Analysis of Students’ Errors in Solving Differential Problems Based on Castolan Theory

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Abstract
Differential material is widely used as a prerequisite for other materials and other courses. But not a few students who find it difficult to learn it. Therefore, the purpose of this qualitative descriptive research is to describe students’ mistakes in solving differential problems based on the Castor theory. Castor’s theory includes conceptual errors, procedural errors, and technical errors. The research was conducted on students participating in basic mathematics courses. Data was collected by test and interview methods. The supporting instruments used were three differential problem tests and an interview guide. The student answers were then analyzed to see which represented the most errors. Test the validity of the data using technical triangulation. Data were analyzed with the stages of data reduction, data presentation, and drawing conclusions. The results of the study found that most students made procedural and technical errors, and a few made conceptual errors. Conceptual errors occur when students incorrectly determine the formula. Strategy errors occur when students use the wrong formula, are unable to complete a simple form, and there is missing data. While the technical errors are mostly due to wrong writing or counting. The cause of this errors is because they do not understand the concept, are not used to solving problems that require high-level thinking skills, and are not thorough.

Keywords: Error analysis; castor theory; derivative functions.
I. INTRODUCTION

Basic mathematics is one of the compulsory courses in the Faculty of Teacher Training and Education (FKIP), especially the Mathematics and Natural Sciences education department. One of the materials discussed in the course is differential. This topic is widely used to solve other problems, especially in derivative applications, for example, determining the gradient of the tangent line of a curve, increasing or decreasing function, speed and acceleration of a trajectory, stationary value of a function and others (Setyaningsih & Kustiana, 2023).

In addition, the differential topic is also provided in other courses such as differential equations, integral calculus, mathematical statistics, and others. However, based on the results of differential problem tests given to the participants in basic mathematics courses, it was found that 68.75% of students obtained results below 65 with an average of 61. Student errors were quite diverse, not just the result of unfamiliarity and situational (Rusyda et al., 2022). As many as 47% of students made concept errors, 32% principle errors, and 21% procedural errors (Hajerina et al., 2022). Other errors made by students in solving differential problems included determining critical points, extreme values, and turning points. They were struggling to understand and apply concepts in solving problems (Sumargiyani & Nafi, 2020). Some of the factors that caused students to make mistakes include lack of accuracy, lack of practice, lack of understanding of the problem and inadequate experience of their previous knowledge (Sukmaningthias, Hasyanah, Sari, & Nuraeni, 2023).

Errors are an indication of difficulties experienced by students (Pujilestari, 2018; Sadijah & Afriansyah, 2023). Students' errors are important to analyze in order to create maximum learning and realize the achievement of learning objectives (Rusyda et al., 2022). Through error analysis, the types and locations of errors made are obtained (Umam, 2014). Furthermore, Rusdianto said that analyzing errors was also useful to develop new insights for teachers/lecturers in overcoming difficulties experienced by students (Raufany & Solfitri, 2019; Mutiarahman, Edriati, & Suryani, 2023). By understanding the topic and types of errors, some countermeasures could be planned, for example by developing better learning strategies.

The errors made by students could be grouped into several types of errors. Castolan distinguished errors into three types, namely conceptual errors, strategy errors, and technique errors (D et al., 2021; Khanifah & Nusantara, 2013; Lenterawati et al., 2018; Noviani, 2019; Sa'aduddien Khair et al., 2018). Meanwhile, according to Waston, there were 8 types of errors, namely: incorrect data, improper procedures, missing data, missing conclusions, response level conflicts, indirect manipulation, skill hierarchy, and other than these 7 categories (Evriyanti et al., 2020). Meanwhile, according to Soejadi, student errors based on mathematical objects were divided into four, namely errors of facts, concepts, principles, and operations (Gustianingum & Kartini, 2021).
This study focused on analyzing student errors based on Castolan's theory that grouped the errors into three types, namely conceptual errors, strategies, and technical errors. Each type of error has its own indicators. In summary, the error indicators according to Castolan are presented in Table 1 below.

