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Didactic Transposition of Straight-Line Equations: from Scholarly Knowledge to Knowledge to be Taught

I Made Suarsana[1], Didi Suryadi[2*], Elah Nurlaelah[3], Al Jupri[4], Estrella R. Pacis[5]
[1,2*,3,4]Mathematics Education, Universitas Pendidikan Indonesia
Jalan Dr. Setiabudhi No. 299, Bandung, Indonesia
[5]Mariano Marcos State University
City of Batac, 2906 Ilocos Norte, Philippines
made.suarsana@upi.edu; didisurya1902@gmail.com; elah_nurlaelah@upi.edu; aljupri@upi.edu; erpacis@mmsu.edu.ph

ABSTRAK

Penelitian ini bertujuan untuk mengkaji proses transposisi didaktik materi persamaan garis lurus dari scholarly knowledge ke knowledge to be taught. Penelitian menggunakan *framework didactical design research* dengan pendekatan kualitatif dan desain fenomenologi hermeneutik. Sumber data adalah *scholarly knowledge* berupa buku geometri analitik di perguruan tinggi dan dokumen knowledge to be taught berupa kurikulum dan buku pelajaran matematika SMP kelas VIII. Beberapa temuan permasalahan yang dapat menjadi potensi munculnya hambatan belajar yaitu pernyataan bentuk umum persamaan garis lurus $y = mx + c$, pendefinisian gradien sebagai rasio jarak atau rasio perubahan, serta penggunaan konteks yang tidak memenuhi asumsi realitas matematika. Temuan ini dapat dijadikan acuan bagi *noosfer* agar lebih berhati-hati dan melakukan antisipasi dalam merancang kurikulum dan buku pelajaran tentang persamaan garis lurus.

Kata Kunci: transposisi didaktik; persamaan garis lurus; pengetahuan ilmiah; pengetahuan yang akan diajarkan.

ABSTRACT

This research examines the didactic transposition process of straight-line equation material from scholarly knowledge to knowledge to be taught. The research uses a didactical design framework with a qualitative approach and hermeneutic phenomenological design. The data sources are scholarly knowledge in the form of analytical geometry books at universities and knowledge-to-be-taught documents in the form of curriculum and grade VIII middle school mathematics textbooks. Some of the problem findings that could potentially create barriers to learning include stating the general form of the straight-line equation y=mx+c, defining gradient as a distance ratio or change ratio, and using contexts that do not meet the assumptions of mathematical reality. These findings can be a reference for the noosphere to be more careful and anticipate when designing curricula and textbooks about straight-line equations.

Keywords: didactic transposition; straight line equation; scholarly knowledge; knowledge to be taught.

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1. INTRODUCTION

Straight lines, called lines, are one of the objects studied in geometry (Sasane, 2015; Kaunang, 2018). If a point is defined as a geometric object with no length or area, then a line is defined as a geometric object with no area (Sasane, 2015). The material about lines in geometry is generally related to the properties and relationships of lines with points, other lines, or other geometric objects arranged in an axiom system such as the Euclidean system. With the discovered the Cartesian coordinate system by Ren é Descartes, a branch of mathematics was developed, namely coordinate geometry, better known as analytical geometry ((Jain, 2005; Suwanto dkk., 2023). Analytical geometry is considered a combination of geometry and algebra, which makes a one-to-one mapping between mathematical equations and locus so that a more systematic and firm geometric problem-solving method is obtained (Suarsana, 2014; Hajizah & Salsabila, 2024). With the development of the branch of analytical geometry, the discussion of straight lines has expanded to the topic of straight-line equations. Knowledge of linear equations plays a significant role in developing other branches, such as calculus, vector analysis, and linear algebra, including in other fields of science such as physics, economics, and engineering.

In learning linear equations, students can visually represent line objects to be more easily understood (Sasane, 2015). Ideas about lines and slopes have been known to students before learning, so when compared to learning other mathematical topics, learning linear equations should have the potential to be easier to understand (Abdussakir, 2009). However, research findings show that learning linear equations still experiences obstacles. In the school curriculum, learning linear equations appears at the junior high school level (phase D) and is grouped in the algebra content element (BSKAP, 2022). An initial understanding of linear equations (algebra), function graphs, and coordinate systems are prerequisite skills that greatly determine students' success in learning linear equations. Inadequate basic knowledge is a major learning obstacle in analytical geometry (Modestou & Gagatsis, 2007). Geometry learning outcomes are highly dependent on students' visual and abstraction abilities, but both of these abilities are still very weak (Fitriyani et al., 2018).

Previous studies on learning barriers in linear equation material have been conducted by previous researchers and found that three learning barriers emerged: epistemic, didactic, and ontogenic. Epistemic learning barriers are in the form of errors or difficulties for students in understanding the concept of gradient, understanding the principle of the relationship between two lines, both parallel and perpendicular, and determining equations and graphs of straight lines ((Kadarisma & Amelia, 2018; Usiskin, 1987; Wantah & Prastyo, 2022). Didactic learning barriers are a lack of opportunities for students to construct their understanding and weak reinforcement of prerequisite material (Wantah & Prastyo, 2022). Ontogenic barriers include low learning and mental readiness of students considering mathematics a complex subject (Putra & Setiawati, 2018; Wantah & Prastyo, 2022).

In previous studies, efforts to uncover learning barriers experienced by students were carried out through a diagnosis of learning readiness, a diagnosis of learning difficulties, and also through in-depth interviews with teachers and students so that knowledge was obtained about the types of barriers in learning linear equations. This study used a different perspective to uncover potential learning barriers. An in-depth study was conducted through content analysis of existing documents to reveal how the transition from scientific knowledge (scholarly knowledge) to school mathematics (knowledge to be taught) or also known as didactic transposition analysis (Bosch & Gasc ó n, 2006).

Theoretical Framework: Didactic Transposition

The term didactic transposition was first coined by Yves Chevallard in 1978 (Bergsten et al., 2010). The idea of didactic transposition put forward by Chevallard was heavily influenced by the view of sociologist Michel Verret (1975) that teaching knowledge in schools should not be equated with the way knowledge is obtained in scientific communities. Verret's idea arose from his anxiety about teaching at universities, while Chevallard's idea arose from his anxiety about teaching at schools (Bergsten et al., 2010). The basic idea in didactic transposition focuses on the fact that the knowledge taught in schools comes from knowledge constructed by the scientific community at universities or other scientific institutions (Chevallard & Bosch, 2020). The objects or subjects of knowledge constructed by this scientific community undergo a shift when they are selected and designed to be taught until they are taught in schools. This transition process is the process of didactic transposition. The didactic transposition process begins with scholarly knowledge (SK) produced by scientists (scholars). SK is transferred by the noosphere (curriculum developers, textbook compilers) into knowledge to be taught (KT).

