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Rethinking How We Teach Derivatives Representation: The Framework of Hypothetical Learning Trajectory

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ABSTRAK

Konsep turunan merupakan prinsip dasar dalam Kalkulus untuk pengajaran matematika di tingkat sekolah dan universitas. Namun demikian, definisi yang mencakup beberapa representasi terkadang membuat konsep turunan sangat sulit dipahami. Tujuan penelitian ini adalah untuk mengkaji representasi konsep turunan yang dipahami oleh partisipan. Penelitian ini menggunakan metodologi kualitatif untuk menyelidiki fenomena tertentu. Data penelitian dikumpulkan melalui wawancara klinis yang dilakukan terhadap mahasiswa (N=5) dari salah satu universitas di Provinsi Jawa Barat. Teknik analisis data menggunakan triangulasi meliputi reduksi data, analisis data, dan penarikan kesimpulan. Temuan penelitian ini menunjukkan bahwa representasi konsep turunan bagi banyak partisipan masih terbatas pada konteks simbolis untuk memecahkan masalah prosedural. Representasi yang terbatas dapat menimbulkan hambatan epistemologis dalam menyelesaikan masalah konseptual. Temuan ini menjadi dasar untuk mengembangkan *Hypothetical Learning Trajectory* (HLT) yang mencakup tujuan, prakiraan proses pembelajaran, dan aktivitas untuk mendorong terciptanya pemahaman melalui representasi konsep yang beragam.

Kata Kunci: *Hypothetical Learning Trajectory*; Konsep Turunan; Representasi

ABSTRACT

The concept of a derivative is a fundamental principle in calculus that is essential for teaching mathematics at both the school and university levels. Nevertheless, definitions encompassing several representations sometimes render the concept of derivatives exceedingly challenging to comprehend. The objective of this research is to examine the representation of the derived notion as seen by the participants. The study employs qualitative methodologies through a case study framework to investigate certain phenomena. Research data collected via clinical interviews conducted with students (N=5) from a university in West Java Province. The data analysis technique using triangulation includes several stages, namely data reduction, data analysis, and drawing conclusions. The findings of this study suggest that the representation of derived concepts for many of participants remains confined to a symbolic context for solving procedural difficulties. The restricted representation can create epistemological obstacles in resolving conceptual issues. These findings serve as the foundation for developing a Hypothetical Learning Trajectory (HLT) design that encompasses objectives, forecasts of the learning process, and activities to promote the creation of understanding through diverse representations of concepts.

Keywords: Hypothetical Learning Trajectory; Derivative Concept; Representation

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1. INTRODUCTION

In recent years, instructional research in calculus has advanced significantly. Research in this domain investigates both students' comprehension and cognitive processes, as well as the efficacy of pedagogical strategies developed by educators to enhance students' understanding construction (Cho & Kwon, 2023; Mart í nez-Planell & Trigueros, 2021; Garc í a-Garc í a & Dolores-Flores; 2021; Chen & Wu, 2020; Kidron, 2020). The derivative is regarded as a challenging concept for students in calculus, as its definition necessitates comprehension of related concepts such as function and limit (Rodr í guez-Nieto & Rodr í guez-V á squez, 2022; Carli, et al., 2020). The concept of a derivative is crucial to comprehend, as its examination encompasses variations in amounts and alterations that are fundamental issues inside calculus (Mkhatshwa, 2024; Nıeto, et al., 2021; Moru, 2020).

The challenge of comprehending the notion of derivatives remains one of the most significant obstacles in mathematics education at both the school and university levels (Thompson & Harel, 2021; Garc í a-Garc í a & Dolores-Flores, 2021; Greefrath et al., 2023). Numerous prior research findings have corroborated that the knowledge imparted by educators and the knowledge acquired by students concerning the concept of derivatives face challenges, particularly in relation to the comprehension and significance of the derivative concept (Shirawia, et al., 2024; Feudel & Biehler, 2021). The findings of this study align with the research by Astuti, et al., (2025), which indicated that students may readily derive a function but encounter difficulties in elucidating the link and interpreting the outcomes of the derivation in different situations.

