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Mapping Cognitive Load and Thinking Zones in Understanding Function Limits

Rina Oktaviyanthi[1*], Ria Noviana Agus[2]
[1*,2]Mathematics Education, Universitas Serang Raya
Taman Drangong, Serang, Indonesia
[1*]rinaokta@unsera.ac.id; [2]riaagus@unsera.ac.id

ABSTRAK

Penelitian ini merespons terbatasnya kajian yang memetakan pola berpikir siswa berdasarkan beban dan zona kognitif dalam memahami limit fungsi secara grafis. Dengan menggunakan teori beban kognitif dan klasifikasi zona (risiko, tantangan, optimal), enam mahasiswa dipilih secara purposif dalam studi kasus kualitatif untuk mewakili variasi kemampuan. Data berupa respons tertulis dan think-aloud dianalisis untuk menelusuri transisi zona kognitif. Hasil menunjukkan pola kognitif yang beragam, dari miskonsepsi hingga integrasi konseptual. Beban tinggi dapat dikelola melalui regulasi diri, sedangkan beban rendah tetap berisiko jika struktur pemahaman belum berkembang. Studi ini menekankan pentingnya instruksi adaptif berdasarkan profil kognitif mahasiswa dan menawarkan kerangka zona berpikir untuk mendukung pembelajaran matematika yang personal.

Kata Kunci: Beban kognitif; limit fungsi; pola kognitif; regulasi metakognitif; zona berpikir.

ABSTRACT

This study addresses the limited research mapping students' thinking patterns through cognitive load and cognitive zones in understanding function limits graphically. Using Cognitive Load Theory and the classification of risk, challenge, and optimal zones, six first-year mathematics education students were purposively selected in a qualitative case study to represent varying academic abilities. Data from written responses and think-aloud protocols were analyzed to trace zone transitions. Findings showed diverse cognitive patterns, from misconceptions to successful conceptual integration. High cognitive load was manageable through self-regulation, while low load still poses risks if the understanding structure is undeveloped. The study highlights the importance of adaptive instruction aligned with students' cognitive profiles and offers a thinking zone framework to support personalized mathematics learning.

Keywords: Cognitive load; cognitive pattern; function limits; metacognitive regulation; thinking zones.

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1. INTRODUCTION

Mathematics learning is a cognitively demanding process that requires not only content understanding but also effective cognitive regulation (Petričević et al., 2022; Seufert et al., 2024). One key factor influencing this process is cognitive load, which refers to the mental effort needed to process and retain information during learning. In mathematics, high cognitive load can hinder performance due to the abstract and multi-step nature of problem-solving (Barbieri & Rodrigues, 2025; Lepore, 2024). Researchers have emphasized the importance of managing cognitive load to support learning outcomes. This has led to the development of instructional strategies aimed at balancing task complexity and student capacity.

Despite this focus, little is known about the detailed cognitive patterns that emerge during mathematics learning, especially how cognitive load interacts with internal thinking zones. Cognitive zones refer to mental states or stages learners experience, such as confusion, effortful thinking, and fluent understanding (Lodge et al., 2018; Phan & Ngu, 2021). This framing of thinking zones can be grounded in Vygotsky's Zone of Proximal Development, which suggests that learners move between different levels of competence depending on the support and challenge they face (Doolittle, 1997; Xi & Lantolf, 2021). Thinking zones, such as risk, challenge, and optimal, can be seen as cognitive manifestations of this developmental space. These zones may correspond to varying levels of cognitive load and influence the quality of mathematical reasoning. Prior research using tools like neuroimaging has explored brain activation during learning, but has yet to fully connect these insights with how students experience and transition through cognitive zones (Schacter, 2025; Zadina, 2023). Understanding these dynamics is essential for designing learning environments that support cognitive clarity.

There remains a significant gap in linking specific levels of cognitive load with transitions between cognitive zones during problem-solving in mathematics. Most studies treat cognitive load as static and do not consider how it fluctuates across stages of learning or between students (Krieglstein et al., 2023; Skulmowski & Xu, 2021). However, in reality, cognitive load is not fixed, it changes dynamically as learners shift between confusion, effort, and understanding, especially in complex topics like function limits. This dynamic nature is crucial because static models fail to capture how students regulate thinking in real-time, a feature that is often overlooked in traditional CLT-based studies in calculus. The lack of individualized mapping obscures how different learners respond to the same mathematical task (Wang & Lehman, 2021; Younger et al., 2024). Moreover, little is known about how shifts between zones, such as from confusion to clarity, are reflected in students' responses (Cross Francis et al., 2022; Li & Lajoie, 2022). Addressing this gap requires a more granular method of tracking cognitive transitions alongside load intensity.