Furthermore, KT is transferred by teaching institutions/educational institutions (schools or teachers) into taught knowledge (TK). Ultimately, this knowledge will become learned knowledge (LK) by students. Therefore, LK originates from a series of long processes in a didactic transposition process, and it is impossible to interpret LK without considering the phenomenon of constructing school mathematics knowledge from SK (Bosch & Gasc ó n, 2006). In this didactic transposition concept, Chevallard offers a model to analyze the didactic transposition process that occurs, which is called the reference epistemological model (see Figure 1) (Chevallard & Bosch, 2014; Chevallard, 2006), where the researcher's position is outside the didactic transposition process.

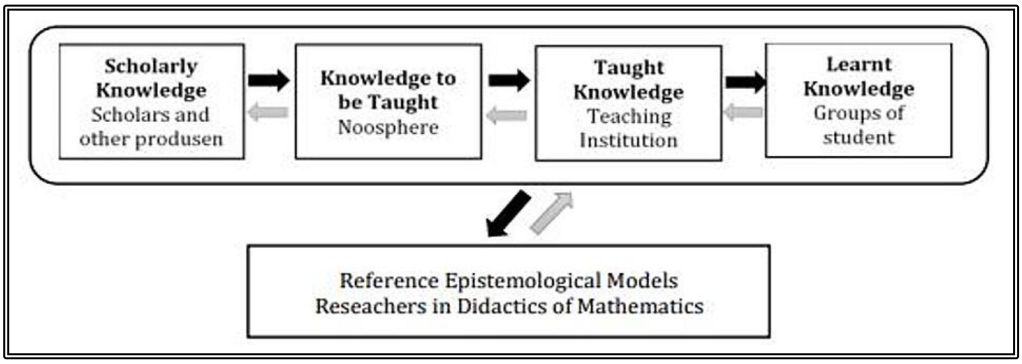


Figure 1. The Researcher's Position is Outside the Transposition Process

2. METHOD

This study uses a didactic design research (DDR) framework and an interpretative paradigm to examine designs to find potential learning obstacles. DDR is a mathematics education research methodology that provides a framework for examining and handling the complexity of learning through critical reflection practices in producing a didactic design (Suryadi, 2019). According to Suryadi (2019), mathematics developed by experts (SK) is a priori or formal knowledge so that when transposition is carried out into curriculum material (KT) and/or learning material (TK), a transposition process needs to be carried out which begins with personalization and recontextualization so that it becomes a posteriori knowledge. Suryadi (2019), in his didactic transposition research, developed an analysis pattern while still referring to the didactic transposition process that has been put forward by Chevallard & Bosch (2014), which consists of 2 stages, namely the prospective stage and the metapedadidactic stage. The prospective stage focuses on analyzing the phenomenon of knowledge transfer from SK to KT, the transition from KT to TK, and the transition from TK to LK. The metapedadidactic stage aims to carry out the didactic transposition process, starting from producing SK to designing the knowledge into KT and TK.

The presentation in this paper focuses on analyzing the transition of knowledge from SK to KT for the material of linear equations in junior high school mathematics lessons. Institutions play an important role in conducting didactic transposition studies to formulate a conceptual relationship between mathematics as a discipline and mathematics as a subject in schools (Chevallard, 1991). This study examines explicitly the didactic transposition that Indonesian institutions have carried out in presenting the topic of linear equations in the mathematics curriculum and mathematics textbooks in junior high schools. The study uses an interpretive paradigm, namely examining the phenomena of reality related to the impact of didactics on a person's way of thinking (Suryadi, 2019). The reality that is the focus of the study includes three things, namely meaning, experience that produces meaning (meaning), and culture that has an impact on the encouragement of the creation of experience in the meaning process. Thus, this study uses a qualitative approach with a hermeneutic phenomenological design.

a. Data Sources

In qualitative research, four methods can be used in data collection: participant observation, document study, in-depth interviews, and artifacts (McMillan & Schumacher, 2001). The primary technique used in data collection in this study is document study. Documents in the didactic transposition analysis of linear equation material can be selected into two, namely (1) documents in the form of primary books from mathematicians to reveal linear equations as SK and (2) independent curriculum documents and junior high school mathematics textbooks issued by the Indonesian government to reveal knowledge of linear equations as KT. The independent curriculum has been implemented since 2022, so for schools that have started implementing this new curriculum, its implementation has entered its second year. The independent curriculum for the junior high school level has been implemented for grades 7 and 8. The topic of linear equations in the independent curriculum appears in phase D of algebra elements with learning outcomes, namely (1) students can distinguish several nonlinear functions from linear functions graphically, and (2) students can present, analyze, and solve problems using relations, functions, and linear equations. In elaborating learning outcomes into learning objectives (syllabus), the topic of linear equations usually appears in grade 8. The Indonesian government has officially distributed grade 8 mathematics textbooks for the independent curriculum in the form of student books and teacher books. In more detail, the documents used as data sources in this study are presented in Table 1.

Table 1. References for Scholarly Knowledge and Knowledge to be Taught

Year	Publisher	Author	Title
1893	Dublin University Press	Casey, John	A treatise on the analytical geometry of the point, line, circle, and conic sections: containing an account of its most recent extensions, with numerous examples
1922	Ginn	Siceloff, Lewis Parker Wentworth, George Smith, David Eugene	Analytic geometry
1986	Wiley eastern Limited	Jain, P K Aḥmad, Khalīl	A Textbook Of Analytical Geometry Of Two Dimensions
2015	World Scientific Publishing Company	Sasane, Amol	Plain plane geometry
2008	Springer	Aarts, J M Erne, R	Solid Geometry

Year	Publisher	Author	Title
2022	Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi	Kepala Badan Standar, Kurikulum, dan Asesmen Pendidikan	Keputusan No. 033/H/KR/2022 Tentang Capaian Pembelajaran Pada PAUD, Jenjang Pendidikan Dasar, dan Jenjang Pendidikan Menengah pada Kurikulum Merdeka
2022	Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi	Mohammad Tohir Abdur R. As'ari Ahmad C. Anam Ibnu Tauiq	Buku Siswa: Matematika untuk SMP/MTs Kelas VIII
2022	Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi	Mohammad Tohir Abdur R. As'ari Ahmad C. Anam Ibnu Tauiq	Buku Panduan Guru Matematika untuk SMP/MTs Kelas VIII

b. Data Analysis

Data analysis in this study is in the form of content analysis of source documents. Content analysis is carried out by systematically examining documents to describe objectively, clearly, and measurably the information/knowledge contained in the document (Nilamsari, 2014). In this study, documents were analyzed descriptively by (1) explaining the concept of line equations in SK and KT documents and (2) describing the transposition of knowledge from SK to KT.

c. Validity

The validity of the analysis results in a study of the meaning and significance of a phenomenon is carried out with a forum group discussion (FGD). FGD is a form of group discussion that aims to gain an understanding of a specific topic through various interpretations from the perspective of experts in their fields (Wong, 2008). FGD involves mathematicians, mathematics learning experts, and mathematics teachers. This FGD is an effort to triangulate the study's results to minimize the subjectivity or bias of researchers.