The research conducted in Indonesia regarding the comprehension of derivatives at both school and university levels indicates that students are proficient in resolving routine problems but lack the ability to articulate conceptual significance derived from the interconnections among concepts such as gradients, functions, limits, and continuity (Prihandhika, 2024; Destiniar, et al., 2021; Lefrida, et al., 2021). This state may create epistemic barriers in the learning process, preventing students from identifying solutions to problems that differ from the previously provided context (Nurwahyu & Tinungki, 2020). Zandieh (2000) proposed a paradigm to investigate students' comprehension of derivatives through numerous representations to get a comprehensive understanding applicable to problem-solving. Zandieh (2000) posits that the framework of derivatives with multiple representations serves as a metric for practitioners and researchers to assess various conditions, including: 1) each individual's comprehension of concepts endorsed by the mathematics community; 2) comparative understanding between individuals; 3) efficacy of teaching strategies predicated on the introduction of diverse aspects of a concept; 4) effectiveness of pedagogical practices in relation to the employed curriculum; 5) and evaluation of fundamental concepts that must be delivered through a meticulously planned set of teaching materials within the curriculum. According to Zandieh, Jahangiri et al., (2022) elucidated that the term representation encompasses both the process and the product, which are essential components for facilitating individual comprehension of mathematical concepts.

Zandieh (2000) presents a taxonomy of representations grounded in three fundamental principles essential for the production of derived ideas: ratio, limit, and function. The three objects are examined in various contexts: the graphical context concerning the slope gradient of the function y=$f(x)$ at the point (a, $f(a)$), the verbal context related to average change (rate of change), the paradigmatic physical context associated with velocity, and the symbolic context involving the difference quotient equation denoted by the notation. Zandieh's assertion is corroborated by other research on mental construction, which demonstrate that participants exhibit varying representations in their understanding of the derivatives (Moru, 2020).

The multirepresentation framework on the concept of derivatives, as articulated by Zandieh (2000), is delineated in Table 1.

Table 1. The Framework of Derivative

Process-object layer	Representation			
	Graphical	Verbal	Paradigmatic Physical	Symbolic
	Slope	Rate of Change	Velocity	Difference Quotient
Ratio				
Limit				
Function				

Consequently, in the instructional process, it is essential to focus on hypothetical learning trajectories to foresee epistemological challenges that may arise when students encounter non-routine problems or conceptual issues necessitating proficiency in diverse representations of derivative concepts (Brousseau, 2002). The Hypothetical Learning Trajectory (HLT) predicts learning scenarios that elucidate the interdependent relationship among three primary components: the cognitive process, objectives, and learning activities, aimed at constructing student comprehension through a series of tasks and instructional mapping (Ivars, et al., 2020). Simon & Tzur (2004) delineate HLT through four principles: 1) HLT is grounded in students' genuine comprehension of a concept; 2) HLT serves as a framework for planning instruction on mathematical concepts; 3) mathematics tasks include directives for developing an understanding of mathematical concepts; 4) adjustments to each component of HLT may transpire during the learning process due to the unpredictability of the forecasts made. HLT represents the correlation between instructional theory and practical teaching experiments. Gravemeijer & Cobb (2006) categorize the function of HLT into three phases: 1) the design phase, which aims to formulate teaching materials that require development or adaptation; 2) the teaching phase, which serves as a guideline for instruction and observation; and 3) the retrospective analysis phase, which is intended to establish the focus for evaluative analysis.

Gravemeijer and Cobb (2006) present an example of designing HLT as depicted in Figure 1.

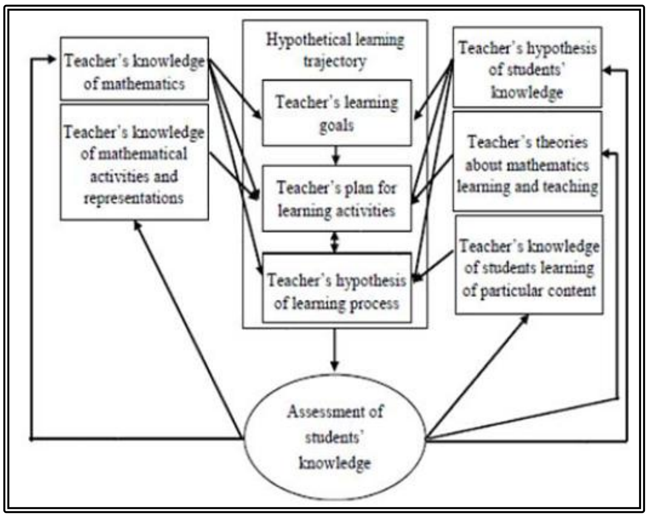

Figure 1. Hypothetical Learning Trajectory

Numerous prior research findings indicate that HLT plays a crucial role in developing learning plans that are more accommodating and proactive in addressing diverse student comprehension responses (Ivars, et al., 2020; Jamilah, et al., 2020). The objective of this project is to develop a Human Learning Technology (HLT) design based on the representation of derived concepts among participants, serving as a reference for creating learning designs. The research topic is: 1) How is the representation of derived notions possessed by participants? 2) How is the design of the Hypothetical Learning Trajectory (HLT) informed by the representation of derived concepts among participants?