This study aims to investigate cognitive pattern recognition in mathematics learning by mapping cognitive load and identifying students' thinking zones. Specifically, it explores how first-year mathematics education students respond to graphical limit problems, a topic known for high conceptual demand. Through written responses and cognitive analysis, students' transitions across zones such as risk, challenge, and optimal will be tracked. The goal is to identify cognitive patterns associated with successful or unsuccessful problem-solving. Unlike previous studies that focus on cognitive load as a singular measure, this research emphasizes the shifting interplay between mental effort and reasoning quality. This understanding will support the design of more responsive, cognitively informed instruction.

In summary, the study bridges cognitive load theory with cognitive pattern analysis to visualize how learners engage with mathematical concepts. By tracing how students think and transition between zones during problem-solving, the research highlights the interplay between cognitive effort and reasoning quality. Findings will inform strategies for reducing unproductive cognitive load and enhancing conceptual understanding. Ultimately, this study contributes a new perspective to mathematics education by offering a framework to recognize and support students' cognitive states. The results have practical implications for adaptive learning and cognitive scaffolding in mathematics instruction.

2. METHOD

This study employed a descriptive qualitative approach grounded in an in-depth analysis of students' cognitive patterns (Azungah, 2018; Elliott & Timulak, 2021). The primary aim was to identify cognitive strategies used in solving mathematical problems involving graphical representations of functions, as well as to map the levels of cognitive load and the thinking zones experienced by each subject. The research design adopted the principles of single-case pattern recognition with cross-subject comparisons (Kratochwill et al., 2021; Shen et al., 2023), focusing on capturing the cognitive processes within the context of specific mathematical tasks.

Six first-year mathematics education students were purposively selected, representing diverse academic backgrounds and varying levels of self-confidence. Academic diversity was based on GPA distribution (ranging from 2.75 to 3.90) and classroom performance. The sample size (n = 6) follows the principle of qualitative saturation, which suggests that meaningful cognitive themes in homogenous groups can emerge with 6 to 12 participants (Braun & Clarke, 2021; Hennink & Kaiser, 2022). The inclusion criteria were as follows: (1) the ability to read and interpret function graphs, (2) prior exposure to the concepts of limits and function values, and (3) willingness to participate in think-aloud interviews.

Three primary instruments were used in this study. First, cognitive stimulation tasks in the form of graph-based problems were designed to elicit students' reasoning processes, particularly their conceptual understanding of discontinuities and limits. These tasks included questions such as determining the value of $f(2)$ and $\lim_{x\to 2} f(x)$, aiming to trigger potential conflicts between the concept of function values and limit values (Oktaviyanthi et al., 2024). The full set of tasks is provided in Appendix A, and a sample item is shown in the enhanced Figure 1. Second, a semi-structured interview guide combined with a think-aloud protocol was employed to capture students' verbalized cognitive processes during task completion (Wolcott & Lobczowski, 2021). The guiding questions encouraged students to articulate their reasoning, explain their interpretation of the graph, reflect on their understanding of limits, and express their confidence in their answers. Third, a cognitive pattern coding sheet was developed to systematically analyze the responses (Boyer et al., 2005). This tool allowed the researcher to examine student behavior and categorize their cognitive states based on five key dimensions: initial response, graph identification, conceptual understanding of limits, confidence, and final conclusion.

