Some of the things that influenced the emergence of representational abilities were high motivation in solving problems, the learning methods presented, and students' abilities in learning mathematics.

Keywords: Algebraic Forms; Ability; Learning Methods; Motivation; Mathematical Representation.
I. INTRODUCTION

Mathematical representation is defined as one of the processes established in learning mathematics with the purposes to understand concepts, principles, facts, and metacognition (Murni, 2014; Hidayat, & Lestari, 2022). Representation is characterized as a link between everyday experiences and abstract mathematical concepts, which are typically difficult for students to grasp in a direct manner (Samsuddin & Retnawati, 2018). There are three types of representation (Schifter & Russell, 2022), namely the representation of numbers in the form of sentences, representations portrayed in visuals, and general representations that allow students to grasp based on their interpretations. In this case, mathematical representation interprets that students begin to have visual thoughts of real-world objects, concrete objects, or arithmetic symbols, and can also take the form of spoken or verbal language, as well as images or graphs to express mathematical concepts. The appearance of these objects becomes the embodiment of ideas or correlations in mathematical concepts based on experience and understanding, which are recognized, to be a meaning that is considered to exist (Goldin, 2014; Pebrianti & Puspitasari, 2023). The goal of learning mathematics (NCTM, 2000) emphasizes the importance of this principle because it can help students understand a concept by actively building information from their experiences. As a result, NCTM identifies representation as one of the five standards in the process of learning mathematics as part of connection, communication, and problem solving. The indicators of mathematical representation ability according to the National Council of Teacher Mathematics (NCTM, 2010) are as follows; (1) create and utilize representations to organize, record, and convey mathematical ideas, (2) select, apply, and translate across representations to solve problems, and (3) use representations to model and interpret physical, social, and mathematical phenomena.

It is, however, difficult to generate representations as part of students’ ideas to solve math problems. Students struggle with incorporating mathematical symbols or images to interpret problem ideas into mathematical concepts (Sari, Darhim, & Rosjanuardi, 2018). Students also find it difficult to generate visual ideas to portray mathematical problems because they are unfamiliar with their learning sources (Loc & Phuong, 2019). There are also several errors in representing symbols, namely; illogical representation of problem situations, problem representations that are not in accordance with mathematical sentences and equations, and representations that do not explain the concepts given in the problem into mathematical sentences (Nurrahmawati, Sa’dijah, Sudirman, & Muksar, 2021). The lack of students’ mathematical representation ability is due to the use of
conventional learning methods that do not involve students in the learning process resulting in a limited space for independence learning (Minarni, Napitupulu, & Husein, 2016). In addition, the complexity of the problem also causes students to find it difficult to envisage what types of visualizations and forms of representation might emerge as a result of the challenges and students' propensity of addressing non-routine problems (Johar & Lubis, 2018). Thus, unique preparation and approaches are required in the teaching and learning process to enable students to generate ideas and interpret problems based on their understanding of the problems they have encountered. Students also should also be given unusual and challenging problems to represent them in the best way they can.

Realistic mathematics learning (RME) is a learning approach that facilitates students in transitioning from casual to formal concepts. The principle of didactical phenomenology is the main concern in realistic mathematics (Larsen, 2018) where learning is directed to create a context to bring up mathematical ideas for students' informal activity models. Bridging between informal to formal concepts is not intended to replace the framework of the mathematics learning process itself but rather to complete and connect the mathematical notions for the future (Putra, 2018). In this case, it means that students can acquire new concepts that may apply in their daily lives through their learning experiences. The informal mathematization method involves translating contextual difficulties into mathematical problems (horizontal mathematization) and then formulating problems into problem solving (vertical mathematization) (Treffers, 1987; Gravemeijer, 1994; De Lange, 1987). (Treffers, 1987; Gravemeijer, 1994; De Lange, 1987). Realistic mathematics learning is recommended for improving critical and creative thinking skills as well as communicative and reasoning skills through the development of well-prepared instructional materials by teachers (Palinussa, Molle, & Gaspersz, 2021).