2. METHOD

This research employed a qualitative methodology utilizing a case study technique to examine particular phenomena concerning the depiction of derived concepts as perceived by participants (Creswell, 2015). Research data were collected from clinical interviews utilizing open-ended questions to thoroughly and meaningfully investigate participants' cognitive processes (Hunting, 1997). Data analysis techniques use triangulation with several stages, namely data reduction, data presentation, and drawing conclusions. This study recruited five potential mathematics teacher students from a university in West Java Province who have completed differential calculus courses which was implemented during February - May 2025. The clinician interview data were further evaluated utilizing the derived concept framework established by Zandieh (2000). The analysis results informed the development of the Hypothetical Learning Trajectory (HLT) design regarding the derivative notion in this study.

3. RESULT AND DISCUSSION

The clinical interviews with five prospective mathematics teacher students revealed that the majority exhibited analogous inclinations in their interpretation of the derivative to derive a function. This section presents a dialogue from a clinical interview conducted with two participants who demonstrated superior performance compared to the other subjects. S1 denotes the first subject, S2 denotes the second subject, and R represents the researcher.

a. Results of Clinical Interview

The responses provided by the two participants during the clinical interview process are delineated as follows:

Conversation between researcher R and initial subject S1

R: what is your comprehension of the concept of a derivative?

S1: the concept of a derivative pertains to the derivation of an exponential function." For instance, for the function $f(x)$=$3x^3$, the derivative is $9x^2$."

R: how do you derive $9x^2$?

S1: I utilize the formula f' (x)= nx^{n-1}, if the function $f(x)$= $3x^3$ is differentiated, then the differentiation process is f' (x)= $3.3x^{3-1} = 9x^2$

R: do you possess an alternative method to derive the function?

S1: indeed, sir, an alternative method involves utilizing this formula $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. however, the derivation process is rather protracted.

R: could you elucidate the derivation of the derivative formula via limits, and clarify its connection to the derivative formula of the function $f'(x) = x^{n-1}$ for the function $f(x)$= x^n?

S1: the derivative formula derived from the limit is obtained from the difference quotient equation $\frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$, which is utilized to determine the gradient value." In the Cartesian diagram, $\frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$ is modified to $\frac{f(x+h)-f(x)}{h}$. However, I do not comprehend the significance of the notation $\lim_{h\to 0}$. Concerning the correlation between the formulas $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and the derivative formula $f'(x)$= nx^{n-1}, I am uncertain, sir.

R: previously, you examined gradients and Cartesian diagrams within a geometric framework. Are there additional contexts you are aware of that pertain to the concept of derivatives?

S1: I don't think so, sir.

Conversation between researcher "R" and initial subject "S2"

R: what is your perspective on the concept of derivatives?"

S2: the concept of derivatives serves to diminish the power of a function, particularly those with powers of two or greater. Furthermore, derivatives are utilized to ascertain the maximum or lowest values of a function.

R: could you provide an instance of the explanation you have offered?

S2: derivative used to diminish the power of a function, consider the following example: if there exists a function $f(x)$= x^4+ x^6, then the derivative of the function is $f'(x)$= $4x^3 + 6x^5$

R: could you elucidate the techniques of its derivation?

S2: I employ the derivative notation f' (x)= nx^{n-1}, sir. Consequently, the process of differentiating the function $f(x)$= x^4+ x^6 is $f(x)$= $4x^{4-1}$+ $6x^{6-1}$. The outcome remains $f'(x)$= $4x^3 + 6x^5$. The number of accents signifies the number of times the function is differentiated. Therefore, if the function $f(x)$= x^4+ x^6 is differentiated twice, utilizing the same methodology as previously, the result is = $12x^2 + 30x^4$.

R: what is your explanation regarding the maximum or minimum function?

S2: a maximum or minimum function is characterized by $f'(x)$=0; it is also referred to as a stationary point.

R: could you elucidate additional contexts pertaining to the concept of derivatives?