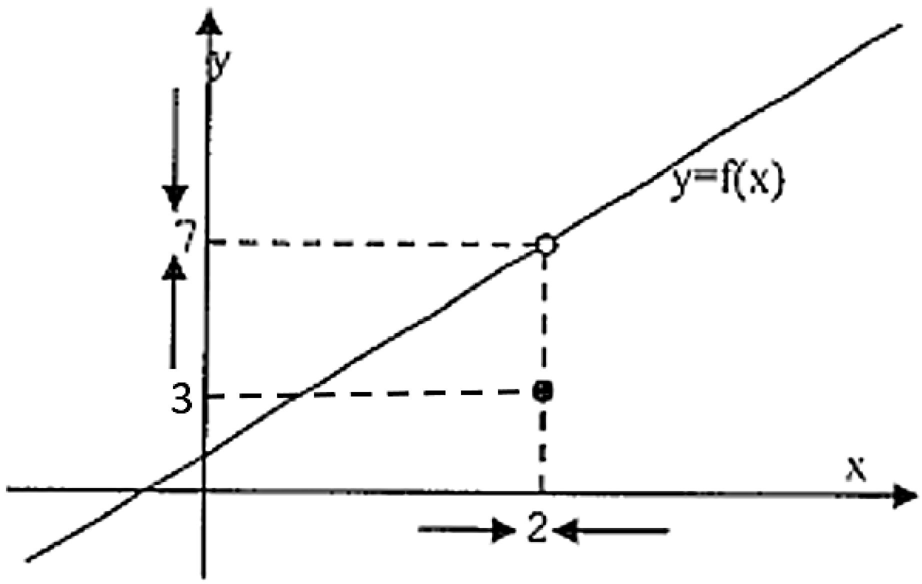

Figure 1. Sample Cognitive Stimulation

Data collection followed a four-step process. First, each subject was individually presented with a graphical mathematics problem and instructed to solve it while thinking aloud. This think-aloud process was video and audio-recorded for accuracy. Second, the entire session was transcribed verbatim to capture all verbal expressions, pauses, and cognitive cues. Third, follow-up interviews were conducted to clarify or probe deeper into students' cognitive strategies, especially when unclear or ambiguous reasoning appeared in the initial transcripts. Finally, all data were organized and prepared for analysis using a cognitive pattern recognition framework.

The data analysis in this study followed a multi-phase approach, beginning with transcript reduction to isolate relevant verbal indicators linked to students' interpretation of the graph, their understanding of limits, and the formulation of conclusions. This was followed by cognitive coding, which categorized each student's thinking processes into specific cognitive zones. To classify responses, the researchers developed a zone-based coding scheme across five cognitive aspects: initial response, graph identification, conceptual understanding of limits, confidence, and final conclusion. Each aspect was aligned with three possible cognitive zones: Risk (Red), Challenge (Yellow), and Optimal (Green), based on the observable indicators expressed by the subjects. This coding structure is shown in Table 1. Each descriptor in Table 1 serves as an operational definition derived from observable verbal and behavioral indicators during problem-solving. Following the cognitive zone classification, each student's responses was mapped to observe transitions between zones. These transitions were used to assess the intensity and direction of cognitive effort, as well as to visualize patterns of cognitive movement (e.g., from risk to challenge, or from challenge to optimal). The strength of these transitions was later visualized in schematic maps using colored and weighted arrows.

Table 1. Cognitive Zone Coding for Each Cognitive Aspect

Aspect	Risk Zone (Red)	Challenge Zone (Yellow)	Optimal Zone (Green)
Initial Response	Confused; unable to start	Hesitant understanding	Calm, immediate understanding
Graph Identification	Misinterprets key points or direction	Partial recognition with uncertainty	Accurately identifies relevant points quickly
Concept of Limit	Fails to distinguish limit from function value	Mixed understanding, needs clarification	Understands limit as approaching from both sides
Confidence	Uncertain, highly hesitant	Somewhat confident, still unsure	Highly confident and logical
Final Conclusion	Incomplete or incorrect reasoning	Tentative but partially supported answer	Logically derived and well justified answer

In addition to zone transitions, students' cognitive load levels were inferred by evaluating their dominant cognitive zones, frequency of conceptual confusion, degree of hesitation, and presence of regressive thinking patterns (Oktaviyanthi, Agus, Garcia, et al., 2024). Based on these indicators, cognitive load was categorized into low, moderate, and high. To strengthen validity, two trained raters independently coded the transcripts, and discrepancies were discussed until consensus was reached. Inter-rater reliability reached a Cohen's Kappa of 0.87, indicating a high level of agreement in zone classification.