Table 1. Types and Indicators of Errors According to Castolan Theory

<table>
<thead>
<tr>
<th>Types of error</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual error</td>
<td>1. Unable to choose appropriate formulas</td>
</tr>
<tr>
<td></td>
<td>2. Unable to apply the formulas correctly</td>
</tr>
<tr>
<td>Strategy/Procedural error</td>
<td>1. Unable to follow the steps correctly</td>
</tr>
<tr>
<td></td>
<td>2. Unable to present the simplest form of the answer</td>
</tr>
<tr>
<td>Technical error</td>
<td>1. Lack of accuracy,</td>
</tr>
<tr>
<td></td>
<td>2. Writing errors</td>
</tr>
</tbody>
</table>

Source: (Aini & Irawati, 2022)

Several studies have been conducted to analyze students' errors in solving differential problems.

The results of the study (Aini & Irawati, 2022) found that the types of students' errors in solving order $n$ inhomogeneous differential equations based on the Castolan stages were conceptual errors in using inappropriate formulas. While procedural errors of the students demonstrated that they were unable to present the answer of the simplest form. Technical errors related to the value of an arithmetic operation, writing errors or errors in moving constants or variables.

The results of a study (Hajerina et al., 2022) on analyzing students' errors in differential calculus courses on derivative topics found concept errors, including not understanding the concept of the quotient rule. Principle errors included not knowing how to solve the problem. While one of the procedural errors was performing operations incorrectly on sum and subtraction of integers.

Another study conducted by (Sumargiyani & Nafi, 2020) showed that in solving differential calculus problems students had difficulty in understanding concepts, applying concepts to calculations, and starting the initial steps of the calculation. As a result of these difficulties, various errors occurred including errors in determining critical points, extreme values, and so on.

Based on these issues, the purpose of this study was to describe students' errors in solving differential problems, especially derivative application material based on Castolan theory.

II. METHOD

This descriptive study used a qualitative approach involving students of basic math courses. The determination of the participants was based on purposive sampling technique with certain criteria (Sugiono, 2016). The participants were selected based on the number of incorrect answers to the questions. The data collection was conducted using test and interview techniques. The test aimed to find the mistakes the students made. While interviews were conducted in addition to testing the validity of the data as well as to find the causes of these errors based on Castolan's theory.

The data obtained were analyzed using the Miles & Huberman model in (Laja, 2022) consisting of three stages, namely (1)
data reduction, (2) data presentation, and (3) conclusion drawing. The data validity test was carried out by triangulation techniques, namely comparing the answers to the test results and interview results. The questions used as test instruments in this study are presented in Figure 1 as follows.

Table 1. The students' errors

<table>
<thead>
<tr>
<th>Item</th>
<th>Types of error</th>
<th>Kolom 2</th>
<th>Kolom 3</th>
<th>Correct answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Concept, strategy, and technique</td>
<td>68,75%</td>
<td>25%</td>
<td>0,00%</td>
</tr>
<tr>
<td>2</td>
<td>Strategy</td>
<td>12,50%</td>
<td>68,75%</td>
<td>6,25%</td>
</tr>
<tr>
<td>3</td>
<td>Concept, strategy, and technique</td>
<td>44%</td>
<td>25%</td>
<td>6,00%</td>
</tr>
</tbody>
</table>

Figure 1. The test instruments

III. RESULT AND DISCUSSION

This research started from providing the differential problem tests to all students participating in basic mathematics courses. The test results were then analyzed to see the types of errors made by students. The results of the analysis found several errors made and grouped based on Castolan theory. The errors based on Castolan theory in solving differential problems are presented in Table 1.

Table 2. Types of the students' errors

<table>
<thead>
<tr>
<th>Item</th>
<th>Type of the students' errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P-1</td>
</tr>
<tr>
<td>2</td>
<td>Strategy</td>
</tr>
<tr>
<td>3</td>
<td>Concept, strategy, and technique</td>
</tr>
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</table>

The following describes the types of errors made by the students and the factors that caused student errors in solving differential problems.

Errors on item 1

Item 1 dealt with determining the value of coefficients for a function to be differentiable at a point. The data in Table 1 showed that the most common errors were strategy errors on item 1.

Furthermore, three student answers with the most errors were selected and represented three types of errors, namely concept errors, strategy errors, and calculation errors. Overall, the types of errors made by students based on Castolan's theory are presented in Table 2.

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Errors on item 1

Item 1 dealt with determining the value of coefficients for a function to be differentiable at a point.

The answer of the first participant (P-1) is presented in Figure 2.
Based on the analysis result on item 1, P-1 could not choose the correct formula. The requirement for \( f(x) \) to be differentiable at point \( x = 1 \) was that \( f(x) \) was continuous at \( x = 1 \) along with left and right differentiable. However, P-1 only investigated the left and right differentiability without investigating the continuity at point \( x=1 \). According to Castolan's theory, this error was a concept error.