S2: that is all I am aware of, sir."

From the dialogue presented, it is known that the participants' understanding of the concept of derivatives tends to be used to solve routine problems, especially in deriving a function. Both participants gave other responses related to the concept of derivatives and explained the concepts of limits, gradients, and minimum/maximum functions. These findings are in line with the results of research by Feudel & Biehler (2021) which showed that the majority of participants could easily derive a function, but experienced obstacles when interpreting the concept of derivatives with other concepts. The findings that have been obtained are also in line with several research results that show that participants still experience obstacles, especially regarding understanding the concept and meaning of the concept of derivatives (Rodr í guez-Nieto & Rodr í guez-V á squez, 2022; Moru, 2020; Thompson & Harel, 2021). In addition, the findings obtained are also relevant to previous studies showing that participants can solve routine problems without being able to provide conceptual meaning based on relationships between ideas such as gradients, functions, and limits (Prihandhika & Perbowo, 2024)

The outcomes of clinical interviews with five participants were subsequently gathered and examined utilizing the derivative concept framework established by Zandieh (2000) to investigate comprehension of the derivative concept through multiple representations, aiming to achieve comprehensive knowledge applicable to problem-solving. The NCTM (2000) asserts that representation is a crucial component in facilitating individual understanding of mathematical concepts, particularly derivative concepts. The representations of derivative concepts demonstrated by subjects are illustrated in Table 2.

Table 2. Analysis of Clinical Interview Results

	Representation			
Process-object layer	Graphical	Verbal	Paradigmatic Physical	Symbolic
	Slope	Rate of Change	Velocity	Difference Quotient
Ratio	○			
Limit	○			○
Function				●

The analysis of the results from the clinical interviews in Table 2 reveals that participants' understanding of the concept of derivatives is represented graphically and symbolically. Their comprehension is confined to the slope context involving ratio and limit, as well as the difference quotient context concerning limit and function. According to Zandieh's framework (2000), the black circle illustrates the predominant representation utilized by participants in elucidating the concept of derivatives. The black circle within the symbolic representation, associated with the difference quotient and the object function, signifies that the majority of participants' understanding of derivatives is primarily based on the general formula $f'(x) = nx^{n-1}$ for deriving the function $f(x) = x^n$.

Meanwhile, the white circle shows a less comprehensive understanding so that participants explain the concept briefly without providing justification for the explanation that has been given. From the results of clinical interviews, participants showed graphical representations by conveying the idea of $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope (gradient of the function). Participants also showed symbolic representations in the context of difference quotients and limit objects by showing the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, as an alternative step to derive a function. From the results of the analysis using the framework presented by Zandieh (2000), it is known that participants still have not mastered all the basic representations needed to understand the concept of derivatives so that there is the potential for epistemological obstacles, namely obstacles that occur when participants are given a context or example of a problem that is different from what is always given in learning (Brousseau, 2005).

The discussion of each representation, as presented by Roundy, et al. (2015), that: 1) the graphical representation of the concept of derivatives examines the slope or gradient of a line. In the ratio object, the slope of the line is located between two points that intersect a curve. While in the limit object and the function object, the idea being studied is about the equation of the tangent line at a point on the curve; 2) The verbal representation of the concept of derivatives examines the rate of change. In the ratio object it is expressed as the average rate of change, while in the limit and function objects it is expressed as the instantaneous rate of change; 3) The paradigmatic physical representation of the concept of derivatives is related to average speed, instantaneous speed, and speed as a function of time; 4) symbolic representation of the derived concept regarding the definition of the formal concept in the notation used, both ratio notation, limit notation and function notation. From the representation conditions owned by the participants, it is necessary to design efforts for Hypothetical Learning Trajectory (HLT) to facilitate the construction of derived concept representations so that the potential for epistemological obstacles to participants in learning can be minimized.

b. Hypothetical Learning Trajectory

Hypothetical Learning Trajectory (HLT) is conceptualized as a description of participants' thinking on certain mathematical concepts based on predictive routes through the design of a series of instructional tasks that will be given to trigger mental processes and achieve learning objectives (Sari, et al., 2025; Utari, et al., 2025; Clements & Sarama, 2004; Gravemeijer, 2004). HLT on derived concepts in this study focuses on learning schemes through instructional tasks that contain all representations of derived concepts, namely graphic, verbal, paradigmatic physical, and symbol representations.