3. RESULT AND DISCUSSION

a. Scholarly Knowledge of Straight-Line Equations

In defining the equation of a straight-line, Siceloff et al. (1922) and Jain & Ahmad began by defining "Locus is the path traced by a moving point under certain geometrical conditions, and Straight Line is the simplest Locus of point in the plane. Furthermore, the equation of a locus is a relation between x and y, which is satisfied by the coordinates of all points of the Locus and by no others. Thus, the equation of a straight line is defined as the relationship between x and y such that the coordinates of all points on the straight line are satisfied.

Casey (1893a) defined the equation of a straight line directly as a relation between the coordinates of a variable point such that it satisfies that the point lies on the line. Casey describes several cases of determining the equation of a straight line from certain conditions owned by the straight line graph. First, when the straight line graph forms an angle α with the positive x-axis and intersects the y-axis at point $(0, b)$ then by using the properties of opposite angles, the relationship between the coordinates of the variable point $P(x, y)$ can be formulated as $y = mm + b$, with $m = \tan(\alpha)$. Second, when the straight line graph intersects the x-axis and y-axis at points $(a, 0)$ and $(0, b)$, respectively, the straight line equation $\frac{x}{a} + \frac{y}{b} = 1$ is obtained by using the similarity property. Third, another form of the straight line equation that can also be formulated based on the relationship between the coordinates of the variable point P(x,y) is $x\cos(\alpha) + y\sin(\alpha) = p$, with p being the distance from the origin to the line, α being the angle of the height line to the positive x-axis. Aarts et al. (2008) used a vector approach to formulate the straight-line equation to obtain a parametric equation. For example, a straight line passes through points $A(x_1, y_1), B(x_2, y_2)$ then the straight line equation AB can be written in $(x, y) = (1 - \lambda)(x_1, y_1) + \lambda(x_2, y_2)$, with $\lambda \in$ a real number.

The concept of a straight line gradient is defined as the tangent of the inclination angle with the formula $m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$ (Siceloff et al., 1922). The inclination angle is formed by the line with the positive x-axis. Thus, the value of the gradient of a line varies greatly; it can be negative, positive, zero, or undefined. A line has a constant slope. Aarts et al. (2008) define the gradient as the direction vector of the line. If a line has a gradient of $m = -2$, then it can be said that the direction of the line is parallel to the vector $\langle -2,1 \rangle$.

b. *Knowledge to be Taught* of Straight-Line Equations

In the independent curriculum document on learning outcomes, it is stated that mathematics is a science or knowledge about learning or logical thinking that is very much needed by humans to live, which underlies the development of modern technology (BSKAP, 2022). Mathematics has a dual function: learning material that must be understood (content elements) and a conceptual tool to construct and reconstruct the material and hone and train the thinking skills needed to solve life problems (process elements). Mathematics learning materials in junior high school (phase D) are organized into five content elements: numbers, algebra, measurement, geometry, and data analysis and opportunities. The process elements in mathematics subjects are (1) reasoning and proof, (2) mathematical problem solving, (3) communication, (4) mathematical representation, and (5) mathematical connections.

Elemen	Capaian Pembelajaran
Aljabar	Di akhir fase D peserta didik dapat mengenali, memprediksi dan menggeneralisasi pola dalam bentuk susunan benda dan bilangan. Mereka dapat menyatakan suatu situasi ke dalam bentuk aljabar. Mereka dapat menggunakan sifat-sifat operasi (komutatif, asosiatif, dan distributif) untuk menghasilkan bentuk aljabar yang ekuivalen. Peserta didik dapat memahami relasi dan fungsi (domain, kodomain, range) dan menyajikannya dalam bentuk diagram panah, tabel, himpunan pasangan berurutan, dan grafik. Mereka dapat membedakan beberapa fungsi nonlinear dari fungsi linear secara grafik. Mereka dapat menyelesaikan persamaan dan pertidaksamaan linear satu variabel. Mereka dapat menyajikan, menganalisis, dan menyelesaikan masalah dengan menggunakan relasi, fungsi dan persamaan linear. Mereka dapat menyelesaikan sistem persaman linear dua variabel melalui beberapa cara untuk penyelesaian masalah.

Figure 2. Phase D Learning Achievements based on Content Elements in the Independent Curriculum (BSKAP, 2022)

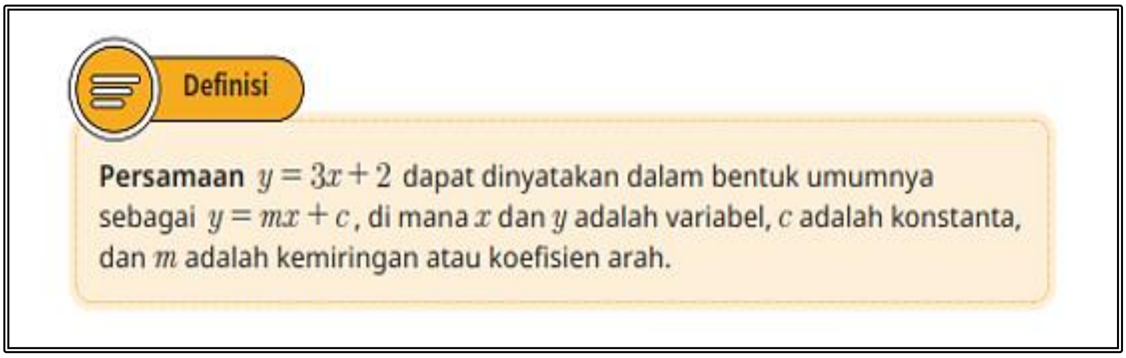
In terms of content elements, the linear equation material in the independent curriculum appears in the algebra element with learning outcomes at the end of phase D. Namely, students can present, analyze, and solve problems using linear functions, linear equations, straight line gradients in the Cartesian coordinate plane (BSKAP, 2022). In the teacher's book document published by the government, this learning achievement is described into nine learning objectives with a flow (See Figure 2), namely (1) Understanding the form of linear equations, (2) Explaining Cartesian coordinates, (3) Drawing straight lines in Cartesian coordinates, (4) Understanding the concept of gradient, (5) Solving problems with the concept of gradient, (6) Determining linear equations, (7) Understanding the concept of the form of a straight line equation, (8) Describing other forms of straight line equations, and (9) Determining the solution to a linear equation (Tohir et al., 2022a). Furthermore, the content analysis of the independent curriculum student textbook was carried out to obtain a description of the knowledge to be taught on the material on straight-line equations. The description of the linear equation material in student textbooks is divided into two subtopics: linear equations and gradients (Tohir et al., 2022b).