Realistic mathematics learning approaches based on collaboration and cooperation models in line with current learning developments are considered to have the most impact (Afriansyah, & Turmudi, 2022). For example, in discussion-based learning, students will become social agents of learning by teacher-designed learning plans to attain learning goals (Haataja, Chan, Salonen, & Clarke, 2022). On the other hand, the use of interactive and adaptive technology in the learning process can improve and develop concept understanding better than traditional learning (Reinhold, Hoch, Werner, Richter-Gebert, & Reiss, 2020). This study assessed how a realistic mathematics learning design could help students create representations of a mathematical problem using a learning
design that was valid and reliable against indicators of students’ mathematical representation skills, as well as a learning video that can construct students’ ability to create representation models.

According to Piaget’s stage of cognitive development, the cognitive growth of students aged 11-15 years is the stage of formal operational development (Mu’min, 2013). Based on this stage, students are just starting to understand concepts originating from semi-concrete objects as they progress towards the abstract thinking stage (Juwantara, 2019). Thus, at this stage students should still be given concrete object stimuli or what students are able to envisage during this phase of their lives to be an introduction to learning mathematics. For example, students will be exposed to abstract symbols for the first time when learning the introduction of algebraic forms, with abilities requiring students to explain algebraic forms and perform operations on algebraic forms (Nasriadi & Sari, 2017). Many students eventually struggle with algebra (Cahyani & Sutriyono, 2018) and even become reluctant to learn math henceforth as a result of their initial impression that is too tough for them (Sari, et al, 2020). Vertical and horizontal mathematization are required to recognize formal concepts through informal models that students recognize. RME is a learning approach that embraces the process of vertical and horizontal mathematization. The RME approach not only instills long-term learning concepts that students will remember but also improves mathematical abilities such as problem solving skills, communication skills, connection skills, reasoning skills and mathematical representation skills.

Therefore, this study aimed to investigate how students' mathematical representation skills were and what types of representation appeared. This study was conducted in face-to-face learning utilizing the RME technique to develop the observation element of representation in learning. The results of this study were expected to reveal the types of representation that appeared allowing learning designers to create an equivalent instructional model in order to meet mathematics learning goals.

II. Method

This qualitative study investigated how the RME technique affected the development of students' mathematical representation skills in learning algebraic forms. Based on the foregoing, this study tried to discover the types of mathematical representation ability that students demonstrated during the teaching and learning process. According to the age of the students, the development of representation ability was noticed from the stage of modeling with concrete operational type to formal abstraction. This study focused on four seventh grade junior high school students in Aceh who were learning the curriculum
of Introduction to Algebraic Forms. The study involved three meetings to conduct during the odd semester of the 2022/2023 academic year.

Data collection techniques in this study were carried out through a questionnaire of students' perceptions of representation skills before learning the material of Introduction to Algebraic Forms, a description question test to find out the form of mathematical representation that emerged, and unstructured interviews with the four participants. Based on the indicators of mathematical representation, a learning design and learning tools consisting of Lesson Plans (RPP), Students’ Worksheets (LKPD), and test questions were made and validated by the experts. The purpose was to allow the process of emergence of the mathematical representation model to appear in the natural design of learning. The data analysis technique, on the other hand, referred to the objectives of the study and was analyzed using a qualitative approach.

Qualitative analysis of this study was carried out through questionnaire results using a Likert scale. This scale was used to measure individual traits using a total score based on categories (Budiaji, 2013). The following are the scores and representation statements used.

Table 1. Likert Scale Mathematical Ability Questionnaire Score

<table>
<thead>
<tr>
<th>Score</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Very Good</td>
</tr>
<tr>
<td>4</td>
<td>Good</td>
</tr>
</tbody>
</table>

The questionnaire results were used to find out whether students had habits and interests in solving problems, as well as plans that were usually made before solving math problems. Furthermore, the results were analyzed to investigate the students' abilities to solve problems, make mathematical models and make mathematical representations. These three instruments were analyzed using qualitative descriptive techniques.