While investigating the left differentiable, P-1 also chose an inappropriate formula. This was in line with the research results (Aini & Irawati, 2022) that in solving inhomogeneous differential equations of order \( n \), students answered the question incorrectly because they used an inappropriate formula. As for the right differentiable, besides choosing the wrong formula, they also wrote the wrong point. It should be \( x1+ \) but written \( x2+ \). As a result, the calculation result was not correct. The reason was they did not remember the conditions of P-1 differentiability, they also often confused the formula for finding the limit value and derivative. This was supported by (Rusyda et al., 2022; Sumargiyani & Nafi, 2020) that errors occurred when applying rules without understanding the basic concepts involved.

Unlike P-1, the errors made by P-2 in solving problem number 1 included strategy errors. The formula was appropriate, but it was incorrectly applied, as presented in Figure 3.
participant used an incorrect formula. P-3’s answer was presented in Figure 4.

Figure 4. P-3’s answer to item 1

Despite providing an appropriate formula, P-3 applied it incorrectly, demonstrating procedural errors. It was supposed to be 

$$a(1+h)^2 + b(1+h) - 1 = \frac{\sqrt{1+h} - 1}{h}$$

and there was an error in the calculation as well. The participant did not write the procedure resulting in an incorrect final answer (Najwa & Sari, 2021). The main reason for this error was that the participant was doing the question item in a haste.

Errors on item 2

Item 2 related to the application of the derivative. The students were asked to use the concept of derivative to determine the velocity of an object when its acceleration was zero. Based on the analysis of the students' answers, the errors found in item 2 included strategy and technique errors. Students are not able to provide final answers or conclusions and are less thorough. The following is a discussion of errors for question number 2.

P-1’s answer to question number 2 is presented in Figure 5.

Figure 5. P-1’s answer to item 2

P-1 chose the correct formula, and followed the correct steps, and the calculation results were also correct. However, P-1 was not able to conclude the velocity of the object when the acceleration was 0. It implied that P-1 did not finish the operation completely. According to Castolan’s theory, this was a procedural error. The student was unable to provide the final answer (Meiliasari et al., 2021) or the conclusion was missing (Waston in (Evriyanti et al., 2020). Meanwhile, a study found (Noviyanti, 2021) that the mistakes made by students while working on calculus III problems were 15.38% relating to the incorrect writing of the final answer.

Meanwhile, P-2’s answer presented in Figure 6 based on Castolan’s theory showed no errors. P-2 chose and used the correct formula, and this answer was the most complete one. P-2 concluded the velocity of the object when the acceleration was 0. He concluded that the velocity was 11 units of speed. He did not choose -16 because it was impossible for the velocity to be negative.
Figure 6. P-2's answer to item 2

Another answer to item 2 was the answer of P-3 presented in Figure 7.

There were errors in the determination of \( t \) value and the calculation of the speed \( v(t) \), namely the writing process \( 0 = t^2 - 5t + 4; \quad 0 = (t - 1)(t - 4) \), it was supposed to be \( t = 1 \) or \( t = 4 \). However, it was written \( t = 1; \quad t = 44 \). P-3 did not carefully perform the operation (technical error). Meanwhile in the calculation of \( v(t) \) value, the participant incorrectly wrote the formula. It was supposed to be\( v(t) = 2t^3 - 15t^2 + 24t \), but written \( v(t) = 2t^3 - 15t + 24t \) considered as an incorrect answer.

Figure 7 showed some errors while determining the value of \( t \) and calculating the speed \( v(t) \) at the stage of \( 0 = t^2 - 5t + 4; \quad 0 = (t - 1)(t - 4) \), it was supposed to be \( t = 1 \) or \( t = 4 \). However, it was written \( t = 1; \quad t = 44 \). did not carefully perform the operation (technical error). Meanwhile in the calculation of, there was an error in determining the value of \( v(t) \) and incorrect writing process. It was supposed to be \( v(t) = 2t^3 - 15t^2 + 24t \), but written \( v(t) = 2t^3 - 15t + 24t \) considered incorrect.

In Castolan's theory, this was included into a technical error. The cause of the error was that students were less careful and wrote the formula incorrectly. In addition, P-3 also did not conclude how much the velocity of the object was when the acceleration was 0, so based on Castolan theory, it was a procedural error. The students were unable to provide the final answer (Meiliasari et al., 2021), including missing conclusions (Waston in (Evriyanti et al., 2020). These answers were made by 75% of the students. In contrast to the results of study (Hajerina et al., 2022) on function derivative material, the number of students who made procedural/strategy errors was 21%. While question number 2 was derivative application material.