Unit Pembelajaran 8.5: Persamaan Garis Lurus

Tujuan Pembelajaran	JP
• Memahami bentuk persamaan linier • Menjelaskan koordinat kartesius • Menggambar persamaan linier pada koordinat kartesius	8 JP
• Memahami konsep gradien • Menyelesaikan permasalahan dengan konsep gradien	6 JP
• Menentukan persamaan linier • Memahami konsep bentuk persamaan garis lurus • Menggambarkan bentuk lain persamaan garis lurus • Menentukan penyelesaian dari suatu persamaan linier	10 JP

Figure 3. Learning Objectives on the Topic of Straight-Line Equations in the Teacher's Book (Tohir et al., 2022a)

The first subtopic defines a straight line equation as an equation whose graph can form a straight line. The general form is $y = mx + c$, where c is a constant, and m is the slope or direction coefficient. If the value is $c = 0$, then the graph of the straight-line equation passes through the origin (see Figure 2). To reach this conclusion, a series of tasks given to students are (i) observing the graph of the equation $y = 2x$ with several coordinate points and a graph with two coordinate points, (ii) observing the graph of the equation $y = 3x + 6$ with several coordinate points and a graph with two coordinate points, and (iii) listing the coordinate points shown in the straight line image, writing them in a table. Based on this series of tasks, students are expected to find the definition and general form of the straight-line equation. A new situation is presented in example 5.1. with a different form of equation from the one defined previously, namely $4x - y = 5$. Students are assigned to fill in the empty cells in the table to find ordered pairs (x, y) that satisfy or are solutions to the equation $4x - y = 5$. A straight line will be formed if the ordered pairs that are solutions to the equation are connected. Thus, the equation $4x - y = 5$ has a solution at every point along the line.

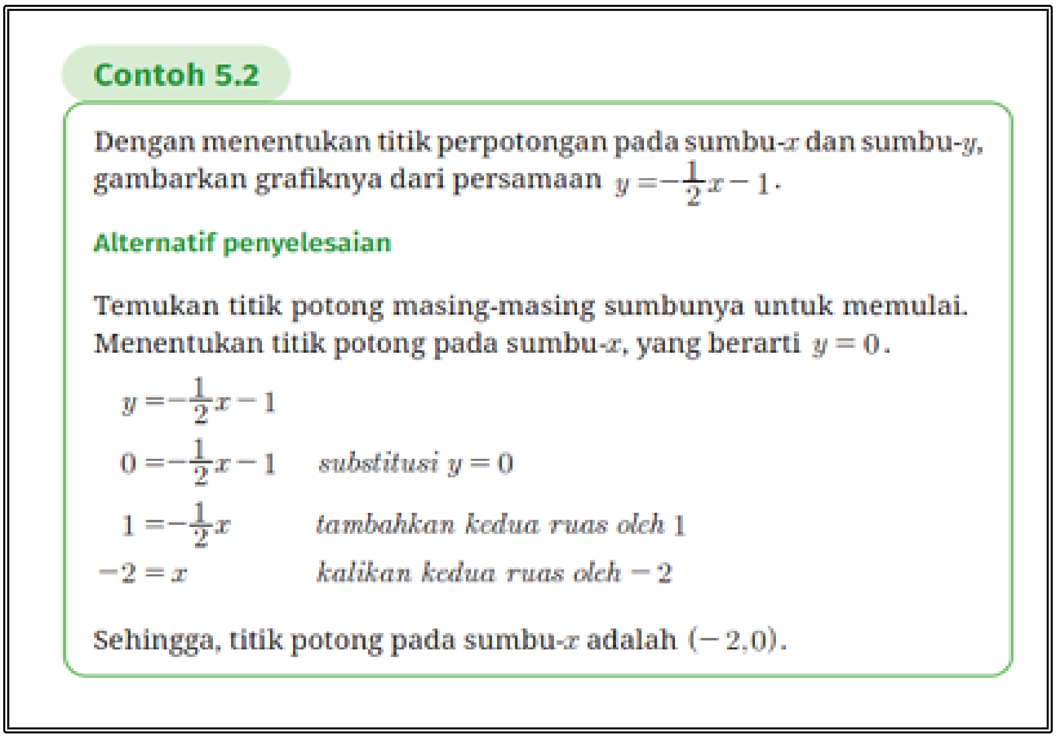


Definisi

Persamaan $y = 3x + 2$ dapat dinyatakan dalam bentuk umumnya sebagai $y = mx + c$, di mana x dan y adalah variabel, c adalah konstanta, dan m adalah kemiringan atau koefisien arah.

Figure 4. Definition of Straight-Line Equation in Student Book (Tohir et al., 2022b)

To get a graph of the solution of a straight-line equation, it is enough to determine two points that are the solution and then connect them with a line. Students expect this conclusion to be obtained through the presentation of example 5.2. (See Figure 4). Given the equation $y = -\frac{1}{2}x - 1$, students are tasked with determining the intersection points on the x-axis and y-axis and then drawing the graph by connecting the two points. Students are also invited to think about how the equation of a line that only passes through one of the coordinate axes, the x-axis only or the y-axis only.



Contoh 5.2

Dengan menentukan titik perpotongan pada sumbu-x dan sumbu-y, gambarkan grafiknya dari persamaan $y = -\frac{1}{2}x - 1$.

Alternatif penyelesaian

Temukan titik potong masing-masing sumbunya untuk memulai. Menentukan titik potong pada sumbu-x, yang berarti $y = 0$.

$$y = -\frac{1}{2}x - 1$$
$$0 = -\frac{1}{2}x - 1 \quad \textit{substitusi } y = 0$$
$$1 = -\frac{1}{2}x \quad \textit{tambahkan kedua ruas oleh } 1$$
$$-2 = x \quad \textit{kalikan kedua ruas oleh } -2$$

Sehingga, titik potong pada sumbu-x adalah $(-2, 0)$.

Figure 5. Example of a Question on Drawing a Straight-Line Graph through Two Points (Tohir et al., 2022b)

In the next section, in the "Ayo Mencoba" section, students are tasked with comparing four linear equations and their graphs in terms of both similarities and differences. If the four equations are related to the form $y = mx + c$, all four have a value of $c = 0$. Graphically, all four pass through the point $(0,0)$. The difference is in the value of m; some are positive, and some are negative; graphically, if $m > 0$, the line is slanted to the right and vice versa.