Table 2. Cognitive Load Categories and Associated Indicators

Cognitive Load Level	Indicators
Low	Dominantly in the optimal zone, no confusion, high confidence

Cognitive Load Level	Indicators
Moderate	Mixed presence of challenge and risk zones, occasional uncertainty
High	Risk zone dominant, frequent misconceptions, unstable or regressive reasoning

3. RESULT AND DISCUSSION

This study focused on recognizing and mapping students' cognitive patterns in solving problems involving graphical representations of functions, specifically related to the concepts of function value and limit. The main instrument was a single graph-based question designed to induce a conceptual conflict between $f(2)$ and $\lim_{x \to 2} f(x)$, thus eliciting varying degrees of cognitive load across subjects. Data were collected through a combination of think-aloud protocols, semi-structured interviews, and cognitive coding sheets. The analysis focused on five key aspects: (1) initial response to the problem, (2) graph identification, (3) understanding of the limit concept, (4) confidence in the answer, and (5) final conclusion. In addition, cognitive zones were mapped as well as an estimation of each subject's cognitive load.

To further explore the cognitive dynamics of each participant, this section presents a detailed cognitive profile for each subject based on their responses, verbal narratives, and zone transitions during problem-solving. These individual profiles are structured into tables that summarize the student's answers, analytic reflections, and the sequence of cognitive zone transitions. The analysis provides insight into how students navigated conceptual conflict, managed cognitive load, and moved between the Risk, Challenge, and Optimal Zones during the graph-based limit task.

Subject 1 – From Challenge to Risk Zone due to Schematic Gap

Table 3. Cognitive Profile of Subject 1

Student's Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x)} =$ ❌ (not clearly answered, misunderstood)	*"I don't quite understand the question. The graph is clear, but I'm confused about what to find."* [Challenge Zone] *"At $\boldsymbol{x = 2}$, there's a point at $\boldsymbol{y = 3}$, so I think $\boldsymbol{f(2) = 3}$."* [Correct Identification] *"But the question also asks for the limit, and I don't really understand how that's different from the function value at that point."* [Conceptual Confusion], [Risk Zone]	🟧 Challenge Zone → 🟥 Risk Zone

Subject 1 demonstrated confusion in understanding the question's intent, despite finding the graph visually clear. This suggests that the subject was operating within the *Challenge Zone*, attempting to process the information but struggling to link it with the intended concept. Although the subject correctly identified that $f(2) = 3$, they failed to comprehend the distinction between function value and limit. When asked to determine $\lim_{x \to 2} f(x)$, the subject did not engage in further analysis, such as tracing the graph from the left and right, but remained stuck in initial confusion. This condition reflects a shift from the *Challenge Zone* to the *Risk Zone*, where the thinking process halts due to unresolved conceptual conflict. Although misconceptions were present, there were no signs of intense mental stress or anxiety, suggesting that the cognitive load was low. The subject's difficulties stemmed not from task complexity, but from a fundamental lack of understanding regarding the concept of limits.

Subject 2 – Reflective Fluctuation Between Challenge and Risk Zones

Table 4. Cognitive Profile of Subject 2

Student's Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x) = 3}$ ❌ (misconception)	*"From what I see, when* $\boldsymbol{x = 2}$*, there's a point at* $\boldsymbol{y = 3}$*, so I think* $\boldsymbol{f(2) = 3}$*."* [Correct Identification] *"But the question also asks for the limit, and I'm not too sure about that. Maybe the limit is the same as the function value."* [Misconception], [Risk Zone] *"I know this might not be entirely correct, but that's what I can conclude from the graph."* [Hesitation], [Challenge Zone]	🟧 Challenge Zone → 🟥 Risk Zone → 🟧 Challenge Zone

Subject 2 began the task with an attempt to interpret the graph, albeit with noticeable hesitation. The subject correctly identified that $f(2) = 3$, indicating a sound understanding of function value. However, when determining $\lim_{x \to 2} f(x)$, they incorrectly concluded that the limit equals the function value. This reflects a common misconception and places the subject in the *Risk Zone*, where incorrect conceptual understanding is taken as a final answer. Interestingly, the subject expressed doubt about the accuracy of their response, suggesting some level of metacognitive reflection. This points to a return to the *Challenge Zone*, where the subject attempted a reevaluation, despite continued uncertainty. The fluctuation between misconception and reflection, along with conscious doubt, indicates a moderate cognitive load. The subject was not in a state of complete confusion but also did not reach accurate conceptual understanding.