The alternative HLT design presented in the image above, begins with a thought experiment and instruction experiment on verbal representation to construct an understanding of the concept of derivatives based on the concept of the result of the difference and the average change through the equation $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. The equation can then be expanded into a graphical representation using the concept of gradient or slope of a line from two points through the notation $\frac{f(x+h)-f(x)}{h}$ until finally the concept of derivative is obtained with the equation notation $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ where f' (x) is the derivative notation while $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is the notation of the gradient of the tangent line at a point provided that the limit exists. The notation of the derivative concept is then simplified to f' (x)= nx^{n-1}. for the derivative of a function $f(x)$= x^n. through a thought experiment and instruction experiment that have been designed in such a way. The alternative HLT designs made based on the findings obtained in the study are presented in Figure 2.

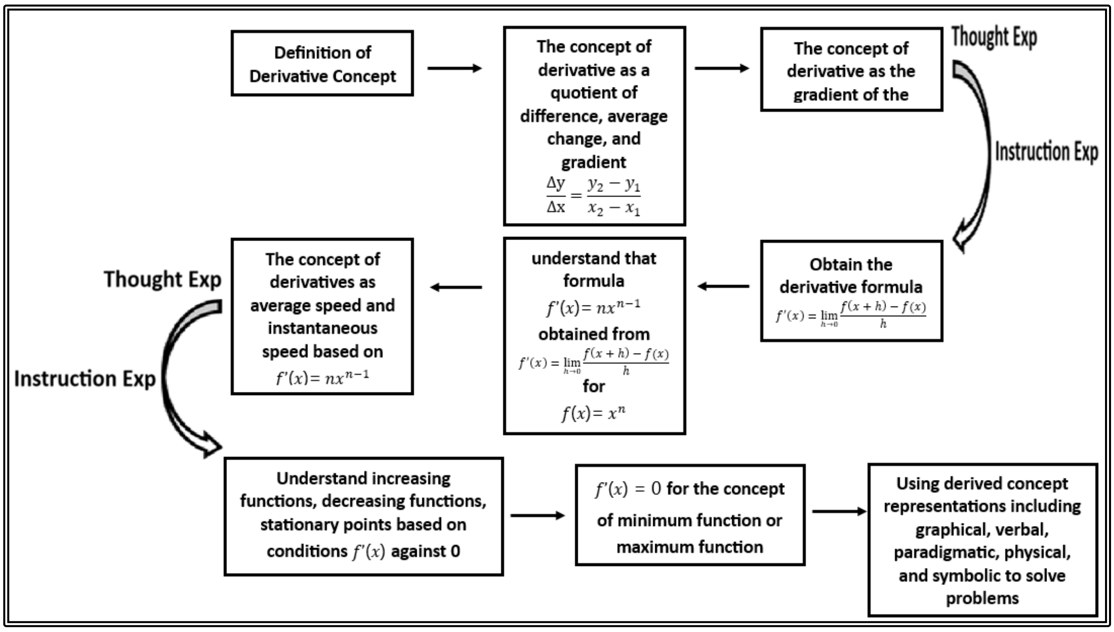

Figure 1. Design of Hypothetical Learning Trajectory

The design of the learning flow aims for participants to have an understanding that the symbolic representation of the derivative concept f' (x)= nx^{n-1} which is often used in problem solving, is obtained from other related concepts. The symbolic representation is then directed into a physical paradigmatic representation such as in the context of instantaneous velocity in physics or the context of marginal value in economics by considering the concept of increasing function, decreasing function, and stationary point that determines the maximum function or minimum function in the derivative concept.

4. CONCLUSION

Based on the results and discussion, it is concluded that the representation of the derivative concept owned by most participants tends to be in the context of symbols whose use is limited to solving routine or procedural problems. The limitations of the representation used by participants in learning and understanding the concept of derivatives have the potential to cause epistemological obstacles when they face non-routine problems or conceptual problems. Therefore, a hypothetical learning trajectory (HLT) prediction design is needed through learning schemes and instructional tasks. The focus of the HLT design that has been made is implemented in stages based on the level of participant mastery of the representation of the derivative concept. Thus, the HLT design that has been made is expected to facilitate participants in constructing a better understanding of the derivative concept so that the potential for epistemological obstacles can be minimized. However, the proposed design still has limitations, especially in the design of tasks that must be given in each learning path. Therefore, the results of this study provide space for other researchers to design a mapping of instructional tasks in the implementation of the HLT design based on the representation of the derivative concept.

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