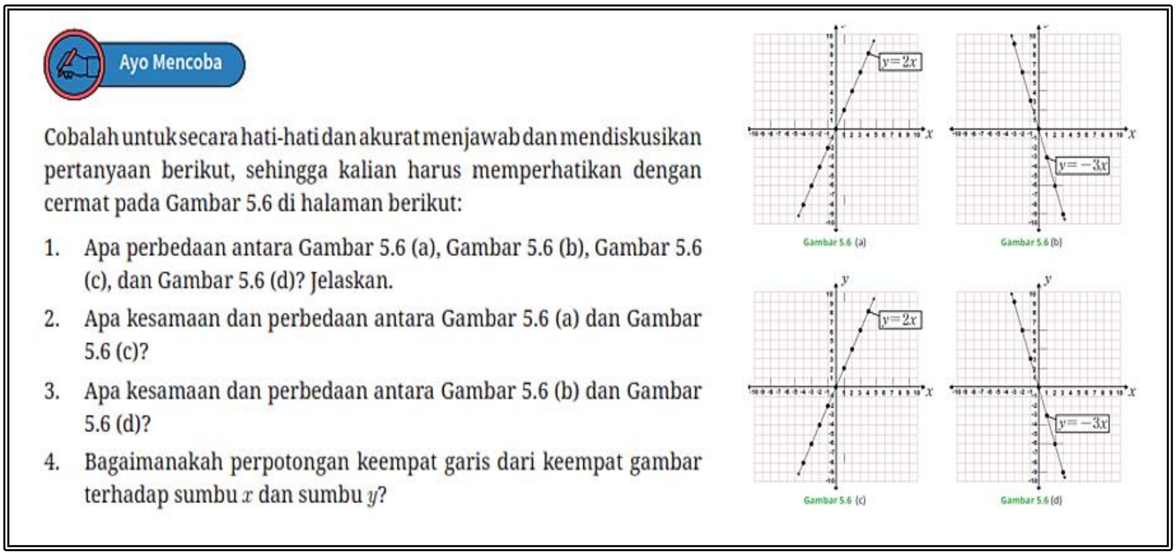
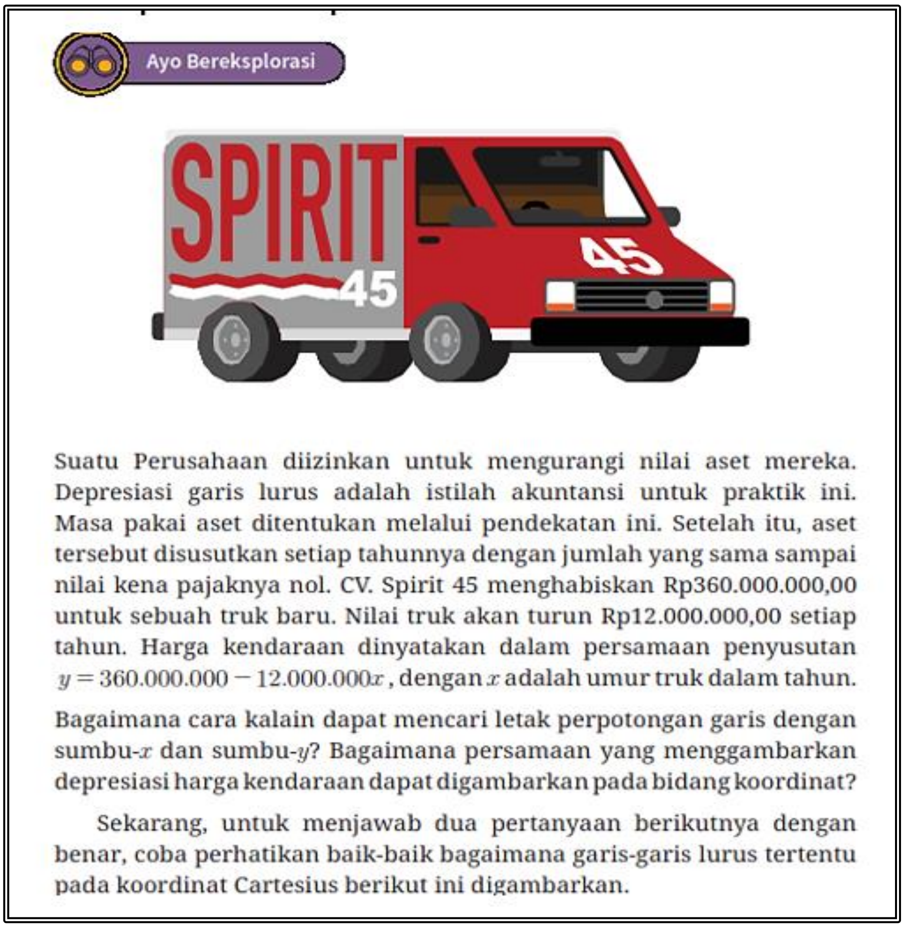


Figure 6. Relationship between the Equation of a Straight-Line and its Graph (Tohir et al., 2022b)

Next, at the end of the first sub-topic description, a contextual problem in the accounting field is presented, known as straight-line depreciation. In the problem, the equation for a company's vehicle price depreciation is $y = 360.000.000 - 12.000.000\ x$. Students are assigned to determine the line's intersection point with the coordinate axis and then draw a graph of the equation on the coordinate plane. The interpretation of the coordinate axes' intersection in the problem context is also described. The points $(0, 360.000.000)$ and $(30,0)$ mean that the vehicle cost in year 0 is 360,000,000 rupiah and in year 30 is 0.



Suatu Perusahaan diizinkan untuk mengurangi nilai aset mereka. Depresiasi garis lurus adalah istilah akuntansi untuk praktik ini. Masa pakai aset ditentukan melalui pendekatan ini. Setelah itu, aset tersebut disusutkan setiap tahunnya dengan jumlah yang sama sampai nilai kena pajaknya nol. CV. Spirit 45 menghabiskan Rp360.000.000,00 untuk sebuah truk baru. Nilai truk akan turun Rp12.000.000,00 setiap tahun. Harga kendaraan dinyatakan dalam persamaan penyusutan $y = 360.000.000 - 12.000.000x$, dengan x adalah umur truk dalam tahun.

Bagaimana cara kalain dapat mencari letak perpotongan garis dengan sumbu-x dan sumbu-y? Bagaimana persamaan yang menggambarkan depresiasi harga kendaraan dapat digambarkan pada bidang koordinat?