III. RESULTS AND DISCUSSION

This study involved four seventh graders of junior high school in Aceh. Four students were selected based on the results of a perception questionnaire which showed that the students tended to illustrate problem solving. Data collection was conducted for three face-to-face teaching and learning processes in the classroom for each stage of development; concrete, semi-abstract, and abstract. There were three stages of data collection, namely giving perception questionnaires before the learning series in three face-to-face sessions, giving questions at the end of the learning series, and open interviews after the intervention stage of the teaching and learning process was completed. The following describes each data collection result for each instrument.
A. Questionnaire and Structured Interview Results

The student perception questionnaire was provided to find out the students' tendencies in learning mathematics skills, especially in mathematical representation skills based on three indicators. The questionnaire was developed in 20 statement items to find out how students understand, implement, and review mathematical problems in mathematical representation. Furthermore, unstructured interviews were conducted to triangulate the data. Based on the results of questionnaires and interviews with four students as the participants, the students' abilities were presented based on indicators of mathematical representation abilities. The four participants have the following questionnaire results:

<table>
<thead>
<tr>
<th>Indicators</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>Average</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.75</td>
<td>4.25</td>
<td>3.75</td>
<td>3.59</td>
<td>4.06</td>
<td>Very good</td>
</tr>
<tr>
<td>2</td>
<td>4.75</td>
<td>4.50</td>
<td>4.25</td>
<td>4.25</td>
<td>4.43</td>
<td>Very good</td>
</tr>
<tr>
<td>3</td>
<td>3.75</td>
<td>4.25</td>
<td>3.75</td>
<td>4.50</td>
<td>4.06</td>
<td>Very good</td>
</tr>
</tbody>
</table>

Description:
Indicator 1: Create and use representations to organize, understand and communicate mathematical ideas.
Indicator 2: Select, apply and interpret across the representations to solve problems.
Indicator 3: Use the representations to model and interpret physical, social and mathematical phenomena.

Based on Table 2, it was revealed that students' representation skills were very good in understanding, applying, and assessing the use of skills in solving mathematical problems. This showed that seventh grade junior high school students already had an overview of the ability to create and represent problems in the form of mathematical models based on the questionnaire results. This depicted the early picture that students began to have the ability of initial abstraction from semi-concrete models through their mathematical representation models.

B. Cognitive Test Results

The development of students' cognitive stages was discovered during the teaching and learning process where the teacher provided learning activities and proposed problems that must be solved through the Students' Worksheet (LKPD). The results of the LKPD were documented and analyzed to find out the strategies used by students in solving problems and the learning trajectory formed during the teaching and learning process. The following is an analysis of the results of student answers on LKPD for each stage of development.

Stage 1. Concrete Operational
At this stage, students were given a very simple illustration of two different groups of objects. The illustration was in
an animal context. Students were instructed to answer questions about simple operations of the same and different objects. Students added the same group of objects and different groups of objects.

Figure 1. Students’ answers from concrete examples of two objects in four concrete models.

From the students’ answers, all four students wrote that group 1 and group 2 were the same object, so the number of group members could be directly mentioned. Whereas two groups with different objects, the number of group members were mentioned respectively. Furthermore, the students were asked about the reason for this phenomenon, and the answer was that the students could not mention both types of objects in one unified universe of speech so they had to be mentioned one by one for each type of group.

The students’ answers above demonstrated explicit reasoning that when the addition operation of two different objects was performed, the result would be each of the objects themselves, whereas when the addition operation of the same objects was performed, the result would be the sum of the two same objects. Based on these results, it was implied that students made a representation of a stimulus in a simple language according to what they understand. For instructional purposes, students had been able to explain which groups had explicit results in addition and which did not. This stage was a basis for students to form representation skills at the semi-abstract operational stage.

Stage 2. Semi-Abstract Operational

In the next stage, the students were instructed to create a simple model through the same context in the previous problem. They were given two conditions where students could fill in the numbers that became the sum of the objects with the sum result. The first condition consisted of the same objects while the second condition consisted of objects from a concrete model as shown below.