Subject 3 – Cognitive Regulation Amid High Load

Table 5. Cognitive Profile of Subject 3

Student's Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x) = 7}$ ✅	*"I find this question quite complex because there's a lot of information to process from the graph."* [Risk Zone] *"After looking at it in detail, I can see that the function's value at* $\boldsymbol{x = 2}$ *is 3, since there's a point there."* [Correct Identification] *"From this, I'm confident that* $\boldsymbol{\lim_{x \to 2} f(x)}$ *is*	🟥 Risk Zone → 🟧 Challenge Zone → 🟩 Optimal Zone → 🟧 Challenge Zone

Student' s Answer	Analytical Narrative	Zone Transition
	actually 7, as that' s the value the graph approaches." [Accurate Analysis], [Optimal Zone]	

Subject 3 initially perceived the graph as complex, indicating that they began in the *Risk Zone* due to early cognitive overload or fatigue. However, the subject was able to self-regulate and proceed with a more structured analysis. They accurately concluded that $f(2) = 3$ and $\lim_{x \to 2} f(x) = 7$, demonstrating a strong grasp of limit concepts. Their thinking pattern transitioned from the *Risk Zone* to the *Challenge Zone*, ultimately reaching the *Optimal Zone* as they successfully integrated graphical data with conceptual understanding. Notably, the subject reconfirmed their analysis with confidence, reflecting cognitive stability while remaining in an active, reflective mode (returning to the *Challenge Zone*). These zone fluctuations suggest that, although the subject experienced a high cognitive load, it was effectively managed through reanalysis and validation strategies, resulting in complete conceptual understanding.

Subject 4 – Stability in Optimal Zone with Minimal Load

Table 6. Cognitive Profile of Subject 4

Student' s Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x) = 7}$ ✅	*"This question is quite easy for me, and the graph is clear."* [Optimal Zone] *"I can immediately see that* $\boldsymbol{f(2) = 3}$ *because the point at* $\boldsymbol{x = 2}$ *is at* $\boldsymbol{y = 3}$*."* *"I see that the graph approaches* $\boldsymbol{y = 7}$ *from both directions."* [Accurate Limit Concept]	🟩 Optimal Zone

Subject 4 showed immediate comprehension of the task and swiftly identified both $f(2) = 3$ and $\lim_{x \to 2} f(x) = 7$ correctly. Their responses reflected high cognitive stability in both conceptual clarity and confidence. The entire thinking process remained within the *Optimal Zone*, with no observed transitions to other zones. There were no signs of confusion, hesitation, or logical errors that would suggest cognitive strain. This indicates a low cognitive load, as the subject was able to process the information efficiently and accurately without significant barriers.

Subject 6 – Optimal Zone with Moderate Load Due to Metacognitive Awareness

Table 7. Cognitive Profile of Subject 6

Student' s Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x) = 7}$ ✅	*"I feel focused enough to solve this question without feeling too pressured."* *"Looking at the graph, I can quickly see that the function value at* $\boldsymbol{x = 2}$ *is 3 because there' s a point there."* *"For the limit, I notice the graph approaches* $\boldsymbol{y = 7}$ *as x approaches 2 from both sides."* [Optimal Zone]	🟩 Optimal Zone

Subject 6 completed the task smoothly and demonstrated accurate understanding of the graph, including correct conclusions for $f(2) = 3$ and $\lim_{x \to 2} f(x) = 7$. The entire thinking process was stable and uninterrupted, consistently placing the subject in the *Optimal Zone*. Unlike Subject 4, however, Subject 6 expressed metacognitive awareness through comments about being focused and not feeling pressured. This indicates active cognitive control over their thought processes, rather than automatic comprehension. As a result, although the subject was in the same zone, their cognitive load was considered moderate due to conscious efforts in monitoring and managing their cognitive processing.