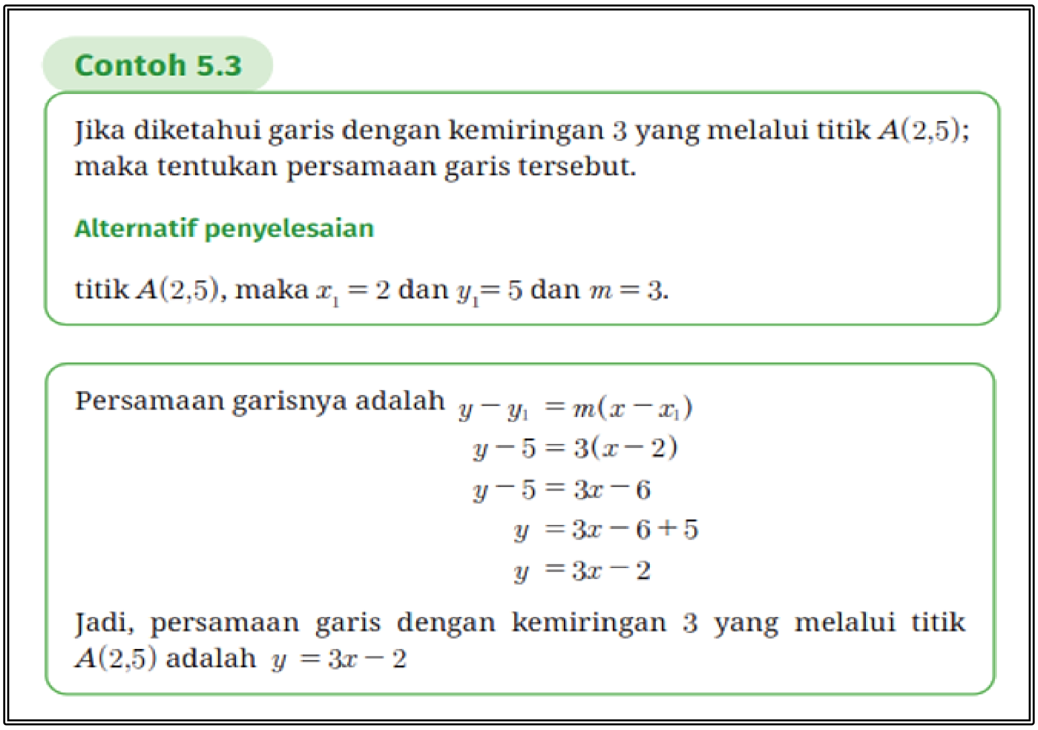
Sekarang, untuk menjawab dua pertanyaan berikutnya dengan benar, coba perhatikan baik-baik bagaimana garis-garis lurus tertentu pada koordinat Cartesius berikut ini digambarkan.

Figure 7. Contextual Problems of Straight Line Depreciation (Tohir et al., 2022b)

In sub-topic 2, the presentation begins by writing the definition of the slope directly as the ratio between the vertical distance and the horizontal distance. However, the formula for the slope is written next, namely:

$$\text{slope}=\frac{\text{change in the length of the vertical side}}{\text{change in the length of the horizontal side}}$$

Next, students are given a series of tasks to understand the definition of gradient better. Given six straight line equations, namely $y = 2x$, $y = -2x$, $y = 2x - 4$, $y = -2x + 6$, $y = x$, $y = 4x + 3$, along with one of the points it passes through. Students are assigned to draw a graph of each equation and then determine the slope/gradient using two methods, namely the value of m and the slope formula. Students are also assigned to interpret the gradient as the direction of the line; for example, if the gradient is $-2 = \frac{6}{-3}$, then it means that another point from the first point on the line is obtained by shifting the starting point 3 units to the left and six units up. From the series of activities, students are expected to conclude (i) if the value of $m > 0$, then the line slopes to the right and vice versa, (ii) the equation of the line passing through the origin with a gradient of m is $y = mx$, and (iii) the equation of the line passing through the point (x_1, y_1) with a gradient of m is $y - y_1 = m(x - x_1)$. Example 5.3 (see Figure 7) is provided to strengthen students' understanding of determining the equation of a straight line, specifically finding the equation of a line with a slope of 3 that passes through point $A(2,5)$. Using the formula $y - y_1 = m(x - x_1)$, the equation of the line is $y = 3x - 2$.



Contoh 5.3

Jika diketahui garis dengan kemiringan 3 yang melalui titik $A(2,5)$; maka tentukan persamaan garis tersebut.

Alternatif penyelesaian

titik $A(2,5)$, maka $x_1 = 2$ dan $y_1 = 5$ dan $m = 3$.

Persamaan garisnya adalah
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 3(x - 2) \\ y - 5 &= 3x - 6 \\ y &= 3x - 6 + 5 \\ y &= 3x - 2 \end{aligned}$$

Jadi, persamaan garis dengan kemiringan 3 yang melalui titik $A(2,5)$ adalah $y = 3x - 2$

Figure 8. Example of a Question to Determine the Equation of a Straight-Line if the Gradient and Points Passed Through are Known (Tohir et al., 2022b)

Contextual issues related to the construction of a wheelchair ramp on the school's back porch are also presented to strengthen students' understanding. The safe slope for a wheelchair ramp is a maximum of 0.15. Students are tasked with investigating whether the slope of the ramp made with the following design meets the safety requirements for users. What is the length of the shortest ramp that meets the requirements?

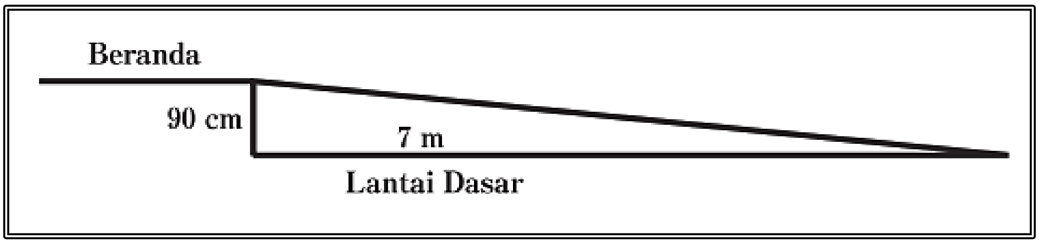
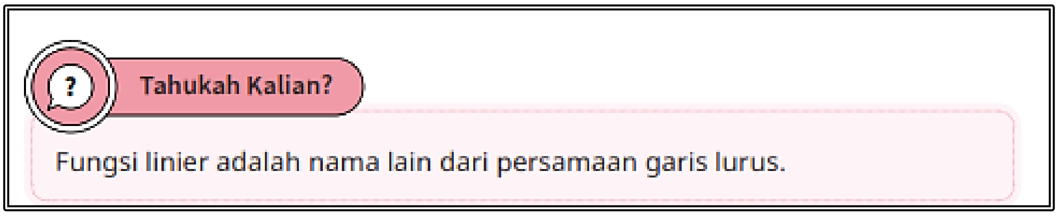


Figure 9. Wheelchair User Path Sketch

The calculation result with the formula is "$slope = \frac{90}{700} \approx 0.128$. The conclusion is that the road built has a slope of less than 0.15, so it meets the safety regulations for wheelchair users. If the length of the ground floor is changed, then the shortest length (x) is 6 m, which can be determined by the method $\frac{90}{x} = 0.15 \leftrightarrow x = \frac{90}{0.15} = 600$.

The next section of the student book, shown in Figure 9, conveys an interesting concept. In this section, the author relates the concept of a straight-line equation to the concept of a linear function. It is written that a linear function is another name for a straight-line equation.


Tahukah Kalian?

Fungsi linier adalah nama lain dari persamaan garis lurus.