Figure 2. Students are asked to create a mathematical model of a concrete condition.
The students began to place the desired number arrangement for two different problems. In the first condition where there were the same objects to be modeled, the students gave a multiplication sign with the meaning "one multiplied duck plus one multiplied duck equals to two multiplied duck" by placing the multiplication sign as a mark that there were several repetitions of the number of objects in question for further counting the number. The students did not place the multiplication sign as a substitute for the variable x but rather interpret that there was one duck plus one duck. This model began to be difficult for students to understand because they had to deal with writing multiplication in a sum operation simultaneously.

The absence of a variable element coded in letter form as the goal of this activity was not achieved. This was also discovered in the second condition in the form of addition of two different objects. The students only wrote "one duck plus one goat equals one duck plus one goat" without any symbols that simplified the form. From the result of the addition, the students realized that they did not write the total number as they did in the previous condition because the objects being added were different, so it was not correct to only mention one of the forms.

This stage implied that the students did not successfully demonstrate the transitioning process from concrete to semi-abstract operational abilities. They were not yet able to imagine what the replacement model of an object that could define the intention was. This meant that students required additional bridges to introduce a "code" that could replace the definition of the number of objects. As it was assumed that the incompleteness at the concrete operational stage would certainly have an impact on the concrete stage and the problem solving ability of students, especially in algebraic form problems.

Stage 3. Abstract Operational

The abstract operational stage was the stage where the students were exposed to the algebraic symbolic forms. The students were no longer given examples of concrete objects or images to make an algebraic math sentence. At this stage the students were instructed to write the form of algebraic operations both those with different variables and the same variables.

![Figure 3. Students were instructed to create an Abstract Form of Algebra.](image-url)
From the students’ answers, the students were able to write the algebraic form model. However, the purpose of this activity was to operate the algebraic form in the form of an answer. Although they did not provide the final result of an algebraic operation, in conclusion students had understood that the coefficients of the equations with the same variables could be summed. Students did not write the final answer because they followed the table instructions. This implied that students understood the previous problem in concrete form, but were not able to solve math problems in abstract form.

IV. CONCLUSION

Mathematical representation ability should be able to appear visually and verbally when solving problems. Based on the results of the study, it implied that the representation ability might appear visually in the form of numbers and descriptions assisted by illustrations that students recognize well. However, this study also demonstrated that students who understood the problem and had concrete operational abilities were not able to solve problems and apply them in abstract operations. The mathematical problems given in this lesson were considered concretely uncomplicated, but could not be able to guide students to a more abstract stage. Challenging problems would encourage stronger motivation so that the students worked harder to find various ways to represent. The results of questionnaires and unstructured interviews showed that the students understood and had very good motivation in solving math problems, but the unchallenging problems did not encourage them in solving problems with mathematical notation and symbols. Real problem-based learning might help students understand some situations and develop students’ mathematical representation models because it dealt with real problems and concrete objects to students. However, real problems were not enough to help bridge the gap in solving problems that used variables. In this study, the participants were seventh grade junior high school students who had just entered the early abstraction stage where there was still a gap with concrete operations. Thus, the students’ difficulties experienced were due to the unfamiliarity of building representation models in solving problems. The learning that had been conducted still dealt with students on procedural concepts that focused on the formula for solving problems. Rarely, were students given open question problems that allow the emergence of several alternative answers to allow the emergence of several possible forms of mathematical representation. Students’ ability might affect the development of mathematical representation skills. Representation in symbolic form appeared more often than representation in the form of images and verbal. In this study, the participants were students with
moderate ability. Among 24 students, 4 students with moderate ability were selected to find out whether mathematical representation skills could emerge and develop well enough for students with moderate ability. Based on the results of this study, it was discovered that the students with moderate mathematical representation ability through symbolic representation had been considered sufficient and well-developed according to their own level. To put it another way, the students with higher ability would have acquired better mathematical representation ability.

REFERENCES