Subject 10 – Emotional Entrapment and Unstable Zone Transition

Table 8. Cognitive Profile of Subject 10

Student's Answer	Analytical Narrative	Zone Transition
$\boldsymbol{f(2) = 3}$ ✅ $\boldsymbol{\lim_{x \to 2} f(x) = 7}$ ❓ (with hesitation)	*"This question makes me dizzy because the graph is a bit confusing."* [Risk Zone] *"I see there's a point at y=* $\mathbf{3}$ *when* $\boldsymbol{x = 2}$*, so maybe* $\boldsymbol{f(2) = 3}$*?"* *"Since the graph seems to rise toward 7… maybe the limit is 7? I'm not too sure."* [Challenge Zone]	🟥 Risk Zone → 🟧 Challenge Zone

Subject 10 began with responses indicating mental pressure caused by the perceived complexity of the graph, as revealed by their statement that the task made them feel "dizzy." This placed the subject in the *Risk Zone*, where high initial cognitive load hindered interpretive efforts. Despite this, the subject attempted to answer by concluding that $f(2) = 3$ and tentatively suggesting that $\lim_{x \to 2} f(x) = 7$, though with evident uncertainty. This effort marked a transition to the *Challenge Zone*, as the subject tried to engage further with the information despite persistent doubts. The instability in their reasoning process and the high degree of uncertainty in conceptual inference suggest a high cognitive load, indicating the need for conceptual scaffolding or intervention to stabilize their cognitive processing.

Across the six subjects, several cognitive patterns were identified. First, all participants correctly identified the function value f(2), indicating that recognizing a point on a graph was a relatively low-load task. However, variation emerged in how students interpreted the concept of limits. Subjects 1 and 2 struggled due to undeveloped conceptual structures, with both demonstrating transitions into the Risk Zone—one staying there (S1) and the other fluctuating (S2). Subject 3 began with high cognitive load but successfully regulated it, transitioning to the Optimal Zone through deliberate reasoning. Subject 4 remained consistently in the Optimal Zone, showing automatic processing with low load, while Subject 6 demonstrated similar understanding with metacognitive engagement, hence experiencing moderate load. Subject 10 exemplified emotional interference, starting in the Risk Zone and failing to stabilize in the Challenge Zone. Overall, the most successful responses occurred when students could manage cognitive load through reflection and schema activation. Conversely, students with undeveloped understanding or emotional overload struggled to shift into productive zones. These patterns illustrate that cognitive success in limit problems depends not just on knowledge but also on regulatory strategies and zone mobility.

Subject 1 initially attempted to understand the task but failed to grasp the concept of limits due to the absence of an operational schema distinguishing function values from limits. Although the graph was clear, the subject transitioned from the Challenge Zone to the Risk Zone, reflecting unresolved conceptual confusion. Based on Cognitive Load Theory (Sweller, 2011, 2020, 2022), the subject experienced low intrinsic and germane load, supporting Novak's view that misconceptions often stem from indistinct conceptual boundaries rather than explicit errors (Novak, 2002; Uher, 2021). Subject 2 showed uncertainty but remained engaged, making the common error of equating the limit with the function value. However, metacognitive awareness emerged as they questioned their answer. This shift from the Risk Zone back to the Challenge Zone suggests learning potential within Vygotsky's (Fletcher, 2024; Seufert, 2018) Zone of Proximal Development. Moderate cognitive load arose from the effort to self-regulate and reflect. Subject 3 perceived the task as complex but successfully used internal regulation strategies to reach accurate conclusions, transitioning through the Risk, Challenge, and Optimal Zones. Their high cognitive load was effectively managed through germane load, illustrating executive control (Friedman & Robbins, 2021) and productive schema construction (Chen et al., 2023). This suggests that success under high cognitive load is not only possible but dependent on learners' ability to monitor, regulate, and restructure their thinking processes, key principles emphasized in adaptive interpretations of CLT (Rosa et al., 2025; Van Merriënboer & Sweller, 2010).