Figure 10. Relationship between Linear Functions and Straight-Line Equations

Next, in example 5.4 and example 5.5. an example of determining the gradient of a line passing through two points $(x_1, y_1), (x_2, y_2)$ is described and calculated using the slope formula $m = \frac{y_1 - y_2}{x_1 - x_2}$. Example 5.7 gives the case of a line parallel to the y-axis, the calculation of which shows that the gradient is undefined. Example 5.8 explains how to determine the coordinates of another point on a line if the gradient and a point are known. By using the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$, without having to determine the equation of the line, the problem can be solved.

The equation of a straight line passing through the points $(x_1, y_1), (x_2, y_2)$ is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$. This conclusion is obtained through a series of tasks to complete the table by (i) determining the gradient m of the line passing through the points $(x_1, y_1), (x_2, y_2)$, (ii) determining the equation of the line whose gradient m passes through (x_1, y_1), (iii) determining the equation $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, and (iv) comparing the equations in steps (ii) and (iii). This formula cannot be used if the two known points are taken from a line parallel to the coordinate axis. The presentation continues by providing alternative solutions if the two points passed by the line are known, namely by (i) using the formula $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, (ii) determining m first then using the formula $y = mx + c$, or (iii) using the definition of gradient and comparing the two algebraic forms, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$.

c. Analysis of Curricular Transposition Phenomenon

The discussion of straight-line equations in SK is classified into the branch of analytical geometry. Analytical geometry is considered a combination of geometry and algebra by making correspondence between mathematical equations and locus so that a more systematic and firm geometric problem-solving method is obtained (Suarsana, 2014). In the curriculum document in Indonesia, the discussion of straight-line equations is classified as an algebraic content element. This indicates that the knowledge provided relates more to variables, constants, equations, and operations. The concepts and properties of straight lines studied are as much as possible related to algebraic forms through the correspondence of point variables with Cartesian coordinates. The definition of straight-line equations has been explicitly given in scholarly knowledge textbooks. According to Casey (1893b), "the equation of a line is such a relation between the coordinates of variable points that if fulfilled, the point must be on the line. Several concepts contained in the understanding conveyed by Casey in defining straight-line equations are (i) relations, (ii) coordinates of variable points, and (iii) straight lines. The coordinates of the variable point for a two-dimensional coordinate system are expressed as (x, y), while for a three-dimensional coordinate system they are expressed as (x, y, z). The definition of a line equation (on a plane) expressed by Casey involves the relation between x and y that is satisfied by all coordinates of points on a straight line. In mathematics textbooks, a line equation's definition has shifted.

The definition of a straight-line equation in knowledge to be taught, according to Tohir et al. (2022b), is an equation whose graph can form a straight line. If we notice a shift in the definition conveyed between Scholarly Knowledge and Knowledge to be taught, we can review the change in the basic concept used to define it. Initially, the relation between the coordinates of the variable point is used in an equation and its graph. The rule of a relation between x and y can appear as an equation; if the ordered pair between (x, y) is depicted on the coordinate plane, then a graph of the relation or equation is obtained. So far, there has been no conceptual conflict due to the shift in the definition of a straight-line equation. The narrowing of the scope of the definition of a straight-line equation from the concept of a relation to the concept of an equation is still coherent. According to SK, the form of the linear equation can be divided into two, namely (i) the form of the linear equation parallel to one of the coordinate axes and (ii) the form of the linear equation not parallel to the coordinate axes.

The general form of the linear equation parallel to the x-axis is $y = a$, and the general form of the linear equation parallel to the y-axis is $x = b$, with a and b being real constants. Meanwhile, the general form of the linear equation not parallel to one of the coordinate axes has the form of the equation $y = mx + c$, with m, c being real constants. The transition that occurs in the textbook is that students are given assignments that lead to the conclusion that the general form of the linear equation is $y = mx + c$. There is an incoherence between the SK version and the KT version. The limitations of the general form presented in the KT version fail to cover all possible linear equations. We know that $x = 1$ is a linear equation parallel to the y-axis. We cannot find the constant values m and c sufficient to obtain the equation $x = 1$. This incoherence has the potential to cause learning obstacles. Moreover, in further explanation, it is stated that the value of m indicates the gradient, and c is the ordinate of the point of intersection of the line with the y-axis. Clearly, in this case, $y = mx + c$ is not the general form of the equation of a straight line, but it is more appropriate to call it the general form of the equation of a line whose gradient is defined. For the case of a line whose gradient is not defined, this does not apply.

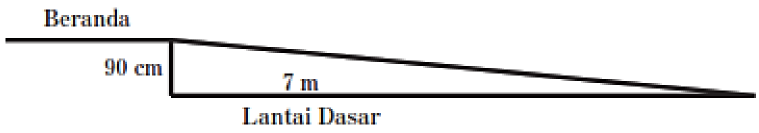
Next is knowledge about gradient. The gradient is also often referred to as the slope or direction of the line. The gradient of a straight line in the SK version is defined as the tangent of the angle formed by the straight line with the positive x-axis. The angle (α) is $0^o \leq \alpha < 180^o$ so that the gradient of the line can be 0 (if $\alpha = 0$), positive (if $0^o < \alpha < 90^o$, undefined (if $\alpha = 90^o$), and negative (if $90^o < \alpha < 180^o$). In textbooks, the gradient is defined verbally as the ratio of the vertical distance to the horizontal distance. This definition is ambiguous and incomplete. Distance is the length of the shortest line segment connecting two geometric objects. The definition given above makes it unclear which vertical distance is meant between which objects. We know that the distance value is always positive; we obtain a non-negative gradient value if we adopt this definition. This is wrong and contradicts the gradient value limits defined in SK. In addition to the verbal definition, the student's textbook also provides the following gradient formula:

$$\text{slope}=\frac{\text{change in length of the vertical side}}{\text{change in length of the horizontal side}}$$

This formula also contains ambiguity. It is unclear what the vertical and horizontal sides are, and which changes in length are also confusing. The presentation of the gradient definition in the textbook can be understood as the author's attempt to connect the concept of gradient with the context of the slope of a ladder leaning against a wall. The author wants to bring the concept of gradient out of its reality, linking it to the context of real life, but this effort instead creates chaos (Hendriyanto et al, 2023). The incoherence of the transition of the gradient concept from SK to KT and the unclear boundaries of the gradient definition presented in the textbook can be potential obstacles to learning.

The author's attempt to remove mathematics from its reality is also evident in the presentation of the contextual problem of the wheelchair ramp on the school's back porch (See Figure 11).

Sekarang pikirkan tentang masalah berikutnya. Gambar 5.9 di bawah ini mengilustrasikan teras belakang sekolah. Hal ini akan dibuat lebih sederhana bagi pengguna kursi roda dengan pembangunan jalur baru. Jika panjang jalan yang akan dibuat 7 meter dari tepi beranda, apakah memenuhi syarat keselamatan bagi pengguna kursi roda?