**Errors on item 3**

Problem number 3 dealt with the derivative of implicit functions. Unlike problems in item 1 and 2, item 3 is relatively new to the participants so there were students who did not answer. Based on the results of the study using Castolan's
theory, three types of errors were found, namely conceptual errors, procedural and calculation errors. The students' answers that showed these errors are illustrated in Figure 8, Figure 9, and Figure 10.

Figure 8 show the answer of P-1's to item 3.

The answer in Figure 8 showed that P-1 works on the derivative of implicit functions similar to explicit functions, by writing $f' = 3x^2 + 2xy + x2y - 6y = 10$. This showed that the student did not understand the concept of implicit function derivatives. Thus, it was included in conceptual errors. The students who were only able to write down formulas to solve problems but could not apply these formulas was categorized as transformation errors (Tangi, Janssen, Benedetti, & Noci, 2021; Alfisyahra et al., 2022). In addition, P-1 could not solve the problem to the simplest form, resulting in a strategy error. To solve the issue, it requires higher-level thinking skills (Kurniawati & Hadi, 2021). In addition to conceptual errors, there were strategy errors in the process. P-1 using the implicit function derivative formula was not comprehensive in both form and content, so there was missing data (Waston in (Evriyanti et al., 2020). In line with the results of Nurlaili's study on error analysis in solving the first linear differential equation, it revealed that there was missing data of 2.85% (Nurlaili & Rifanti, 2020). $3x^2$ was the derivative of $x^3$ derived towards $x$, so it was supposed to be $3x^2 \cdot dx$. $2xy$ was the derivative of $x^2y$ towards $x$, so it was supposed to $2xy \cdot dx$. In this test item, P-1 did not determine the derivatives of $x^2y$ towards $y$, the derivatives of 10, and the derivative of $2y^3$, so it was categorized as technical errors. The main cause of this error was that P-1 were unfamiliar with the derivative of implicit functions. The participant was struggling to complete the operation after transforming the function into an explicit one.

P-2’s answer is presented in Figure 9.

P-2's answer demonstrated that he did not understand the concept (conceptual error), P-2 derived each term but did not pay attention to the terms. This was a strategy error. The first step to solve the derivative of the function should be $3x^2 \cdot dx + 2x \cdot dx \cdot y + x^2 \cdot dy + dx \cdot y^2 + 2xy - 6y^2 = 0$. According to P-2's answer, it was found that both the writing and the result had not been written correctly (technical error). P-2 did not determine the derivative of $x^2y$ towards $y$ nor write the derivative of 10. Like P-1 did, the cause of this error was that the students had not
understood the concept of the derivative of implicit functions.

Participant P-3 solved the derivative of this implicit function better. The errors that occurred included strategy errors as presented in Figure 10.

![Figure 10. P-3’s answer to item 3](image)

The strategy error could be observed from several answers containing incorrect derivatives, namely the derivative of $x^3$ that was supposed to be $3x^2 \cdot dx$ derivatives $x^2y$ towards $y$ that was supposed to be $x^2 \cdot dy$. However, P-3 stated $x^2y \cdot dy$ along with the derivatives of $2y^3$ towards $y$ that was supposed to be $6y^2 \cdot dy$ as $6y \cdot dy$. These errors demonstrated lack of accuracy that was categorized as technical errors. Accordingly, they affected the final results.

**IV. CONCLUSION**

After analyzing the results using Castolan theory, the study concluded that the errors made by students on differential application material included conceptual, procedural, and technical errors. Conceptual errors were present in determining the conditions of a differentiable function at a point, and the concept of implicit function derivatives. Procedural errors were demonstrated when the students could not solve differential problems in the simplest form, either the problem of differentiable function requirements or solving the implicit function derivative, and concluding the final result of the answer. Technique errors were mostly writing errors due to lack of accuracy. These errors occurred because the students did not understand the concept and their thinking skills were not in the higher hierarchy, as well as lack of accuracy and lack of practice. Based on the results and conclusions, it was expected that further study might be able to study different problems and materials, so that students' understanding of differential topics was enhanced to minimize errors.

**REFERENCES**


AUTHOR’S BIOGRAPHY

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