Subject 4 worked accurately and effortlessly within the Optimal Zone, indicating minimal cognitive load and well-developed schemata for limits and function values. This aligns with Bruner's (Bruner & Haste, 2010; Bryce & Blown, 2024) concept of symbolic representation and reflects the profile of a mature, cognitively stable learner. Subject 6, despite reaching similar conclusions as Subject 4, demonstrated active metacognition by reflecting on their thinking state. This placed them in the Optimal Zone with moderate cognitive load, as their self-monitoring added germane load (Sweller, 2018, 2022), supporting long-term learning. Subject 10 reported feeling "dizzy" from the graph, signaling extraneous load. While correctly identifying $f(2)$, they remained uncertain about the limit, shifting from the Risk to the Challenge Zone without reaching full understanding. This indicates unmanaged high cognitive load, confirming that emotional stress can intensify cognitive burden in the absence of regulatory strategies. This aligns with CLT's assertion that extraneous elements, such as unclear representations, can overload working memory and inhibit schema development (Barbieri & Rodrigues, 2025; Van Merriënboer & Sweller, 2010). Instructionally, such learners benefit from scaffolding and conceptual visualization to stabilize their thinking.

This study reveals a variety of thinking patterns and cognitive loads among students solving limit problems based on function graphs. Each subject exhibited different dynamics of cognitive zones, ranging from the *Risk Zone* (high confusion), the *Challenge Zone* (active thinking), to the *Optimal Zone* (stable understanding). These findings address the central research question: *How are cognitive load and cognitive zone patterns formed as students solve limit problems using graphs?* Subjects 4 and 6, who operated in the *Optimal Zone*, completed the task with strong and confident understanding. Conversely, Subjects 1 and 10 showed zone instability and high cognitive load that hindered information processing. The zone transition patterns, as seen in Subjects 2 and 3, demonstrate that shifting between zones is not a sign of failure but part of active regulation in thinking (Block, 2023; Hayes & Hofmann, 2021). These patterns indicate that internalizing the concept of limits depends heavily on initial schematic structures, cognitive regulation, and reactions to task-related pressure (Champ et al., 2022). Misconceptions, such as equating function value with limit, emerged in subjects who failed to navigate from *Risk* to *Optimal Zone*.

These findings reinforce earlier research that immature schematic structures lead to misconceptions about limits and continuity (Guerra-Reyes et al., 2024). However, this study offers new insights beyond binary correct/incorrect outcomes by analyzing zone transitions and cognitive dynamics. In contrast to findings suggesting that high cognitive load usually impedes performance (Sinha & Kapur, 2021), the case of Subject 3 illustrates how regulation strategies can transform high load into productive reasoning. This supports a more nuanced interpretation of CLT, one that sees germane load and cognitive regulation as mediators of success under demanding conditions. Moreover, it is important to acknowledge that cognitive load levels in this study were inferred qualitatively, based on behavioral cues, verbal reports, and zone transitions, rather than measured through standardized instruments. While these indicators align with CLT principles (Sweller, 2011, 2020), they remain interpretive in nature. Additionally, the small sample size (n = 6), while allowing for deep cognitive mapping, limits generalizability. Therefore, the results should be viewed as exploratory insights, paving the way for future studies that employ quantitative measures or physiological data (e.g., eye tracking, EEG) to validate cognitive load more objectively.

In sum, this study contributes to the theoretical integration of cognitive load theory with cognitive zone mapping, emphasizing how students regulate their cognitive states during high-concept mathematics tasks. On a practical level, the findings suggest that adaptive instruction, based on students' cognitive trajectories, could enhance mathematics learning. Educators can use zone diagnostics to identify students in the Risk Zone, offer scaffolded support, and visualize transitions to foster metacognitive awareness and conceptual clarity.

4. CONCLUSION

This study reveals that students' cognitive patterns in understanding the concept of function limits through graphical representations vary significantly and are influenced by transitions between thinking zones: Risk Zone, Challenge Zone, and Optimal Zone, as well as the levels of cognitive load involved. Participants who demonstrated stable understanding were situated in the Optimal Zone with low cognitive load, whereas those who experienced mental pressure and misconceptions moved between zones with varying intensity. Transition patterns such as risk – challenge – optimal illustrate that thinking dynamics are not linear, but are shaped by students' cognitive regulation abilities and existing schematic structures. These findings affirm that high cognitive load does not always lead to negative outcomes, especially when accompanied by reprocessing strategies and metacognitive awareness. The findings are specific to students' reasoning in solving limit function problems, and therefore, should not be generalized to all mathematics topics. The implications of this research highlight the importance of designing adaptive learning approaches based on students' cognitive zones to optimize limit concept comprehension. It also suggests the development of formative assessments that not only evaluate final outcomes but also map the thinking process. Future studies could explore different function types, such as continuity or asymptotic behavior, and include students from various educational levels. Additionally, integrating neurocognitive tools like EEG may provide objective insights into the validation of cognitive zones and transitions.