Gambar 5.9 serambi belakang sekolah

Berapa panjang jalan terkecil yang dapat dibuat agar dapat diakses oleh pengguna kursi roda?

Perhatikan dengan cermat pada Gambar 5.9 di atas; menunjukkan bahwa beranda naik 90 cm di atas permukaan tanah, dan jalan memanjang 7 m, atau 700 cm, dari bibir beranda. Persamaan berikut dapat digunakan untuk menghitung kemiringan jalan yang akan dibangun.

$$Kemiringan = \frac{perubahan\ panjang\ sisi\ tegak\ (tinggi\ beranda)}{perubahan\ panjang\ sisi\ mendatar\ (panjang\ jalan\ dari\ bibir\ beranda)}$$

$$= \frac{90}{700}$$

$$= \frac{9}{70} \approx 0,128$$

Dengan demikian, karena jalan yang sedang dibangun memiliki kemiringan kurang dari 0,15; maka sudah sesuai dengan peraturan keselamatan pengguna kursi roda.

Figure 11. Wheelchair Path Slope Problem and Its Solution

As seen in Figure 11, the slope is calculated by determining the ratio of 90 cm to 7 m so that $m = \frac{9}{70}$ is obtained. We know that the concept of slope in SK is the tangent of the angle formed by a line (wheelchair path) with the positive (imaginary) x-axis. If the angle of the triangle is above α (the acute angle at the base of the path), it is clear that the slope, according to the definition of scholarly knowledge, is $m = \tan(180 - \alpha) = -\tan(\alpha) = -\frac{9}{70}$. There is a difference in the slope calculation results between textbooks and analytical geometry books. When observed closely, the calculation of the gradient in contextual problems often disregards the concept of direction, implying that the slope referenced in real-life scenarios is typically the absolute value of the slope as defined in scholarly knowledge. This has always been a concern for Hendriyanto et al. (2023). The decontextualization efforts by the noosphere, which aim to build meaningfulness in learning, actually impact new chaos. This also has the potential to create learning barriers.

Incoherence was also found between SK and KT, as stated in Figure 9. The student's textbook states that "linear function is another name for a straight line equation." This statement implies that a linear function is a straight-line equation or vice versa. Both are considered the same thing. Is that true? If we look closely, it is true that the graph of a linear function is a straight line, so it can be said that the linear function equation is a straight line equation. However, as explained previously, not all straight-line equations can be expressed as a linear function $y = mx + c$, for example, the straight-line equation $x = 1$. $x = 1$ is not a linear function. This is also a counterexample that proves that the statement "linear function is another name for a straight line equation" is a false statement.

The last finding related to the incoherence of the transition from SK to KT is related to the meaning of the gradient value. The textbook states, "If a line has a negative slope, the shape of the line is always slanted to the left." This statement contains ambiguity. The textbook does not accurately define the concept of slanting to the left and right, leaving it unclear whether the slope is benchmarked from the vertical position or determined by other provisions. In the SK, it is clear that the gradient value, whether positive, negative, zero, or undefined, depends on the tangent value of the angle formed by the line to the positive x-axis. If a line has a negative slope, the angle formed by the line to the positive x-axis (α) is $90^0 < \alpha < 180^0$.

These findings challenge curriculum developers, textbook writers, and teachers when translating scientific knowledge about linear equations into teaching linear equations. The linear function approach in explaining linear equations in textbooks can be understood as an effort to link the material on linear equations with students' initial knowledge of linear functions. However, it is wrong to conclude that linear equations are linear functions. The transition of the gradient definition from tangent value to distance ratio also requires caution. The gradient is not a comparison of distances that only have magnitude but a comparison that has magnitude and direction. Indeed, in junior high school, students do not yet have the prerequisite knowledge about trigonometric ratios or vectors. An alternative approach that can be used is point translation because, in actual translation, apart from talking about changes in distance, it also considers direction.

4. CONCLUSION

Several problems can become learning barriers in transferring knowledge of linear equations from scholarly knowledge to knowledge to be taught. First, there is an incoherence in understanding the general form of linear equations. In the knowledge to be taught, the general form of linear equations is written as $y = mx + c$ to identify linear equations with linear functions. This is wrong because, in scholarly knowledge, the more appropriate general form of linear equations is $ax + by = c$. Second, there is ambiguity in the definition of linear gradients listed in textbooks, both verbal definitions and formulas. The gradient is defined as the distance ratio to the formula for the change in the length of the vertical side divided by the change in the length of the horizontal side. This definition is not as accurate as the definition of gradient (scholarly knowledge), which is the tangent of the angle formed by a line to the positive x-axis. The inaccurate definition of gradient in knowledge to be taught impacts the meaning of gradients, which are also ambiguous, namely that if the gradient of a line is negative, the line slows to the left. Third, there is "chaos" due to the noosphere's efforts to package knowledge from mathematical reality using contextual problems. The noosphere needs to explain assumptions or limitations so that the real-world context used fits the limitations of the mathematical knowledge being learned.

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BIOGRAPI PENULIS

	**I Made Suarsana** is an Indonesian mathematics education scientist, a doctoral student at Universitas Pendidikan Indonesia, and associate professor at Ganesha University of Education. He is an active researcher and writer with published papers in Scopus indexed journals such as the International Journal of Emerging Technologies in Learning, International Journal of Instruction, and Journal on Mathematics Education. Currently he is the Editor-in-Chief of the International Journal of Natural Science and Engineering.
	**Didi Suryadi** is an Indonesian mathematics education scientist and professor at Universitas Pendidikan Indonesia. He is an active and productive researcher with 142 documents published in Scopus indexed journals/conferences. Most contributed topic is mathematics education, i.e didactical design research, Praxeology, the anthropological theory of didactic, and functional thinking.
	**Elah Nurlaelah** is an Indonesian mathematics education scientist and associate professor at Universitas Pendidikan Indonesia. He is an active and productive researcher with 30 documents published in Scopus indexed journals/conferences. Most contributed topic is mathematics education, i.e learning outcome, word problem, dynamic geometry and computing.

Al Jupri

is an Indonesian mathematics education scientist and professor at Universitas Pendidikan Indonesia. He is an active and productive researcher with 84 documents published in Scopus indexed journals/conferences. Most contributed topic is mathematics education, i.e algebra learning, computational thinking, didactical desain, and geometri learning.

Estrella R. Pacis

is an Associate Professor V and the College Secretary of the Mariano Marcos State University College of Teacher Education in Laoag City. She specializes in Mathematics, Mathematics education, assessment and curriculum audit. She received her degrees Bachelor of Science in Education majors in Mathematics and Physics and Master of Arts in Education major in Mathematics from the Mariano Marcos State University. Meanwhile, she obtained her Doctor of Philosophy in Mathematics Education from the Philippine Normal University, Manila.