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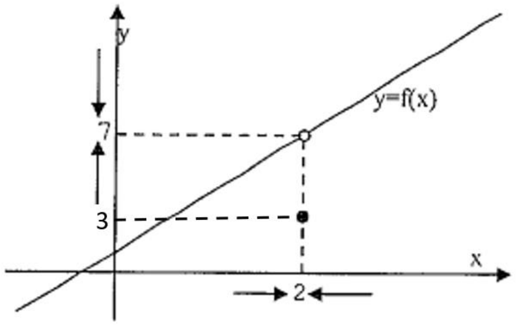
AUTHOR BIOGRAPHY

	**Dr. Rina Oktaviyanthi, M.Pd.** Born in Lebak Banten, on 12 October 1985. Faculty member at Universitas Serang Raya. Completed undergraduate studies in Mathematics Education at Universitas Pendidikan Indonesia, Bandung West Java, in 2007; completed graduate studies in Mathematics Education at Universitas Negeri Surabaya, Surabaya East Java, in 2011; and completed doctoral studies in Mathematics Education at Universitas Pendidikan Indonesia, Bandung West Java, in 2019.
	**Ria Noviana Agus, M.Pd.** Born in Klaten Central Java, on 8 November 1986. Faculty member at Universitas Serang Raya. Completed undergraduate studies in Mathematics Education at Universitas Muhammadiyah Surakarta, Surakarta Central Java, in 2008 and completed graduate studies in Mathematics Education at Universitas Sebelas Maret, Surakarta Central Java, in 2010.

Appendix A — Graph-Based Cognitive Stimulation Tasks

Task 1: Understanding Function Values vs. Limits

The following graph shows the function $f(x)$. Study the graph carefully and answer the questions.

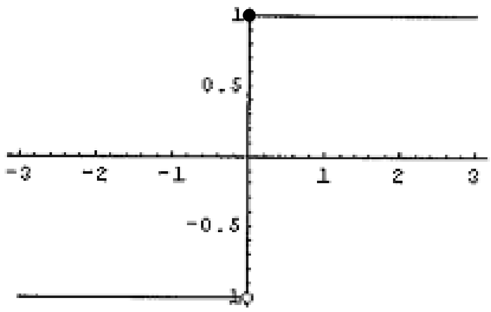


Questions:

1. What is the value of $f(2)$?
2. What is $\lim_{x \to 2} f(2)$?
3. Is $f(x)$ continuous at $x = 2$? Explain why or why not.
4. Describe how the graph helps you determine the answers above.

Task 2: Left and Right Limits

Observe the graph of $g(x)$ below.

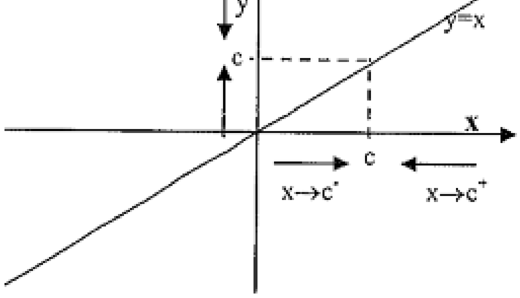


Questions:

1. Find $\lim_{x \to 0^-} g(x)$ and $\lim_{x \to 0^+} g(x)$.
2. Does $\lim_{x \to 0} g(x)$ exist? Why or why not?
3. If $g(0) = 1$, does that affect the existence of the limit? Explain.

Task 3: Identifying Types of Discontinuities

Observe the graph below, which represents a piecewise function. The graph shows two different paths approaching the point $x = c$ from the left and right. A straight line $y = x$ is also shown as a reference.



Questions:

1. Based on the graph, estimate the values of $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$. Explain your reasoning using the direction of approach.
2. If the function is defined such that $f(c) = c$, is the function continuous at $x = c$? Why or why not?
3. If there is a discontinuity, identify its type (removable, jump, or infinite discontinuity) and explain how you determined this from the graph.
4. What modification would be necessary to make the function continuous at $x = c$?